Some Aspects of the Development of the Girder Bridge

1820 - 1890

by

James Simpson Shipway

BSc, FICE, FIES, FIStructE

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Heriot-Watt University

Department of Civil and Offshore Engineering

Riccarton, Edinburgh EH14 4AS

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DECLARATION

Except where reference is made of the work of others,
this thesis is believed to be original and has not
been submitted for any other degree.

J.S. Shipway

James Simpson Shipway (Candidate)

Professor P W Jowitt (Supervisor)

20th October 2002

Date

"It was oft-times the dictum of a learned judge, that a re-statement of the obvious was frequently to be preferred to the elucidation of the obscure."

Anon

Acknowledgements

I wish to acknowledge with much gratitude the warm, friendly encouragement given over a long period of time by my Supervisor, Professor Paul W Jowitt, and for the many valuable suggestions made during his assessment of the work.

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My friend Professor Roland Paxton was endlessly supportive.

Most of all I am grateful to God, without Whom this thesis would never have been started, let alone completed.

J S Shipway

Preface

It might at first be thought that the development of the girder bridge is a well-worn path which has had many expositors in the history of bridge engineering, and there is little new to say in another study.

However, this thesis seeks to get away from a catalogue of more facts and dates, and attempts to give an informed analysis and opinion on the various types of girder bridge as they emerged, based of course on known facts.

Secondly, students of bridge history will know how rarely, if ever, accounts of early bridges give any calculations for forces or stresses in the members. Thus the Gaunless bridge of 1823 is often described and admired, but to the writer's knowledge no calculation has ever been published to evaluate its safety or otherwise. Similarly in the Chester Dee bridge disaster of 1847, in the painstaking inquest as to the cause of the failure, the assessors made no calculation (or none was published) and as a result came to a wrong conclusion as to the sufficiency of the structure if it had behaved as planned.

Often in engineering innovation in structural design, an idea looks good until figures are put to it. Then it is really seen for what it is worth. So in this study an attempt (sometimes somewhat approximate) has been made to evaluate forces and stresses in members, and throw some light on the structural adequacy of these early girders and bridges.

In this way it is hoped that the thesis presents some original material not hitherto explored, and of some interest to others beside historians.

J S Shipway

Abstract

The thesis explores very briefly the background history of bridge construction from Roman times, and the development of masonry arch bridges to suit the spreading network of roads, both in Britain and overseas. The need for the girder bridge made itself felt with the coming of the railways, and early examples in timber, mainly in America, soon gave way to iron.

The girder and beam were mostly used on their own as bridging solutions, but also gave an impetus to suspension bridge design in their application as stiffening trusses.

The bridge girders could not be effectively and safely used without appropriate calculation, and early efforts to solve this problem were devised by Louis Navier in France and Eaton Hodgkinson in England, on which some comment is made.

It was not long before various types of girder began to emerge, often named after their inventors, or patentees.

The thesis explores firstly the development of the girder bridge in Britain from 1820 to the 1860s, and then the corresponding development in America over the same period.

The suspension bridge, the arch bridge and the tied arch are not included, as they are not strictly speaking, girder bridges. The girder bridge is typically a structure with near parallel flanges in which the load is shared between each, and there are web members joining the flanges to resist the shear loads. There are some bridges which appear to deny classification, but a little careful assessment will usually allow a distinction to be made, as in the case of Brunel's bridge at Saltash.

The study continues up to the advent of cantilever bridges, and in particular the Forth Rail Bridge, completed in 1890. This bridge was described by its designers, not as a cantilever as it is commonly known nowadays, but as a "continuous girder bridge".

After 1890, design principles and methods of calculation were well developed. Further innovation did not take place until more recent times, and it has been chosen to end the study at that date.

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Note that the structures described were all constructed using Imperial Units. The conversion factors to metric are as follows:

1 inch	25.4 mm
1 foot	0.305m
1 ton	1.016 tonne
1 ton/ft	3.335 tonne/m
1 ton/sq.ft	109.4 kN/sq.m
1 ton sq.in	15.44 N/sq.mm
1 lb/sq.ft	0.049 kN/sq.m
1 mph	1.609 km/h

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CHAPTER 1

EARLY DAYS

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Chapter 1
Early Days

Introduction

Bridges in primitive forms have existed since earliest times. They have been constructed through need, probably beginning with tree-trunks over streams and the discovery of the arch and the use of stone, thence to the use of timber, iron and steel. Masonry bridges were durable and the stonemason could choose from material which was often readily available, though it had to be quarried, cut and dressed.

A peak of masonry bridge building occurred in Roman times. The semi-circular arch was well understood by their engineers and widely used; some have survived even to the present day. The stone was accurately cut and often used without mortar in the joints, the construction relying on the forces arising from gravity alone to hold it together. Their great aqueducts were built with admirable precision to near-level gradients. Sometimes however, the Romans had difficulty with their foundations - the pump had not been invented and construction in water required cofferdams and tedious methods to obtain dry or semi-dry conditions.

The semi-circular arch had its limitations in that any significant length of span generated a correspondingly large rise in height, which was often unacceptable, requiring steep, embanked approaches and undesirable gradients.

Other arch forms developed - segmental and semi-elliptical - which offered lower rise-span ratios but gave rise to greater thrusts, requiring greater care in the assessment of foundation requirements.

Finally, the material of a masonry arch necessarily limited the span, otherwise the bridge became a victim of its own ponderosity. Any span over 150 feet required particular skill in the design of the centering, and the foundation forces were difficult to accommodate unless the bridge was located on rock. Since rock was often a fortuitous luxury, the spans of masonry bridges were generally limited to less than 200 feet and few of this size were constructed.

The limitations of the masonry arch generated a search for other materials, and this led firstly to designs in timber and secondly the adoption of iron.

In America, timber was plentiful, and timber bridge construction developed rapidly in the early railway era. It was not so plentiful in Britain and few timber bridges were built. Brunel's Cornish railway viaducts of the 1850s belonged to a later date, and employed chiefly imported timber from the Baltic States and from America.

In Britain, a maritime nation, shipbuilding had a long history and the construction of iron ships from c 1815 onwards gave an impetus to the use of iron in bridges. This impetus was repeated 50 years later with the introduction of steel to shipbuilding c 1860, and the Forth Bridge of 1883 - 1890 was the first major bridge in the UK to use steel for its construction.

The loads imposed on bridges also had an influence on their development. For centuries the maximum loading on bridges was the horse and loaded cart, stagecoaches, occasional herds of farm animals and occasional crowds of pedestrians. There was no dynamic loading to speak of, and generally the applied load on a masonry arch was only a fraction of its self-weight - deflection problems did not exist.

With the coming of railways the picture changed dramatically. The requirement for gentle gradients required bridges matching the adjacent gradient of the line. This in its turn often led to problems of clearance and headroom below. Construction depth took on significant meaning. The loads from railway engines far exceeded the horse and cart, and the dynamic effect of the sudden arrival of a train on a bridge generated its own problems, to which hammerblow added a new dimension.

In addition to these requirements there was the further necessity for bridges of longer spans. The railways did not follow the contours of the land in the same way as a canal or road; they required a direct line. A road could end in a ferry crossing, but the railway required a bridge, particularly across the wide rivers of America and India. Often multiple spans were required, and the need for long spans to economise on foundations for the supporting piers. Sometimes the need for navigation clearance for shipping, both height and width, was the criterion for the construction of a large-span bridge, and it often happened that the best geographical location presented the worst problems for the bridge-builder, particularly in foundations.

So it was that the coming of the railways required a new attitude to bridge design; new materials and new structural forms, and a new understanding of structural behaviour. The stage was set for a transition from the shapes of the past to the forms of the future, and thus the girder bridge came gradually into being.

Bridges of Other Types Before the Girder Bridge

The advent of the girder bridge is foreshadowed by the early works of Appollodorus and Palladio. The former is said to have designed the arched, triangulated timber bridge shown on Trajan's column in Rome. It appears to have consisted of a series of significant spans between massive stone piers. (Fig 1). The Italian architect, Andrea Palladio, produced designs for similar timber triangulated trusses in 1570, but none seem to have survived. (Fig 2). It is a mystery how this type flowered briefly with Palladio but then seems to have disappeared for 250 years. (Hopkins, 1970).

In the canal era in England, Thomas Telford produced designs for aqueducts having spans of cast-iron rather than the usual masonry spans with a lining of puddled clay. His Longdon aqueduct for the Shrewsbury Canal crossed the river Tern in Shropshire in 1795 on a series of shallow cast-iron strutted spans supporting a cast-iron trough which held the water of the canal. His magnificent aqueduct at Post Cysullte on the Ellesmere Canal was of similar construction, having 19 spans of 53 ft, completed in 1805. At first sight, these aqueducts appear as hybrid arch and beam structures, but closer inspection shows the absence of a true girder, the support to the cast-iron trough being provided by the arch.

Other bridge types performed well before the advent of railways, notably suspension bridges and the cast-iron arches of Telford. Telford displayed a mastery of the use of cast-iron which was unequalled at the time, and several of these bridges survive to the present day and can be seen at Craigellachie (1814), Tewkesbury (1826) and Galton (1824). That at Tewkesbury, with a span of 170 ft is particularly impressive. Telford's bridges were road bridges, designed before the coming of the railways, but there is little doubt that they could have been equally well designed for the heavier loading.

Early suspension bridges often suffered from lack of stiffness, which led to undulations of their decks in high winds. Dynamic oscillation or vibration was a new phenomenon and little understood. Some bridges were stiffened by additional cables radiating from the tops of the towers, others had cables on the underside to suitable anchorage points. Such bridges were quite unsuitable for railway traffic, and this was proved dramatically by an attempt to use a suspension bridge over the Tees in Co Durham for the Stockton and Darlington railway. The bridge, of 281 ft span, gave rise to a wave 2 ft high in the deck in front of a locomotive (Berridge, 1969). Remedial measures were ineffective and the

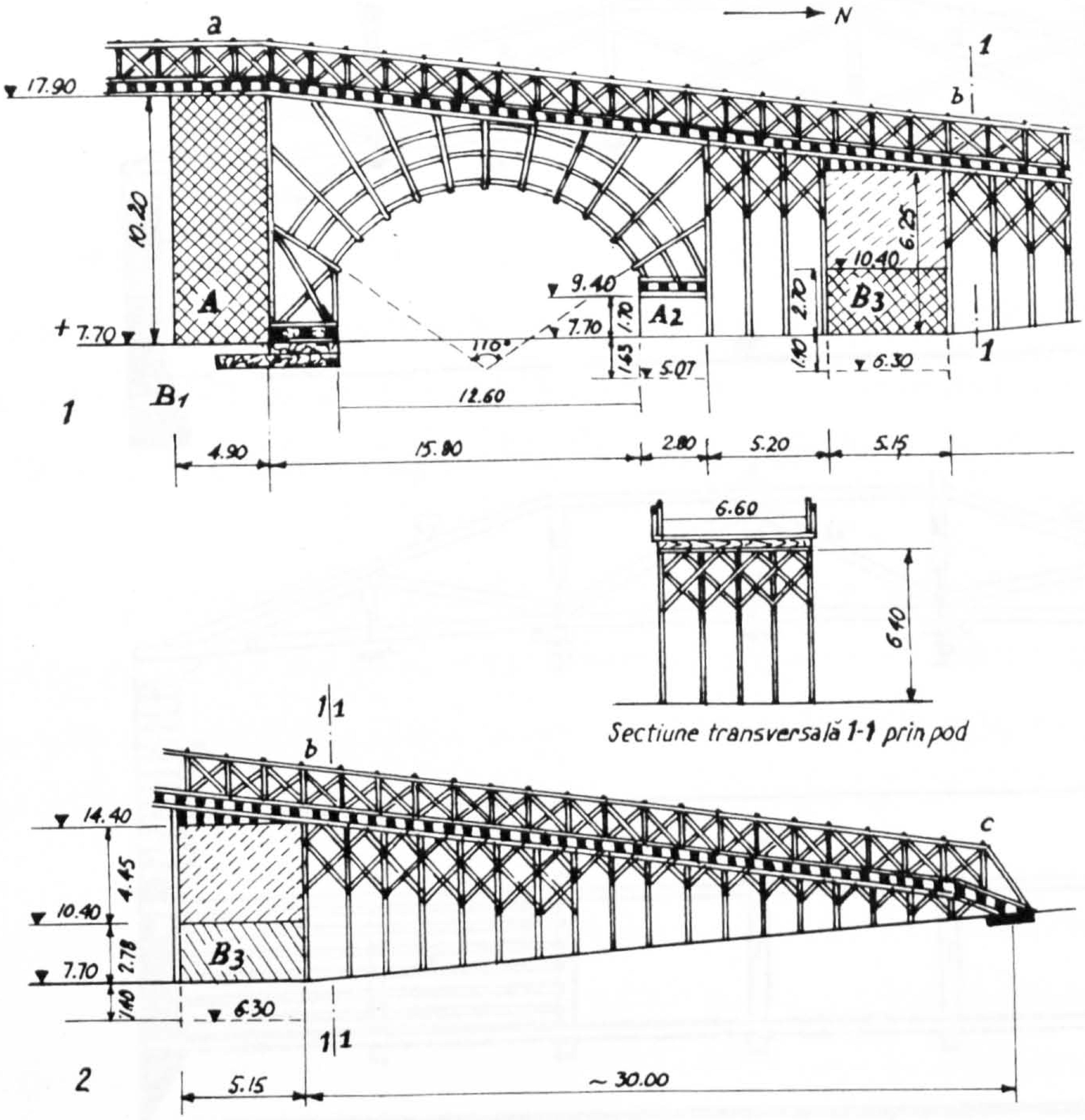

Fig 1 Apollodorus' trusses - Trajan's bridge over the Danube. (Hopkins, 1970)

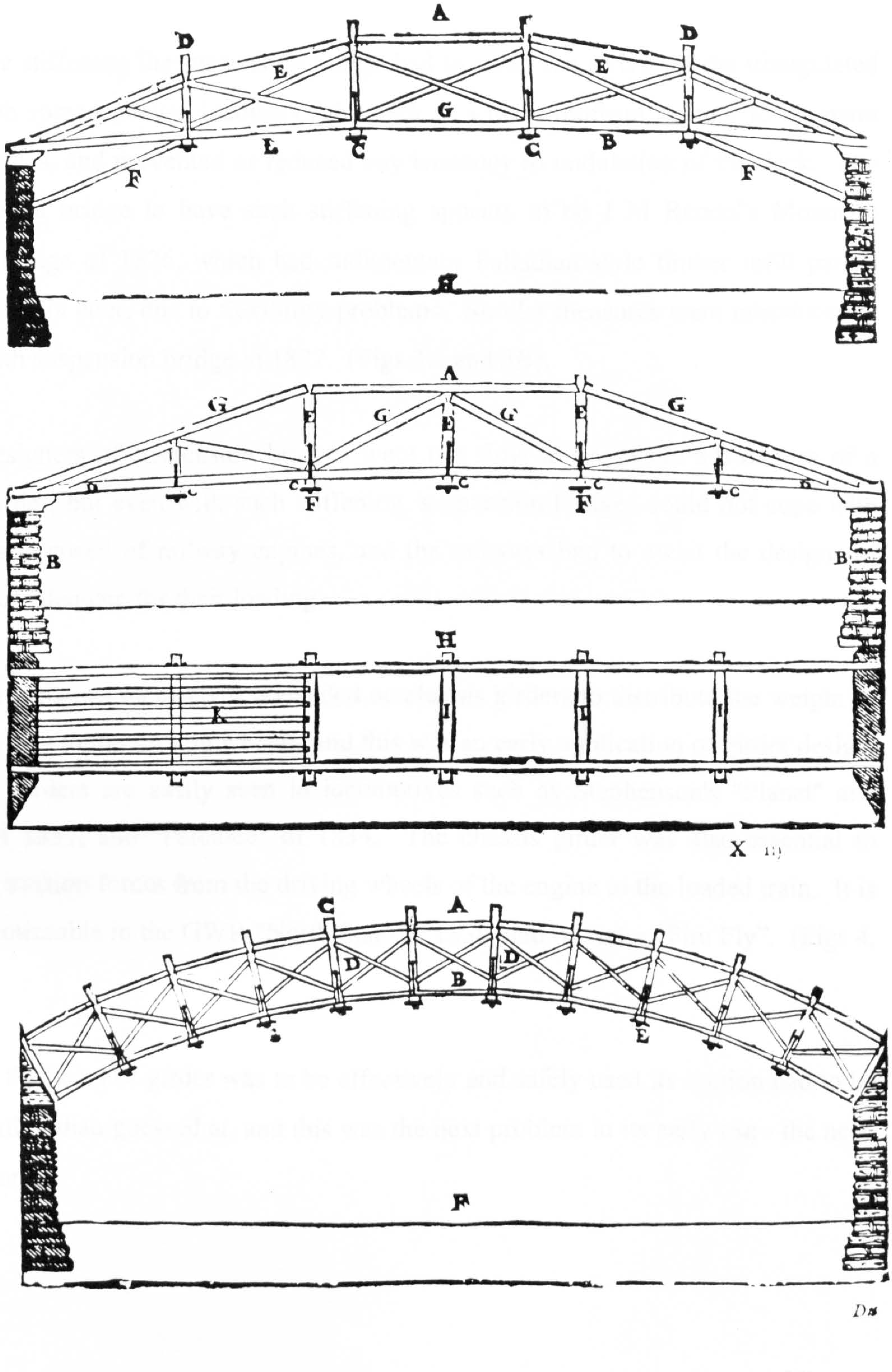

Fig 2 Typical trusses of Andrea Palladio. (Hopkins, 1970)

bridge proved to be quite inadequate for rail traffic, though it behaved satisfactorily as a road bridge.

For effective stiffening the suspension bridge had to await the arrival of the triangulated girder, which spread uneven loading of the deck in a more uniform manner to the main chains or cables, and prevented or reduced any tendency to undulation of the deck. The first suspension bridge to have such stiffening appears to be J M Rendel's Montrose suspension bridge of 1826, which had rudimentary Palladian style timber infill panels added some years later, due to flexibility problems. Similar measures were introduced at Hammersmith suspension bridge in 1827. (Figs 3A and 3B).

Thus the designers of suspension bridges were not slow to grasp the advantages of a stiffening girder, but even with such stiffening, suspension bridges could not cope with the weight and speed of railway engines, and the railways had to await the designs of girder bridges adequate for their loading.

Early railway engines themselves depended on chassis girders to distribute the weight of the boiler evenly to the driving wheels, and this was an early application of girder design. The chassis girders are easily seen in locomotives such as Stephenson's "Planet" and "Samson" of 1831, and "Patentee" of 1834. The chassis girder was also essential to transmit the traction forces from the driving wheels of the engine to the loaded train. It is even more noticeable in the GWR "North Star" of 1838, and its sister "Fire Fly". (Figs 4, 4B).

However, if the beam or girder was to be effectively and safely used its section had to be calculated rather than guessed at, and this was the next problem in its early use - the need for calculation.

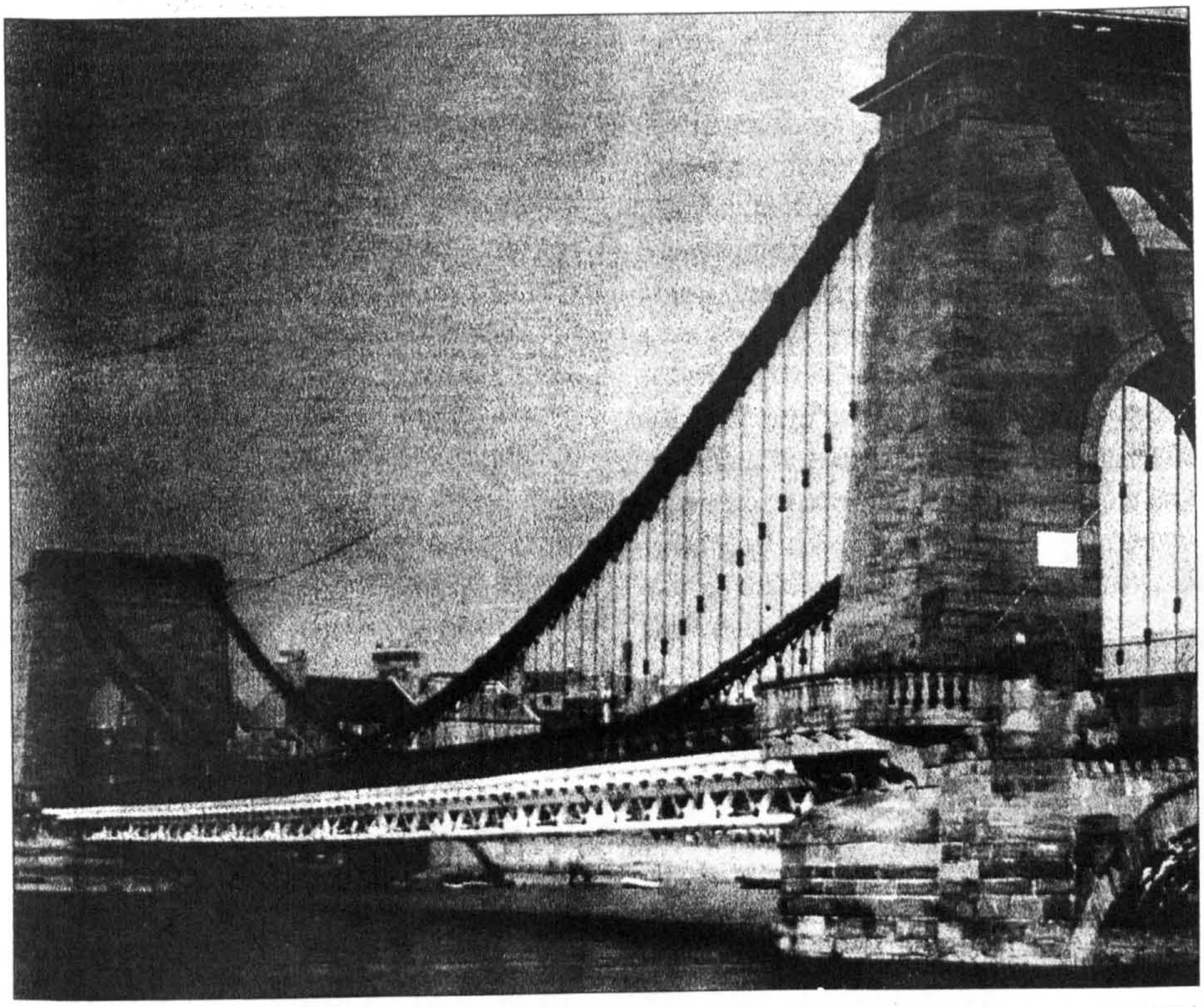

Fig 3A Montrose Bridge 1826 - 1937. Timber trussing below deck level.

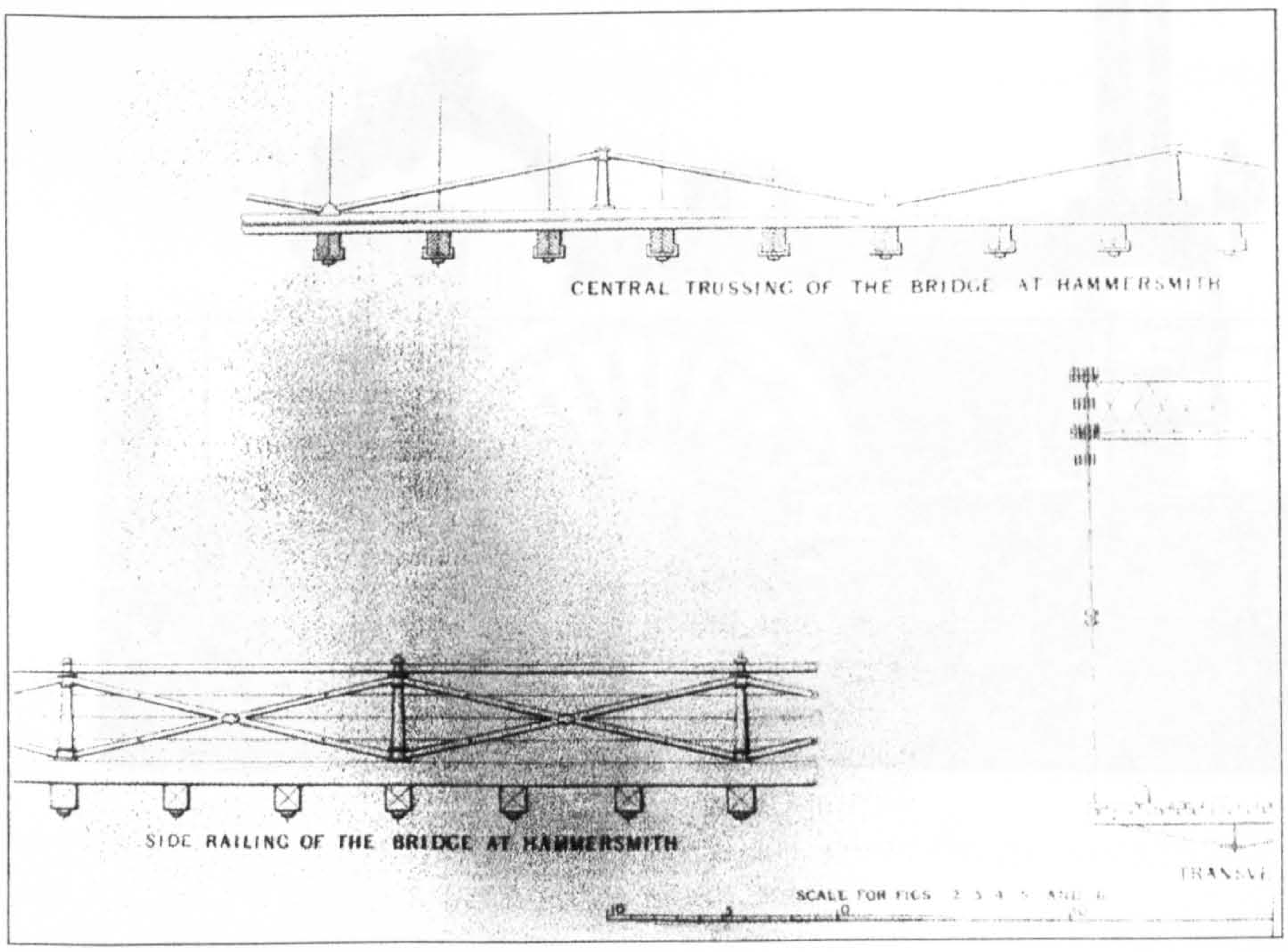

Fig 3B Hammersmith Bridge 1827. Side and central trussing.

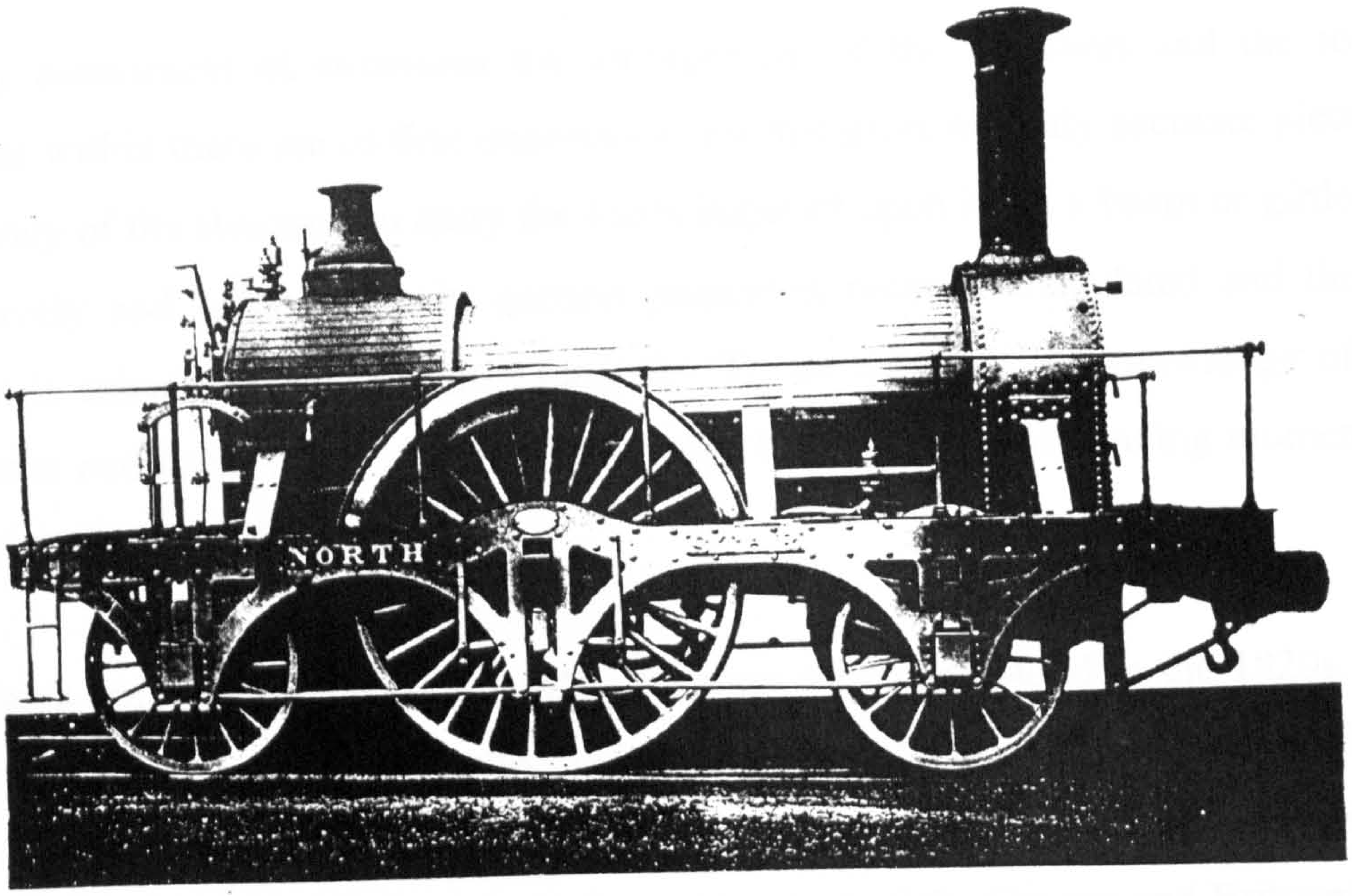

Fig 4 Girder Framing to "North Star" Locomotive.

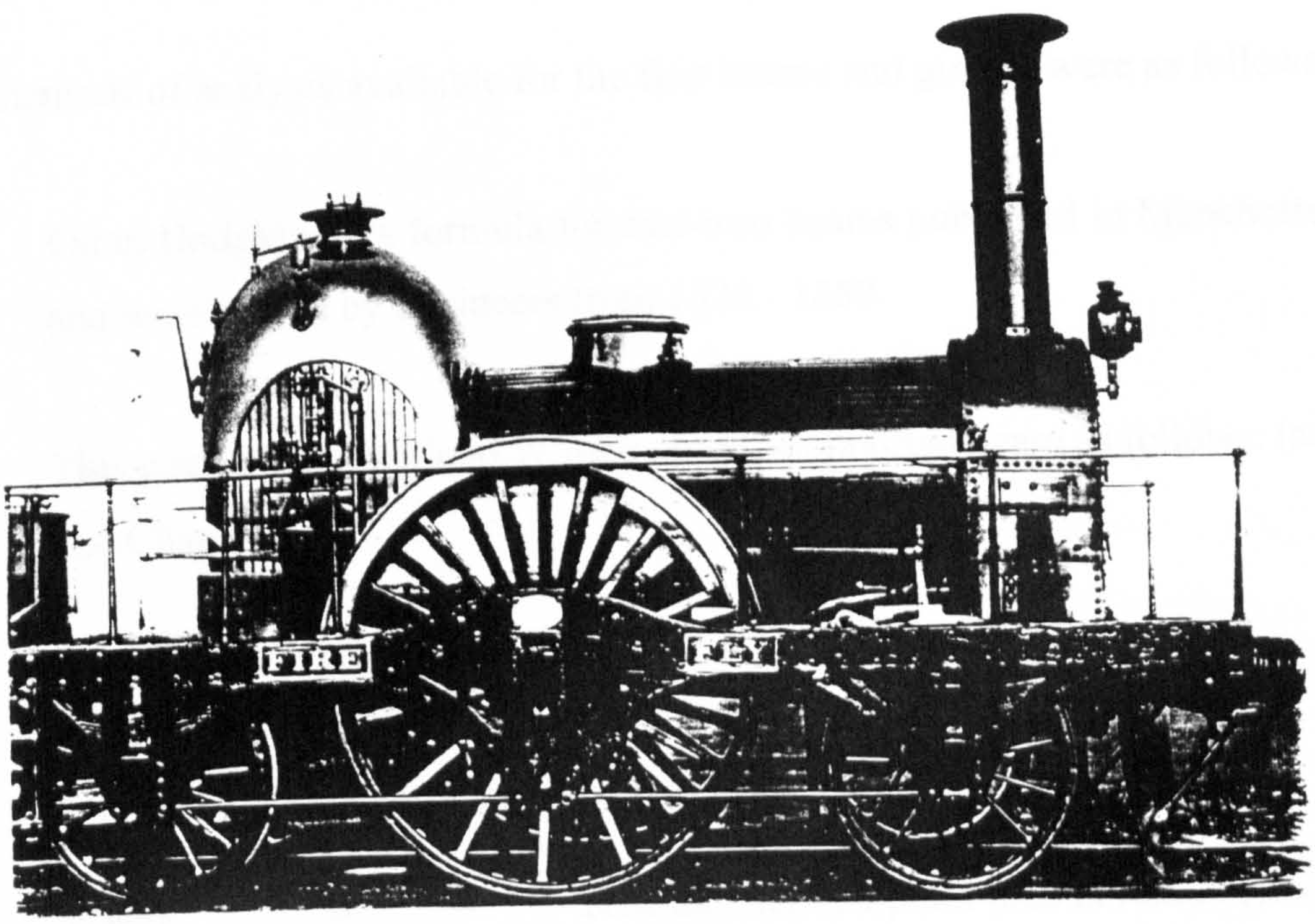

Fig 4B Similar framing to "Fire Fly" locomotive.

Methods of Calculation of Early Bridge Girders

In any assessment of structures the arrangement of the members and the forces and stresses within them are of first importance, for this gives a highly accurate picture of the adequacy of the structure to carry the loads imposed upon it. If a beam or girder is to be effectively and safely used, its section properties must be calculated and the stresses within it calculated rather than estimated. To do this requires knowledge of bending moments and shear force, and the relationship between applied bending moment and the elastic modulus of the section.

Although tensile and compressive stresses were well understood in the 1820s, and used for example in suspension bridge chain design, there seems some uncertainty as to whether the concept of bending stress was equally well understood, and perhaps was not fully appreciated by engineers until the construction of the Conwy and Britannia Tubular bridges between 1846 -50. However, for triangulated girders a rudimentary system of graphic statics could be used for statically determinate structures.

The methods of analysis available for the first beams and girders were as follows:

1. Eaton Hodgkinson's formula for cast-iron beams published in Manchester in 1822 and widely used by engineers from 1825 - 1850.

   This gave a breaking load W for a simply-supported beam as follows: (see Fig 4C and Chapter 2).

$$W = \frac{26\,A\,d}{L}$$

Where			
	W	=	centrally applied point load at failure (tons)
	A	=	area of lower flange only (sq in.)
	d	=	depth of beam overall (in.)
	L	=	span of beam (in.)
	26	=	a constant depending on the ultimate strength of cast-iron

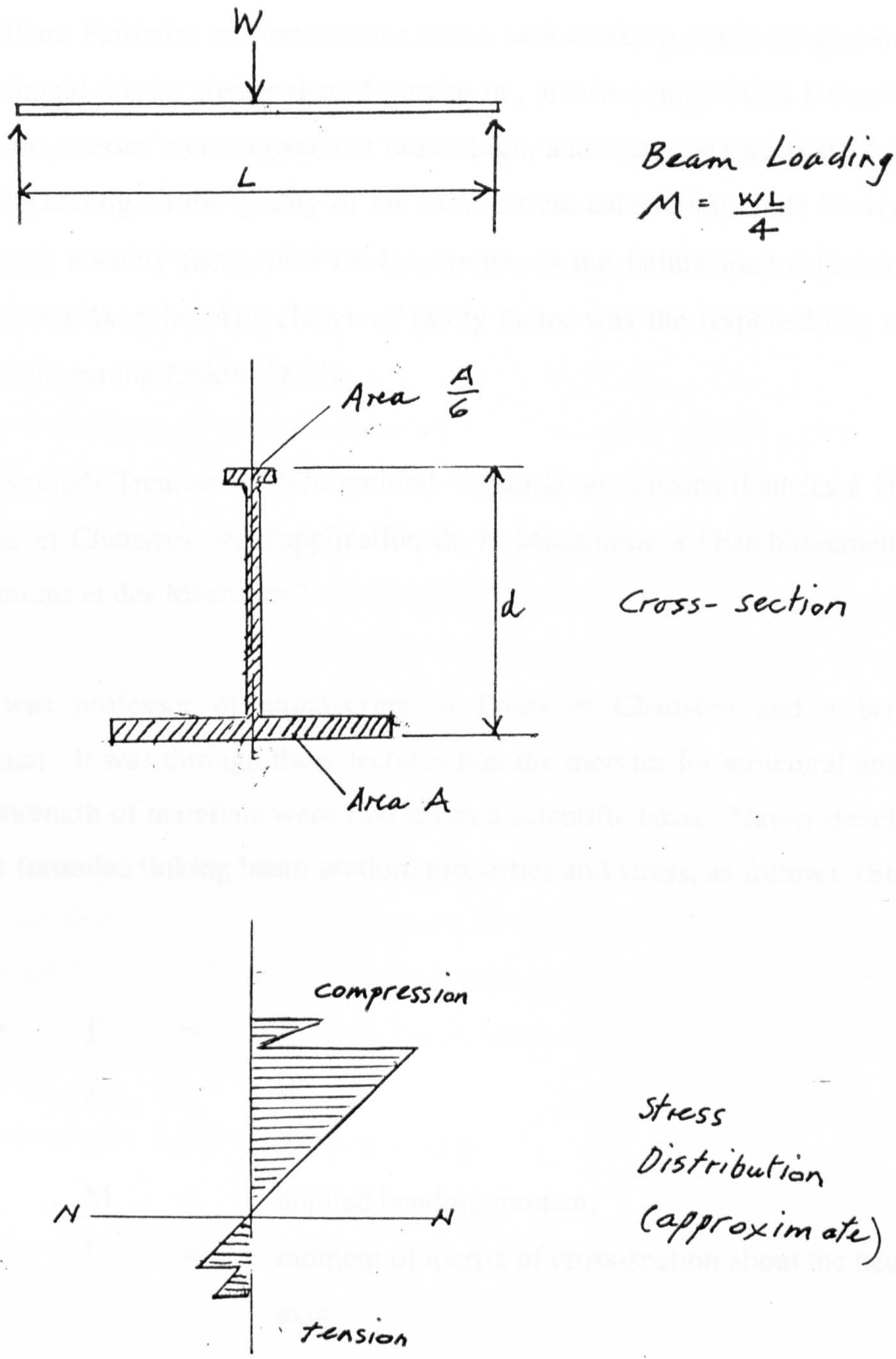

Fig 4C Hodgkinson's Formula and simple beam theory.
W is the failure load at midspan.
The tension flange was 6 x area of the compression flange.

This was applied together with the general rule that the area of the lower flange should be six times the area of the upper flange, owing to the low strength of cast-iron in tension compared to compression. Hodgkinson carried out a series of tests with William Fairbairn and established that a safe working stress for cast-iron in tension should not be greater than 2 tons/sq in., and in compression 8 tons/sq in. Later these stresses were considered rather high, and many engineers used lower values, depending on the quality of the iron. Stress calculation could be avoided by applying a safety factor of 3 or 4, or more, to the failure load calculated by Hodgkinson's formula. The choice of safety factor was the responsibility of the engineer. (Fleeming Jenkin, 1878).

2. C L M Navier's Treatise of 1826 entitled "Resumé des Leçons données à l'Ecole des Ponts et Chausées, sur l'application de la Mecanique à l'Establissement des Constructions et des Machines."

Navier was professor of engineering at Ponts et Chausées and a brilliant theoretician. It was through these lectures that the theories for structural analysis and the strength of materials were first given a scientific basis. Navier developed the basic formulae linking beam section, properties and stress, as follows: (Straub, 1952).

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

Where	M	=	applied bending moment
	I	=	moment of inertia of cross-section about the neutral axis
	f	=	stress at a given distance from the neutral axis
	y	=	distance of given fibre from the neutral axis
	E	=	Young's modulus of elasticity for the material
	R	=	Radius of curvature of bending of the beam

Navier also derived formulae for the deflection of a beam under various forms of loading.

3. Triangle and polygon of forces.

The science of graphic statics had its origins early in the 19th Century. The Frenchman Poinsot ("Eléments de statique" 1804) and the German Mobius (1827) made use of graphic methods for the solution of statical problems (Straub, 1952), though the science did not come to full fruition until Karl Culmann published his "Graphische Statik" in 1864. However, an early understanding of the triangle of forces would have allowed analysis of early framed girders such as the Pratt or N-truss, and the Warren girder.

4. Methods of analysis of continuous beams

These were not fully understood until the 1850s or later, though Navier had tackled the problem by formulating and integrating the differential equation of the elastic line in order to obtain an equation for beam deflection, from which he determined the integration constants from the support conditions.

Much thought was given by Hodgkinson to the analysis of continuous beams relating to the structure of Stephenson's Britannia bridge (1848 - 50) and by Brunel in his analysis of the approach spans for his Chepstow bridge (1852), and the minor deck spans of the main 300 ft truss of that bridge. Both reached acceptable approximate solutions by different routes. (Clark, 1850).

The Use of Calculation for Early Bridge Structures

With the developing use of cast-iron for bridge beams, Hodgkinson's formula was widely used, and his name became associated with the form of beam having a wide lower flange for tension stresses and a narrower upper flange taking compression, namely, a "Hodgkinson girder". With framed structures it is less easy to find out to what degree they were calculated, and how much depended on rule of thumb.

The Bavarian engineer, Karl Culmann (1821 - 1881) who had trained at the Karlsruhe Polytechnicum and had had practical experience on the Bavarian railways in the design and construction of important railway structures, undertook a journey to America and England in 1849 to widen his experience, and later published a study of American and

English bridges. (Culmann, 1851). Culmann had a better theoretical training than many American and British engineers, and his observations are of interest. He studied timber structures in America in some depth, and iron bridges in both countries, including the early works of Long, Howe, Pratt and others in America. In general, Culmann was impressed by American bridge construction and by the resource of their engineers, but in his opinion insufficient importance was attributed to preliminary theoretical analysis of structures. He commented on the widespread American view that when a bridge failure occurred it was not due to a lack of calculation resulting in unsound work, but due to defects arising in service. These views were expressed as late as 1851. The situation at that time in England was rather different, with massive bridge works being undertaken by Robert Stephenson at Conwy, the Menai Straits and at Newcastle, and by I K Brunel at Chepstow, and bridge engineering was no longer in its infancy.

The earliest girder bridge structures worthy of note in Britain date from the 1820s, mainly in the realm of cast-iron beams, but including the unique Gaunless bridge of 1823 in Durham. Design was of course limited by the limitations of the materials, and by the practicability of pouring the cast-iron into moulds to form members of the desired sizes, and by ignorance and lack of understanding in deriving solutions to problems.

Properties of Cast and Wrought Iron

Some of the most significant work in producing wrought iron was made by Henry Cort (1740 - 1800) who patented a manufacturing process in 1784. It was known that the strength of an iron-carbon mixture increases with the carbon content until specks of free graphite begin to form at carbon contents above about 1.7%. Commercially produced cast-iron usually had about 2 - 3% carbon and the resulting graphite inclusion (in effect holes) produced a much reduced tensile strength, although the compressive strength was not affected, and even increased. Thus cast-iron needed care if it was to be used for beams.

Wrought iron was worked or hammered to get rid of the carbon inclusions, rendering it malleable. Cort's process used a reverbatory furnace, and the iron was re-heated and melted until the carbon had burned off. It was then poured out into "puddled balls" and mechanically hammered and processed through grooved rollers to produce a refined product. Wrought iron was now able to be produced reliably in quantity and its price dropped as a result. It was dependable when subjected to tension, and this made the concept of beams and girders of equal strength in tension and compression a distinct reality. It had one disadvantage - the size of the pieces of wrought iron obtained from the mill was quite small - plates were about 8 ft x 2 ft, and therefore a girder of any size had to be made up by joining many such pieces together - either by riveting, bolting, or later, welding.

Working Stresses in Wrought and Cast-Iron

The structural strength of cast-iron varied according to the processes of manufacture, and tests by Hodgkinson showed an ultimate direct tensile stress of 7 to 10 tons/sq. in. In compression the values ranged from 35 to 50 tons/sq. in. and in bending tensile flexure approximately 15 tons/sq. in.

Applying a factor of safety of 6 to the flexure stress, (which was the custom at the time) gives a working stress of 2.5 tons/sq. in. in the tension flange. For the upper compression flange the stress could be considerably higher, but was necessarily limited by lateral stability considerations.

Young's modulus (E) of cast-iron was in the range 4000 tons/sq. in. to 6,000 tons/sq. in.

For wrought iron it was customary to use a working stress in both tension and compression of 4.0 tons/sq. in. to 5.0 tons/sq. in. Young's modulus varied from 10,000 to 12,000 tons/sq. in.

But these design values for the two irons varied in use from engineer to engineer. No limits were laid down by the Board of Trade until the 1850s, and these were no Codes of Practice to give guidance until the advent of the 20th Century.

It is interesting to note that Karl Culmann in his report on American and English bridges of 1851 (see page 13) says:

"The average loading of iron in all the bridges which I had an opportunity to see and calculate" quotes 4.73 tons/sq. in. in compression and 5.89 tons/sq. in. in tension in wrought iron.

The use of cast-iron for beams had a major disadvantage which eventually led to its demise. This was the possibility of blowholes or other defects arising from the casting process, where air could be trapped in a pocket, out of sight and invisible, and seriously weaken the beam or girder. I K Brunel refused to use cast-iron for this reason (its unreliability) in any of his railway works, but he was the only engineer to do so (Report, 1849). In general the use of cast-iron for beams enjoyed a wide popularity up until 1850. It had a further slight disadvantage in that the maximum practical length of beam which could be cast was about 40 ft, and larger spans had to be made up by joining. For railway use the maximum size of such joined beams was limited to about 90 ft span.

Early Iron Bridges in Britain: 1820 - 1830

In considering early bridges only girder or beam bridges are relevant for the purpose of this thesis. The elegant cast-iron arch bridges of Thomas Telford, c. 1800 - 1825, his Conwy and Menai Straits suspension bridges of 1825 - 1826, and Abraham Darby's arch at Iron Bridge of 1779 are not considered. Neither are similar arch and suspension bridges elsewhere in Europe or America.

The earliest girder bridge for a railway was a lenticular truss bridge over the river Gaunless in Durham. Hodgkinson had barely published his cast-iron beam formula in 1822 when the Gaunless bridge made its appearance the following year. But it was not a cast-iron beam, but a framed structure.

The Gaunless Bridge of 1823

Before studying and analysing this remarkable bridge it will be necessary to have a look at the loading for which it was designed. The earliest railways in Britain had their origins in "plateways" which were built from the purpose of transporting coal. The coal was moved in horse-drawn wagons or "chaldrons". There was a limit to the weight a horse could pull, and the chaldrons were small and limited in number in each train. (Fig 4A).

At the time of its completion in the early 1820s the Stockton and Darlington Railway was planned to move coal in this way. The earliest chaldrons varied in size and shape, but by the beginning of the 19th Century most collieries in the North had standardised on the "Newcastle chaldron" of about 1.10 tons weight carrying about 2.64 tons of coal. (Such a chaldron can be seen in the National Railway Museum at York). An average train for one horse was four chaldrons.

For the Gaunless bridge the most onerous load would be a train of chaldrons with one centred on the middle of a span. The wheels would be on the Stephenson gauge of 4 ft 8 ½ in., and spaced about 5 ft apart longitudinally. The load from each wheel would be spread slightly by the decking. Because of the slow speed of horse haulage, impact forces would be insignificant.

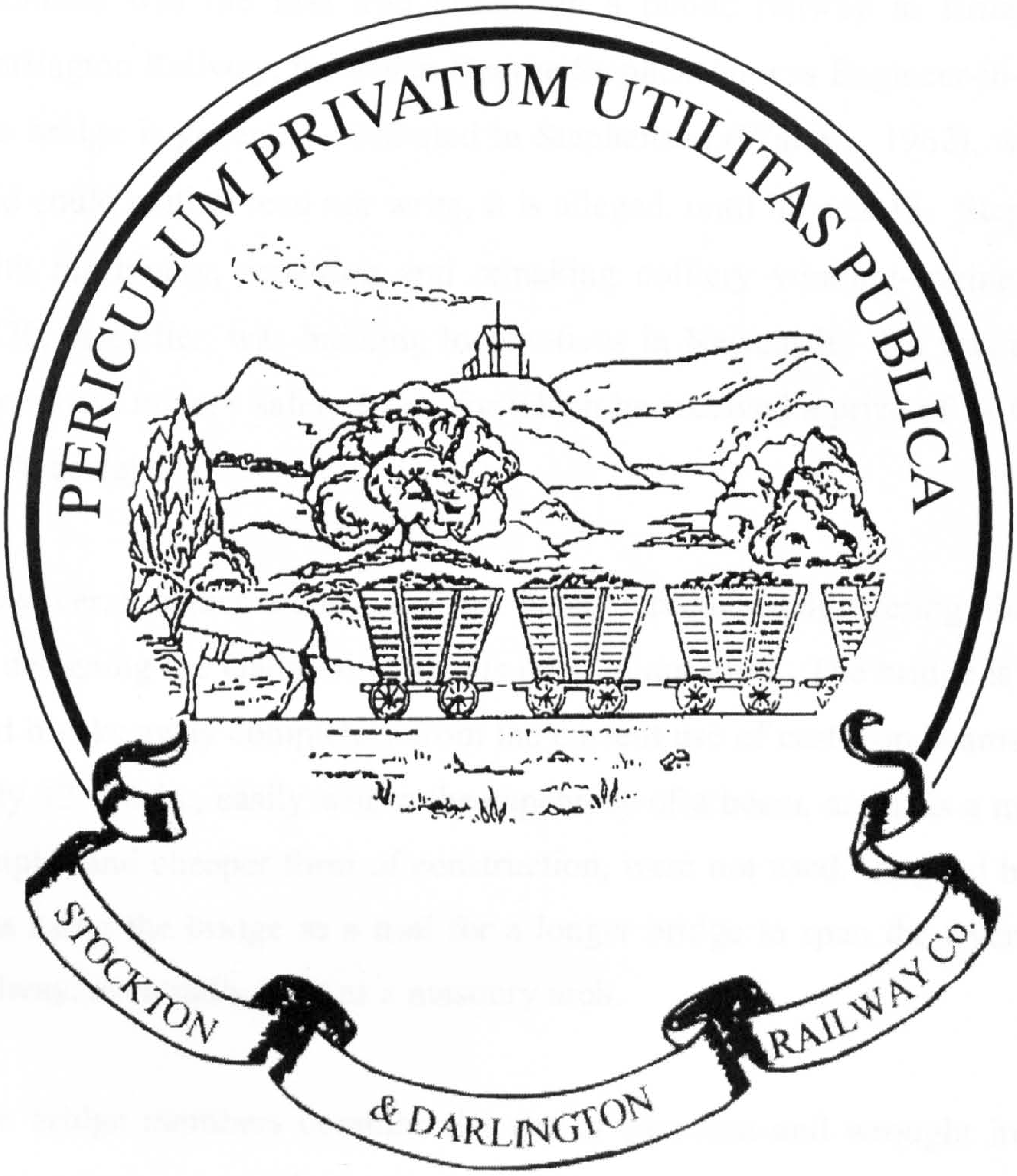

Fig 4A Drawing of seal of Stockton & Darlington Railway showing horse haulage and chaldrons (Beckett, 1984)

During its long life (it was replaced in 1901) the Gaunless bridge would be subject to greater loading, but the above was the loading condition obtaining at the time of its design.

Gaunless was the first iron bridge on a public railway in Britain, the Stockton and Darlington Railway, for which George Stephenson was Engineer-in-Chief. The design of the bridge is generally attributed to Stephenson (Walters, 1962), who was born in 1781 and could neither read nor write, it is alleged, until he was 18. Stephenson was a wizard with machinery, repairing and remaking colliery winding-engines, and eventually by 1820, or earlier, was building locomotives in Newcastle. He was also the inventor of a successful miner's safety lamp, for which he received a prize of £1,000, a very large sum in those days.

However, whether Stephenson had innate structural engineering ability and was capable of designing the Gaunless bridge is open to question. The bridge is strikingly innovative and breaks away completely from the current use of cast-iron beams. The four spans are only 12 ft 6 in., easily within the capability of a beam, and it is a mystery why beams, a simpler and cheaper form of construction, were not used. Legend has it that Stephenson was using the bridge as a trial for a longer bridge to span the river Skerne on the same railway, eventually built as a masonry arch.

The bridge members combine the use of cast-iron and wrought iron, and the designer apparently knew which members would be in tension and which in compression, and used either cast or wrought iron appropriately. The ironwork was manufactured by John and Isaac Burrell, iron-founders, Newcastle, who may have assisted with the design. The site was subject to flooding, and an additional span was added following a flood in October 1824. The iron-column piers supporting the trusses may have been chosen to avoid restriction of the waterway. The entire bridge structure is preserved in the National Railway Museum.

There are four lenticular truss spans of 12 ft 6 in. The top and bottom chords are curved members of 2 ½ in. dia solid wrought iron bars, separated by verticals of cast-iron shaped like timber stair balusters. The verticals are carried up beyond the top chords and are constantly in compression from any load on the bridge. The deck also provided lateral

Fig 5 Gaunless Bridge, West Auckland, County Durham (Walters, 1962)

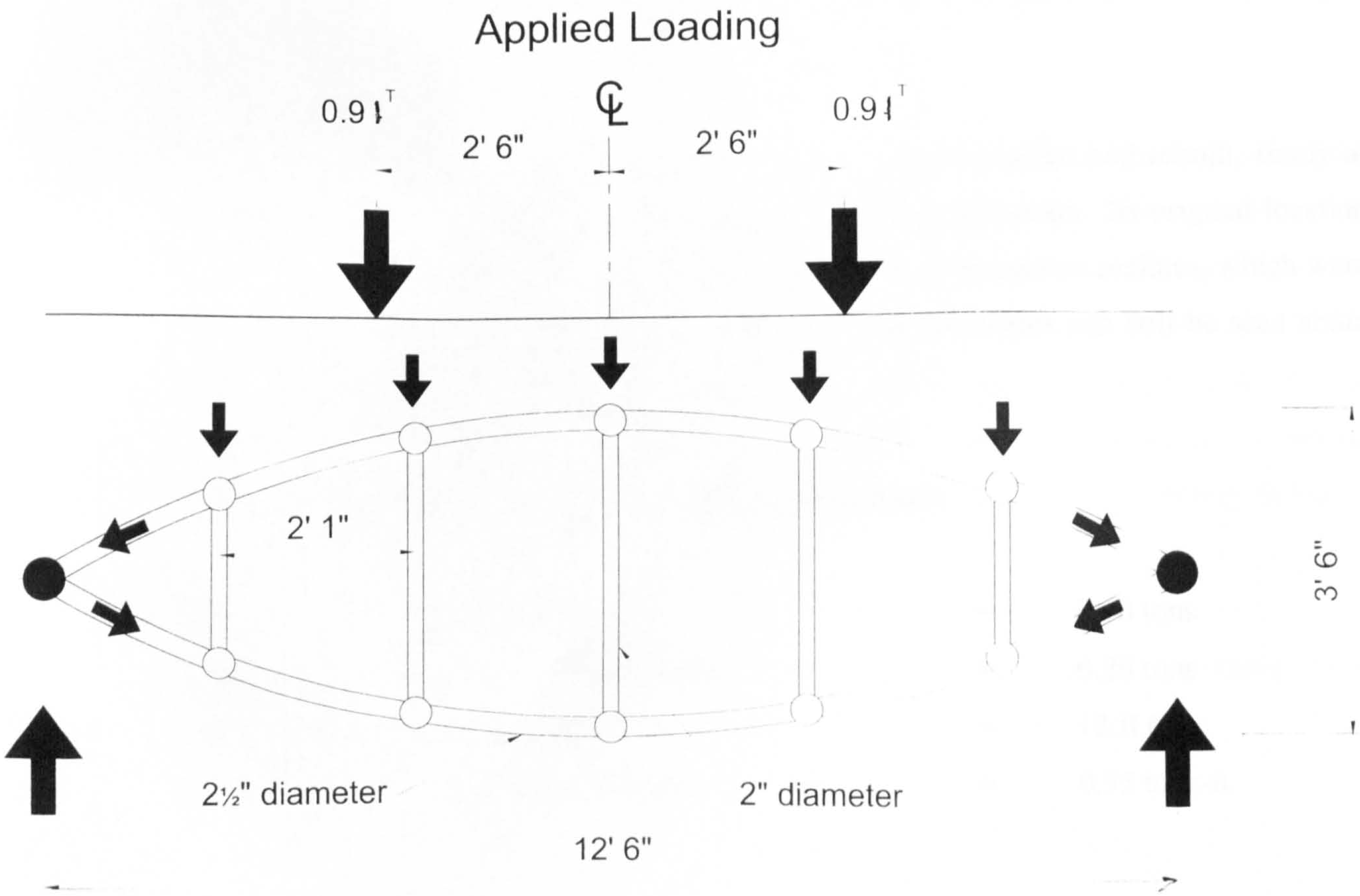

Fig 6 Gaunless Bridge. Applied loads and forces in truss

restraint to the top chord. The design showed considerable insight into the nature of forces and force action giving rise to bending, tension and compression in the members.

The two truss members are spaced 4 ft 10 in. apart, or very nearly under the 4 ft 8 ½ in. gauge of the rails. This is a convincing arrangement and avoids bending moments in the members of the timber deck. The loading from the rails is taken directly into the main girder members.

The main chords in elevation are curved to 12 ft radius, the top chord acting as an arch whose outward thrust is restricted by the inward pull of the lower chord, the two being strongly braced apart by the verticals. The rise of the arch, 1 ft 9 in., is the same as the sag of the lower chord. Difficulties of connecting the solid curved members are solved by the use of spherical bosses on the verticals through which the curved chords pass.

There are no diagonal shear members in the web to take care of distortion of the panel shapes arising from uneven loading. Any such forces arising would be small, and resisted by the Vierendeel effect of the connecting bosses. (The Vierendeel girder had no panel diagonals and relied on the stiffness of the joint connections for stability). On Brunel's lenticular truss bridge at Saltash, 460 ft span, it will be noted that such shear bracing is also very light.

The bridge remained in place until 1901, when it was dismantled and rebuilt, firstly at Darlington Station, then latterly in the National Railway Museum. Its original location was on the level stretch of line between the Etherley and Brusselton inclines, which were suited to horse haulage. The original masonry of the abutments can still be seen about 100 yards east of West Auckland Station.

Calculation relating to Gaunless Bridge (approximate):

Dead load of each lenticular truss	=	0.40 tons
Dead load of deck, rails, etc (each truss)	=	0.20 tons
Span of simply - supported truss	=	12 ft 6 in.
Dead load bending moment at midspan	=	0.95 tons-ft

Applied load from chaldron train (chaldron full - 3.64 tons)

Say 4 point loads, 0.91 tons each, 5 ft apart:

Live load bending moment at midspan	=	3.41 tons-ft
Therefore total bending moment at midspan	=	4.36 tons-ft

Since the construction is rudimentary and lack of fit is a possibility, a simple calculation is adequate in the circumstances and the bending moment is assumed to be resisted by a couple at midspan arising from the forces in the upper and lower chords.

Area of each chord, 2 ½ in. diameter	=	4.91 sq. in.
Lever arm	=	3 ft 6 in.
Therefore direct force in. each chord	=	1.25 tons
Therefore direct stress in. each chord	=	0.26tons/sq. in.

The inclination of the chords to the horizontal at the ends is about 34 degrees and this gives rise to shear forces in the chords which are very similar to the direct forces at the midspan due to bending.

There is a slight curvature in the chords between panel points, but the resulting bending effect does not contribute materially to the direct stresses.

The above assumes that the verticals distributed the wheel loads to each chord of the trusses, but this may not have been the case. Lack of fit in joints could apply the entire load to one chord, and in this case the single chord would act as an arch or a tie between the supports. Resistance to the resulting thrust or pull would have to be provided by the opposite chord, which would afford some relief. However, the factor of safety was ample, considering that the wrought iron could safely accept 5.0 tons/sq. in. in tension. The upper chord was well restrained by the deck members against buckling failure in compression.

The Gaunless bridge could be said to be well designed, and was a landmark in the development of the girder bridge. It was almost the first of all such bridges.

Design of a Typical Hodgkinson Beam - Water Street Bridge, Manchester (Figs 7, 8)

One of the earliest uses of the cast-iron beam for a railway bridge was the construction of Water Street bridge, Manchester, in 1828 - 1830 for the Liverpool and Manchester Railway. The bridge spanned Water Street adjacent to the station and warehouse at Liverpool Road, Manchester. Here again George Stephenson was Engineer-in-Chief, but the successful innovative early truss for the Gaunless bridge of 1823 was not repeated, perhaps for reasons of headroom. Instead a simpler form of cast-iron beam was used. The bridge lasted until 1905 and was evidently well designed, bearing in mind the increase in traffic and heavier loading which developed over these 75 years (Beckett, 1984).

The bridge had a span of 24 ft. 9 in. on a skew of 39°. The two main beams were 13ft. 6 in. apart, shaped in the usual inverted T form in cross-section, but not so pronounced as in later designs (see Figure 8). The beams were not parallel-sided, but had a top flange curved in elevation (hogbacked) and a lower flange which in plan was wider at the centre than at the ends, the flange edges being on a curve. The maximum area of cross-section was therefore at the centre where the bending moment was greatest, and the shape led to some reduction of material and consequent economy.

The deck was carried on transverse cast-iron beams spaced at 2 ft. 9 in. centres. The curvature in plan of the lower flange of the main girder must have necessitated a variation in seating for the cross-girders, but the rather narrower flange (narrower then the usual inverted T-section) must have reduced the torsion effect of the cross-girders. The cross-girders were also Hodgkinson-type cast-iron beams.

The spaces between the beams were bridged by brick jack-arches, making for a heavy self-weight of the bridge, but the construction of the jack-arches, abutting the main girders, would have to some extent resisted twisting of the girder, and again assisted in reducing torsion. The curved upper surface of the jack-arches would require to be infilled to provide a level bearing for the rails, and this infilling may have been gravel or ballast, which would be lighter and cheaper than concrete.

The Water Street bridge was manufactured under William Fairbairn's supervision and the design is usually attributed to him rather than George Stephenson. (Tomlinson 1914).

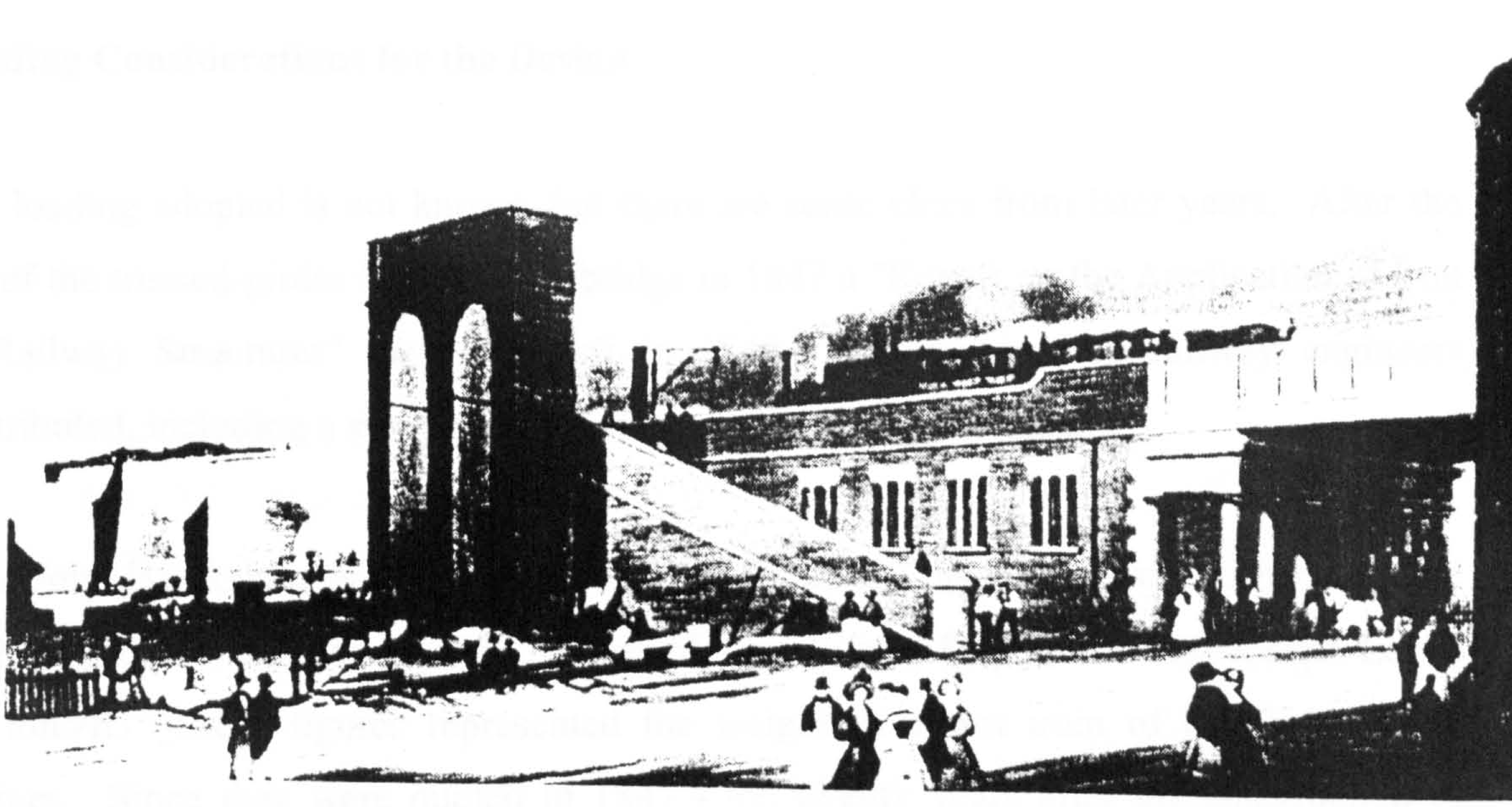

Fig 7 Water Street bridge of 1830 on extreme right (Beckett, 1984)

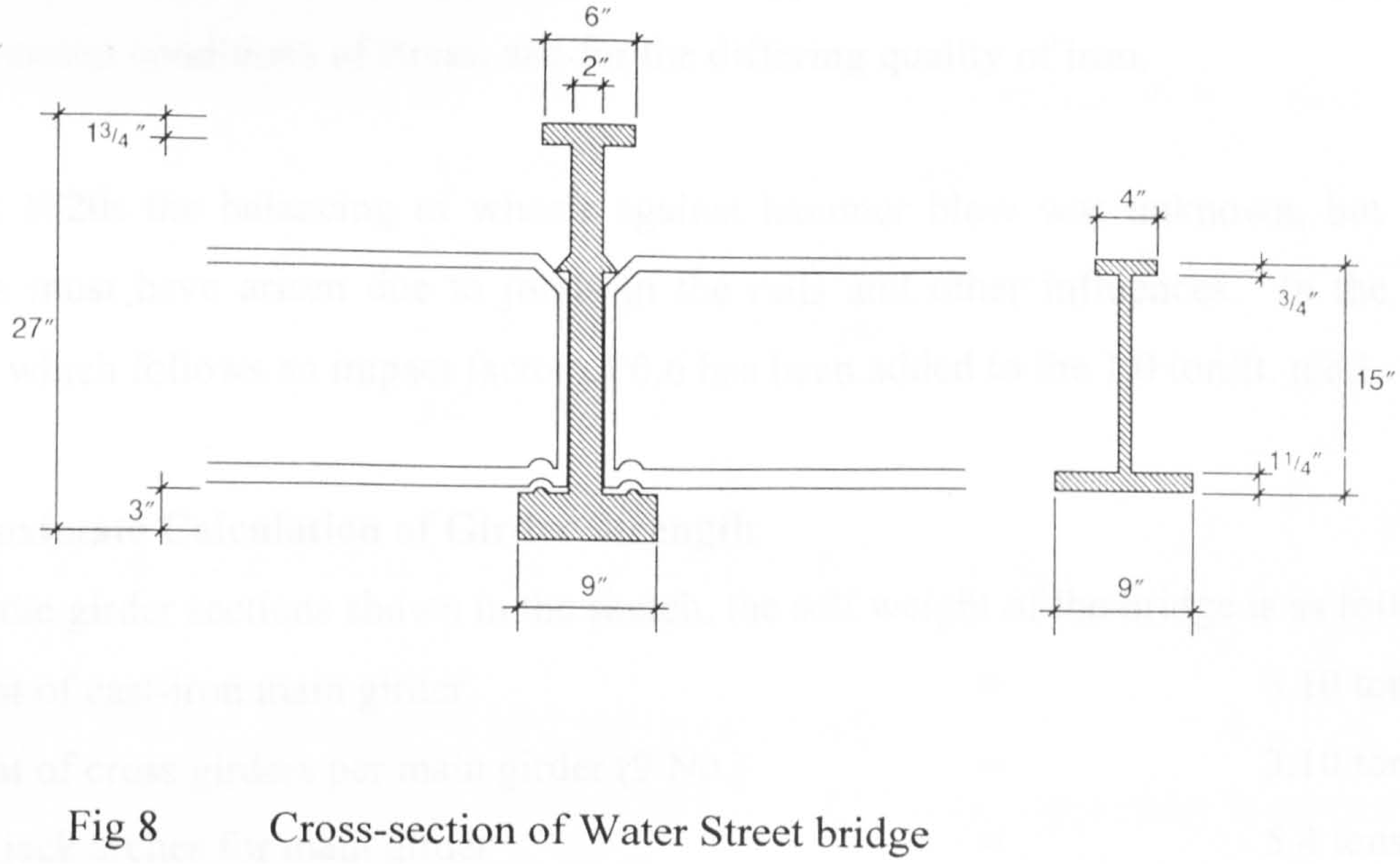

Fig 8 Cross-section of Water Street bridge

Main girder and deck girder (after D Beckett)

Loading Considerations for the Design

The loading adopted is not known, but there are some clues from later years. After the fall of the trussed-girder Chester Dee bridge in 1847 a "Report on the Application of Iron to Railway Structures" was produced in 1849. Various leading railway engineers contributed, including a statement of their opinions on railway loading.

Thus John Hawkshaw is quoted as adopting 1.75 tons/ft. as a uniformly distributed live load; Brunel and Fairbairn used 1.50 tons/ft. and Robert Stephenson and Joseph Locke 1.0 tons/ft. These figures represented the weight of a test train of coupled railway engines. Since they were quoted in 1847 - 49, twenty years after the construction of Water Street bridge, it is reasonable to assume that the primitive engines of the Liverpool and Manchester Railway weighed somewhat less, but for the purpose of a design check the figure of 1.0 tons/ft. has been adopted, bearing in mind also the long life of the bridge and its adequacy for the heavier loading of later years.

In calculating girder sections to resist the loading a factor of safety had to be chosen, but in the early days there was no common practice. Some engineers applied a factor as high as 7.0 to the breaking load derived from the Hodgkinson formula, others as low as 3.0. Generally a F.O.S. of 5.0 appears to have applied. Factors varied for tensile and compression conditions of stress, and for the differing quality of iron.

In the 1820s the balancing of wheels against hammer blow was unknown, but impact effects must have arisen due to joints in the rails and other influences. In the design check which follows an impact factor of 0.6 has been added to the 1.0 ton/ft. u.d.l.

Approximate Calculation of Girder Strength

From the girder sections shown in the sketch, the self weight of the bridge is as follows:

Weight of cast-iron main girder	=	3.10 tons
Weight of cross girders per main girder (9 No.)	=	3.10 tons
Brick jack arches for main girder	=	5.4 tons
Infill, say gravel 5" deep average	=	3.2 tons
Rails, sleepers, etc per girder say	=	1.0 tons
		15.80 tons

Therefore self weight = 15.80 tons/girder	=	0.64 tons/ft.
Live load = 0.50 x 1.6	=	0.80 tons/ft.
Total	=	1.44 tons/ft

Girder properties:

The girder is 27 in. deep, and the neutral axis is 11.50 in. above base

Moment of inertia about n.a.	=	7110 $in.^4$
Z of upper flange	=	459 $in.^3$
Z of lower flange	=	618 $in.^3$

Stresses in girder, extreme fibres:

Bending moment at midspan $= \dfrac{WL}{8} =$ 87.3 ton/ft.

Therefore tensile stress - lower flange $= \dfrac{M}{Zl} =$ 1.69 tons/sq. in.

Compression stress - upper flange $= \dfrac{M}{Zu} =$ 2.28 tons/sq. in.

This figure of 2.28 tons/sq. in. can be judged to be safe against lateral instability, as the 27 in deep main girder is restrained in position by the 15 in. deep cross-girders and by the brick jack arches. The design is generally conservative and safe.

It is of interest to check the girder using Hodgkinson's early formula: W = 26 Ad/L

Where	W	=	breaking load at midspan (tons)
	A	=	area of lower flange (sq. in.)
	d	=	overall depth of girder (in)
	L	=	girder span (in)

Here A = 27 sq. in.; d = 27 in; L = 24.9 in - 297 in; W = 63.8 tons

Applying F.O.S of 5, design W = 12.2 tons (point load)
For a u.d.l. to produce the same midspan BM, W - 24.4 tons. Therefore W as uniformly distributed load w = 24.4/24.75 = 1.0 ton/ft. run.

This figure is compatible with the railway design loading of the 1820s, and indicates that the design was safe according to Hodgkinson's method.

To sum up, provided the cast-iron was flawless and free from defects, the cast-iron beam was adequate in its use. The next step was to try and improve on the limiting beam span of around 40 ft. imposed by the technique employed in the casting of C.I. beams.

CHAPTER 2

THE TRUSSED GIRDER FIASCO

Chapter 2

The Trussed Girder Fiasco

Note on Hodgkinson's Formula

Although Hodgkinson's formula is widely quoted in literature of the period, nowhere does the derivation of the formula appear. It may be that its derivation was well understood and required no explanation, but on the other hand it may have been blindly applied, since his standing as a theoretician was high.

The basis of the formula is that the bending moment at the midspan of a beam with a point load is $\frac{WL}{4}$, and this BM is resisted by a couple at the midspan. W = breaking load, L = length of span.

The couple is formed by the tension force in the lower flange acting in opposition to the compression force in the upper flange. These two forces are separated by a lever arm which should be the distance between the centroids of the forces in the two flanges. However, Hodgkinson takes as his lever arm (d) the overall depth of the beam and ignores any contribution to the strength by the web of the beam. In this way, the resistance to bending is overestimated due to the increase in lever arm, but diminished by neglecting the contribution of the web.

Hodgkinson had finally to decide a breaking stress for the cast-iron in tension, which he seems to have chosen as 6.5 tons/sq. in., which compares with the general accepted range of 7.0 to 10.0 tons/sq. in.

Thus we have: F= force from couple = 6.5 Ad, where A= lower flange area.

$$\frac{WL}{4} = F \times d = 6.5 \times A \times d$$

$$\text{Therefore} \quad W = \frac{4 \times 6.5 \times A \times d}{L} = \frac{26\,Ad}{L}$$

For a u.d.l. it was only necessary to substitute $\frac{WL}{8}$ for $\frac{WL}{4}$. The value of the "safe" load for the Water Street bridge was a u.d.l. of 1.0 ton/ft run from Hodgkinson, but in the parallel calculation using M - Zf, an additional factor of 0.6 had been added to the 1.0 ton/ft u.d.l. to allow for impact. This additional allowance for impact could be said to be covered by the blanket F.O.S. of 5.0 which was applied to the breaking load of W = 63.8 tons obtained in the basic Hodgkinson calculation.

The Water Street bridge was unusual in that it had a web 2" thick joining the flanges, and its area was 44.5 sq. in. The area of the lower flange was only 27 sq. in. comparison. The difference in lever arm between overall depth and the depth between the centroids of the flanges was 27 in. to 24.5 in. Neglecting the web area therefore lent an additional safety factor much greater than the assumption of overall depth for the lever arm in place of the distance between the centroids of the flanges. In this the strength of the beam was underestimated using Hodgkinson's method.

The Bolted Two-Piece Cast-Iron Girder

In seeking to increase the span of cast-iron beams, the obvious solution was to bolt sections of beams together to form longer lengths. (Sutherland, 1964).

Vertical flanges were cast onto the ends and bolted through with wrought iron bolts. Usually two sections of beams were used, with the joint occurring at midspan where the bending moment was greatest. This left something to be desired, since the stresses in the flanges at the vertical joint were significant and not easily calculated using the rudimentary methods of the time. Many years later, at Inverythan in Aberdeenshire in 1882 on the Great North of Scotland Railway, a bolted cast-iron beam bridge failed at the centre joint, resulting in several deaths and injuries. (Berridge, 1969).

It was perhaps fortunate that because many engineers had little trust in calculations, they opted to load test cast-iron beams before they left the manufacturer's yard. Thus the failure load would be calculated, and the engineer would specify a test load perhaps 50% or more above this figure. In this way it was hoped that any blow-holes or defects in the casting would be revealed and the load-carrying capacity of the beam would be determined with some accuracy. This system must have worked well, since there were relatively few failures of cast-iron beams compared to the numbers in use.

Similarly, bolted cast-iron beams were load-tested and the strength and capability of the joint proved. But this solution to the problem of increasing the span of the beam was not entirely convincing. The increase in span was accompanied by an even greater increase in bending moment (the BM increasing according to the square of the span) and consequently a practical problem arose in designing an adequate bolted joint. Sometimes additional material in the form of cast-iron "bosses" were bolted onto the top flange of the girder over the joint, and extended for some distance on either side of it, increasing the depth of the beam locally.

The difficulties of the bolted beam for significantly greater spans than 60 ft. paved the way for the introduction of the trussed girder as a seemingly better method of increasing the practical span of beams. It first made its appearance in Britain in the 1830s.

The Trussed Cast-Iron Beam or Compound Girder

The first trussed cast-iron girders were apparently designed by Charles Blacker Vignoles (1793 - 1875) to carry the North Union Railway over canals in Lancashire in 1831 (Beckett, 1984). Their spans were of the order of 45 ft. The general form of the girder is shown in the illustration. (Figs 9, 10).

Sections of cast-iron girder were bolted together, usually in groups of two, or three, and rectangular wrought iron rods or chains added, with the aim of assisting the lower flange in resisting the tension arising towards midspan. It seems that the tension in the chains could be adjusted slightly by screwed threads at the anchorages, or by other screws pressing down on the chains at the point where they changed direction, usually at the third-points of the beam.

This idea of strengthening a material weak in tension by the addition of another material strong in tension is seen clearly in modern times in the introduction of steel bars into mass concrete, and even more so in the introduction of high tensile wire or bars (which were afterwards tensioned) in the use of pre-stressed concrete. This strengthening of concrete by pre-stressing was introduced in Britain approximately one hundred years later in the 1940s, and in France rather earlier. It is a system which has worked well, and produced concrete beams and bridges of unimagined slenderness and elegance. But it required accurate calculation, and the disposition of the reinforcing tendons within the beam was crucial.

Thus the principle of strengthening the weak cast-iron by tensioned (or untensioned) wrought iron rods was sound in principle but it required care in its application if it was to be effective, and accurate calculation of the forces in the rods in order to know exactly what their contribution was to the strength of the beam. In reality, in the early 1830s engineers were unable to tackle this calculation, and the application of the so-called strengthening seemed to be little more than an inspired guess. The position of the point of anchorage of the rods at the end of the beam in relation to the neutral axis was important. If the rods were anchored too far above the neutral axis it was likely they would induce bending moments of the wrong order. If anchored too low they would be ineffective unless they were prestressed by tensioning. Tensioning the rods introduced a new situation which it seemed was understood even less than the position of the rods.

Lack of fit of the various parts introduced a further complication, and was an unknown quantity.

This somewhat haphazard scenario saw the emergence of the trussed girder. Strangely, it seemed to work, and gained popularity, though discerning engineers such as Brunel avoided it completely.

After Vignoles' early use of the girder, there was an interesting incident in one built at Tottenham across the river Lea in 1839. It had a span of 60 ft. and was in two halves with a bolted joint at midspan. One of the halves failed owing to a defect in the casting. But, it was reported that "the tension rods, however, supported the girder and held the parts so tightly together that the trains passed over as usual until the accident was repaired". (The mind boggles at the apparent laissez-faire attitude to possible serious disaster in those days). This bridge was on the Northern and Eastern Railway. (Sutherland, 1964).

Another girder in three parts spanned 63 ft. across the Minories in London for the Blackwall Railway, and another of 60 ft. span across the Lea at Rye House again for the Northern and Eastern. Both were approved by the Board of Trade inspectors in 1841. Several trussed girders were built for the London and Birmingham Railway (1836), the York and Newcastle Railway (1841), and the York and North Midland Railway (1844 – 45). These latter three were all railways for which Robert Stephenson was the engineer. Owing to the immense pressure he was under at the time due to the burden of railway work, it seems unlikely that he was involved in any of the detailed design or calculation, but was assisted by G.P. Bidder.

The Trent Valley Railway had 19 bridges of plain, cast-iron Hodgkinson-type beams, and six trussed girders. They had various spans of 44 ft. at the Oxford canal to three spans of 70 ft. at the river Tame. The trussed girder was also used across the Tees at Stockton in 1842 where three spans of 89 ft. replaced Captain Brown's ill-considered suspension bridge.

Abroad, the trussed girder made its debut in Italy with spans of 96 ft. across the Arno at Pisa, and across the Ombrone at Grosseto, but these bridges had horizontal tendons rather than the sloping form. The reason for this change of practice is not known. They were Stephenson bridges.

The trussed girder was also employed in the design of supporting beams for the floors of warehouses, notably in the No. 6 Boat Shed at Portsmouth Harbour, which is still in use at the present time. The building dates from 1845, and the beams are on a 40 ft. x 10 ft. grid, spanning 38 ft. 9 in. clear. The beams are 2 ft. 9 in. deep, and cast on the web of each are the words: "Load on this girder should not exceed 40 tons". The beams have inclined wrought iron ties consisting of two 4 ½ in. x 2 in. wrought iron flats underneath and clear of the lower flanges. Tapered keys are introduced, either to pre-tension the ties or to correct lack of fit; probably the latter. Elaborate pinned support points are provided at the one-third span position, apparently to minimise friction during tensioning. (Otter, 1994).

The last use of the trussed girder appears to be the two spans of 75 ft. each which were erected over the Ouse at York for the York to Scarborough railway in 1845 (Fig 9), followed by the three 98 ft. spans of the Chester and Holyhead Railway bridge over the river Dee. This bridge was opened in 1846 and functioned for seven months before its collapse in May 1847, which resulted in 5 deaths (Fig 11). The subsequent Inquiry ruled that the failure was due to the inadequacy of the design, and the trussed girder principle was suspect. Robert Stephenson narrowly escaped a manslaughter charge at the inquest. In effect this disaster spelled the end of the use of the trussed girder. No more were built and existing bridges of the type were substantially strengthened.

The strengthening took the form of the addition of a low-rise cast-iron arch on top, spanning from anchorage to anchorage, transforming the structure into a kind of hybrid tied arch. The construction of these additional arches must have brought its own problems in obtaining an accurate tight fit for the arch to function. Some spans were also shored by timber supports below. (Fig 10).

Since the collapse of the Dee bridge resulted in the abandonment of an interesting structural system, and of a landmark in the design of girder bridges, it is of interest to examine the design of this bridge in detail, and follow the course of the Inquiry.

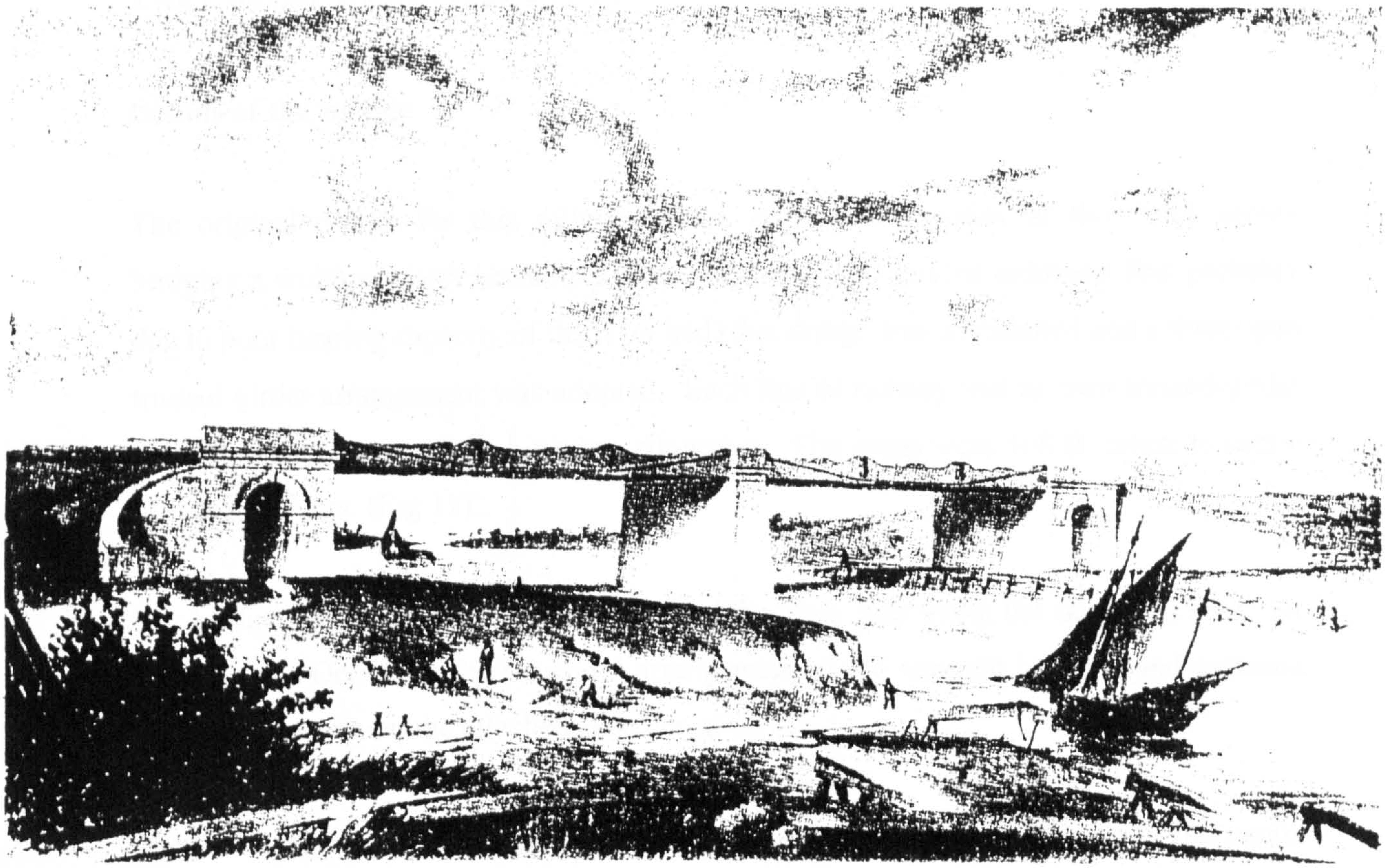

Fig 9 Trussed girder bridge over Ouse at York, 1846.

Two spans of 75 ft.

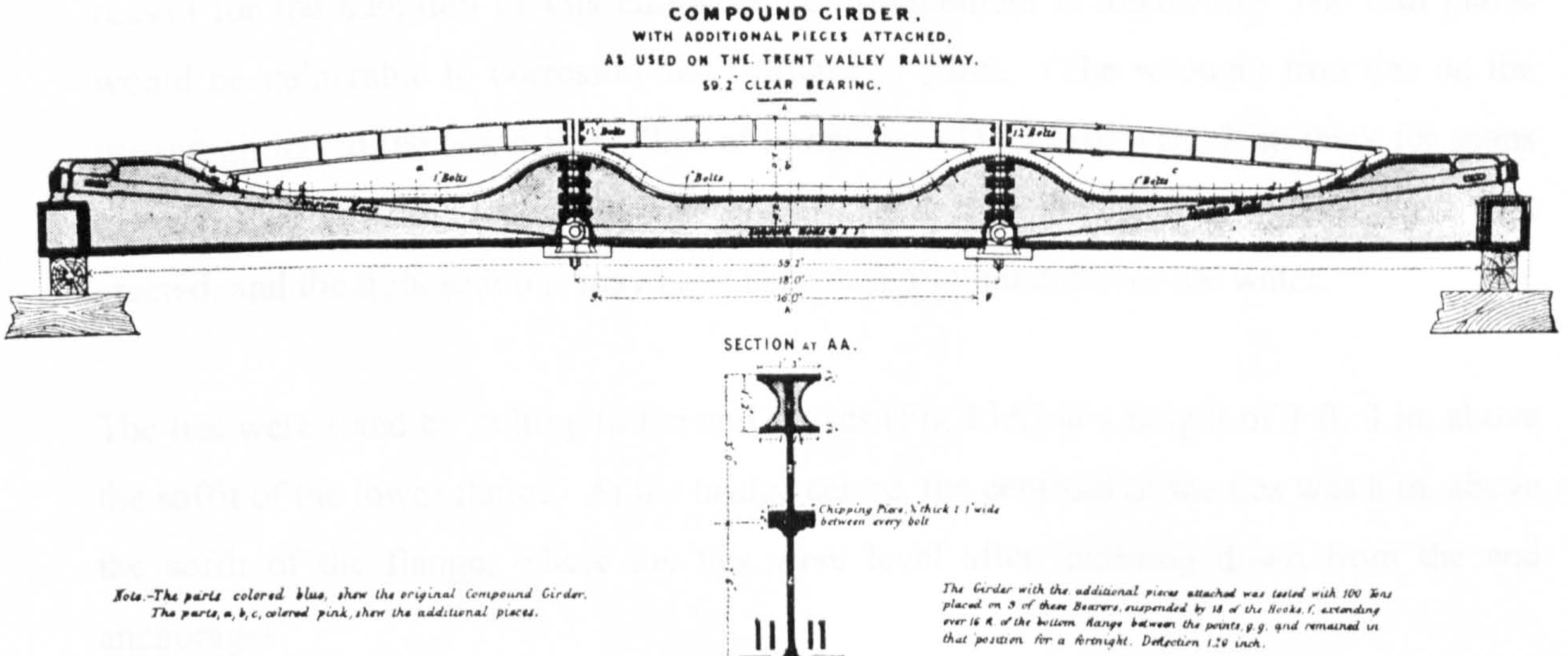

Fig 10 Elevation and cross-section of 60 ft. span trussed girder.

The top castings added after Dee bridge failure of 1847.

(Sutherland, 1964)

The Chester Dee Bridge Failure of 1847

Details of the Bridge

The original design for this railway bridge comprised a series of five brick arches bridging a width of approximately 250 ft. However, for reasons unknown (but probably due to poor bearing capacity of the river bed) this design was abandoned and a three-span trussed girder arrangement was adopted. Each line of railway had its own trussed-girder span; thus there were 12 such girders altogether. The spans were 108 ft. centre to centre of masonry piers. (Fig 11).

The bridge was on a skew of 51 degrees and the clear span along the skew was 98 ft. for each girder. Details of the structural arrangement of the wrought iron ties and cast-iron girders are shown on the attached sketches. (Fig 12).

The 98 ft. spans were composed of three girder sections bolted together. The centre section was 32 ft. 8 in. long, and the depth at the centre was 3 ft. 9 in. The depth was increased locally at the joints and at the ends by substantial cast-iron "bosses" bolted on to the top flanges and extending some way along the length of the beam.

The wrought iron ties were in the form of thin plates 6 in. deep and $^5/_{16}$ in. thick. There were 16 in each tie, 8 on each side of the girder web, giving a total area of 30 sq. in. The reason for the adoption of this cumbersome arrangement is unknown. The thin plates would be vulnerable to corrosion and difficult to paint. (The wrought iron ties on the trussed girders at the No. 6 Boat Shed at Portsmouth Dockyard were 2 in. thick for spans of 40 ft.). The Chester ties may have been added after the cast-iron girders were first erected, and the light sections may have been easier to handle over the water.

The ties were fixed by bolting to the end bosses (Fig 13A) at a height of 7 ft. 3 in. above the soffit of the lower flange. At the bridge centre, the centroid of the ties was 8 in. above the soffit of the flange, where the ties were level after inclining down from the end anchorages.

The deck loading was applied to the girder through timber cross-members bearing on the lower flanges of the cast-iron girders. (Fig 13). This was a common arrangement in bridge

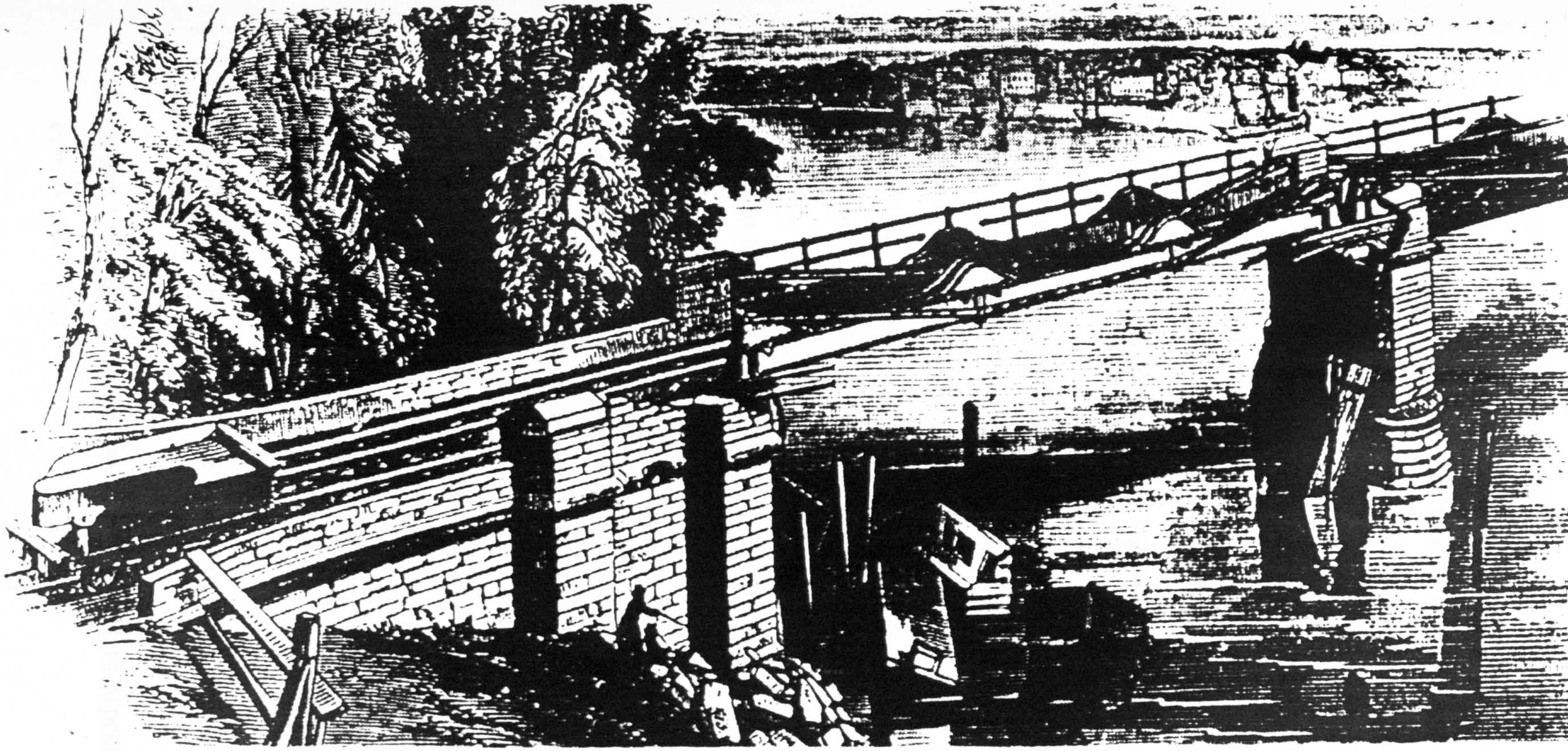

Fig 11 Failure of Chester Dee bridge, 1847. (From the Illustrated London News, May, 1847)

design of the 1830s and 1840s, and the resulting torsion stresses seem to have been regarded as of little consequence, although William Fairbairn was aware of the dangers and designed cross-girders attached to both sides of the flange by hook-bolts. Robert Stephenson himself declared, when a contributor to the 1849 Report on the Application of Iron to Railway Structures, "The torsion is not enough to be noticed". At the Dee bridge the flange bearing width available for the timber deck beams was approximately 9 in. at the end sections. At the centre section this width was much reduced owing to the presence of the wrought iron ties immediately above the flange, and the bearers had to be fitted with a special iron shoe bearing on the outer edge of the flange. (Fig 13). This detail exacerbated the torsion effect deriving from the deck loading, unfortunately at the girder mid-point where the stresses were highest and the buckling tendency was greatest.

Adjustment and pre-stressing of the ties was by four vertical eyebolts which were located at the joints of the cast-iron girder sections where the ties changed direction. (Fig 13A). Tightening of these bolts moved the ties downwards by a maximum of 2.5 in. relative to the centre section of the cast-iron girder. The forces generated in achieving this deflection depended on the values of Young's Modulus (E) obtaining for the wrought and cast-iron, since the eye-bolts not only forced the ties downwards, but their reaction also tended to force the girder upwards. In the following calculation it has been assumed that the total travel of the eyebolts was 2.5 in., and the amount of vertical movement of ties and girder respectively has been based on the relative Young's modulus for each material. The force involved in tightening the eyebolts manually (in the absence of power tools) must have taxed human strength considerably.

For the purpose of calculation (E) for wrought iron has been assumed to be 12,500 tons sq./in. and for cast-iron 6,000 tons/sq. in. These values are at the upper end of the range of values for the materials and imply good quality. This may not have been achieved in each case.

The Structural Action of the Trussed Girder

Before moving to calculation there are certain aspects of the girder arrangement to be considered.

Fig 12 Forces arising from inclined ties at Chester Dee bridge.

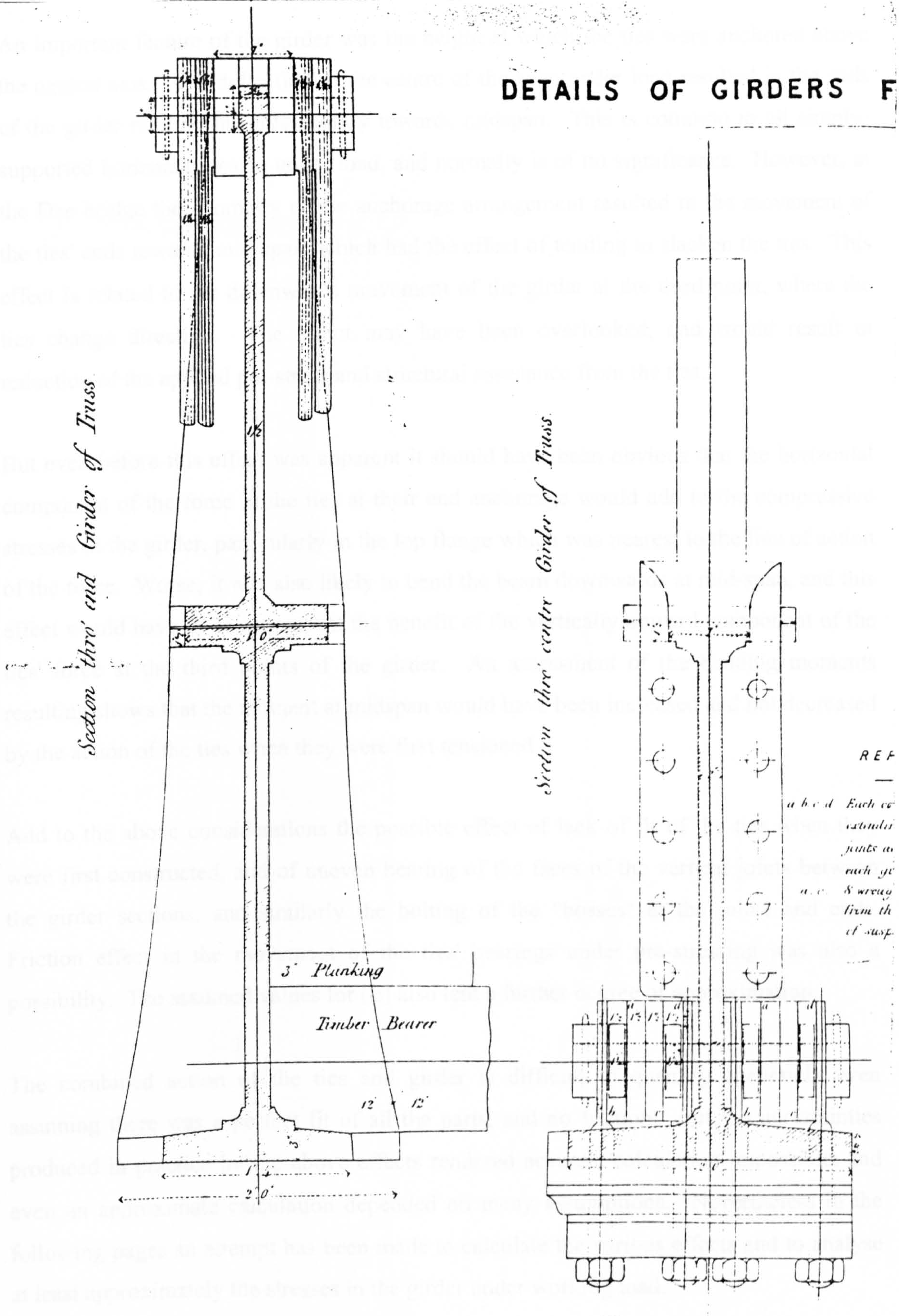

Fig 13A Chester Dee Bridge. Cross-section at centre of girder and chain ties. (From an original drawing from the archives of the Institution of Civil Engineers)

An important feature of the girder was the height at which the ties were anchored above the neutral axis. Any deflection of the centre of the span under load resulted in the ends of the girder rotating inwards slightly towards midspan. This is common to all simply-supported horizontal beams under load, and normally is of no significance. However, at the Dee bridge the geometry of the anchorage arrangement resulted in the movement of the ties' ends towards mid-span, which had the effect of tending to slacken the ties. This effect is related to the downwards movement of the girder at the third point, where the ties change direction. The effect may have been overlooked, and would result in reduction of the applied pre-stress and structural assistance from the ties.

But even before this effect was apparent it should have been obvious that the horizontal component of the force in the ties at their end anchorage would add to the compressive stresses in the girder, particularly in the top flange which was nearest to the line of action of the force. Worse, it was also likely to bend the beam downwards at mid-span, and this effect would have to be set against the benefit of the vertically upward component of the ties' force at the third points of the girder. An assessment of the bending moments resulting shows that the moment at midspan would have been increased and not decreased by the action of the ties when they were first tensioned.

Add to the above considerations the possible effect of lack of fit of the ties when they were first constructed, and of uneven bearing of the faces of the vertical joints between the girder sections, and similarly the bolting of the "bosses" at the joints and ends. Friction effect in the movement of the ties' bearings under pre-stressing was also a possibility. The assumed values for (E) also lent a further degree of approximation.

The combined action of the ties and girder is difficult to quantify accurately even assuming there was a perfect fit of all the parts, and no friction. But the uncertainties produced in practice by the above effects rendered accurate calculation impossible, and even an approximate calculation depended on many assumptions. Nevertheless in the following pages an attempt has been made to calculate the various effects and to analyse at least approximately the stresses in the girder under working load.

The Inquest Report by James Walker and Captain J L A Simmons after the disaster mentions the torsion effects deriving from the deck loading applied to the projection of the lower flange. They measured the twist of the cast-iron girder on an undamaged

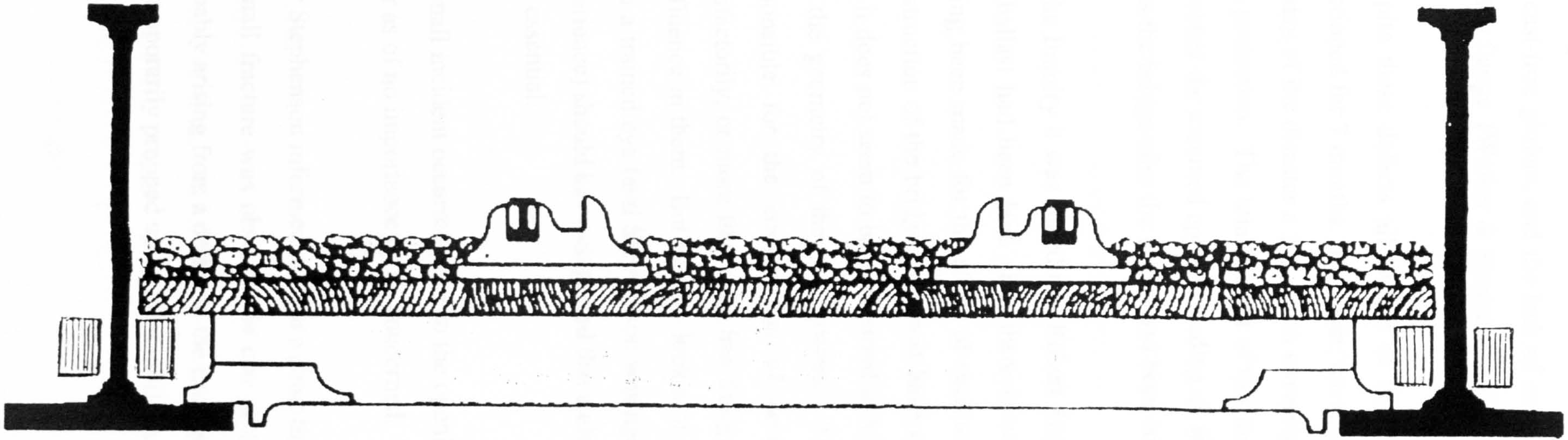

Fig 13 Eccentric bearing of deck on top of girder flange at centre of girder, Chester Dee bridge.

section of the bridge, and found it to be significant. They also observed that some of the cast-iron girders were not cast truly straight, but had deviations to the extent of 3 in. in the top flange. The torsion effect was an added disadvantage in considering the stability of the cast-iron girders, and the lack of straightness was conducive to lateral instability of the top flange. (Walker & Simmons, 1847).

Despite these defects all twelve of the trussed girders of the Dee bridge apparently functioned for 7 months. However, the timber deck was considered a fire hazard, and on the day of the disaster a 5 in. thick covering of broken stone ballast was laid on the deck as a protection. The total weight of this layer for a 98 ft. span was about 25 tons, and it increased the assumed applied loading on the 98 ft. span by some 25%. The first train to cross the bridge after the ballast had been laid caused the collapse of a span.

At the Inquiry it was stated that Robert Stephenson had been present at the bridge when the ballast had been laid. But there is no mention of any checks on the calculations having been made for the effect of such an increase in load. Also Stephenson saw the construction of the bridge, and must have seen the working drawings before it was built. Yet it does not seem to have occurred to him to question the structural action of the ties, nor the geometry of the arrangement. This may have been because he was already responsible for the construction of several such bridges which were performing satisfactorily, or more likely, he had delegated the design task to his assistants and had confidence in them. But even to look at the structural arrangement of this trussed girder with a trained eye (and Stephenson was aged 45 at the time and a civil engineer of long experience) should have suggested that a closer examination of the structural arrangement was essential.

A small incident occurred prior to the opening of the bridge which was seemingly passed over as of no importance by all concerned. Walker and Simmons reported:

"Mr Stephenson informed us, that a short time before the bridge was opened to the public, a small fracture was observed in one of the girder pieces near its joint, and therefore probably arising from a defect in the casting – which induced him to order that opening to be temporarily propped up by piles until a new casting was made and substituted".

It seems this failure did not significantly weaken the girder which may have been held together by the ties until it was repaired. This function of the ties, in holding together a defective cast-iron beam, seems to have occurred more than once, and was regarded as an acceptable raison d'etre of the structural system.

Approximate Calculation and Analysis of the Dee Bridge

A full calculation for the trussed structure is given in Appendix A, and principal values and steps only are given below. Units are in imperial measure, as was the original design. No allowance has been made for hammer-blow or impact effects and the design live load has been taken as Stephenson's value of 1.00 ton/ft. run. (From Walker's and Simmons' Inquiry information the adequacy of the cast-iron girder was based on the Hodgkinson formula, which derives failure loads for static point loads, without impact).

It has further been assumed that the girder was perfectly put together i.e. there was no lack of fit in any of the parts, no friction effect in the pinning of the ties, and the full pre-stress was correctly applied. It has already been explained that this may not have been so in the actual construction, and in fact was unlikely.

Dimensions of the cast-iron girder are shown in Figs 12, 13A. Section properties for the girder at midspan are as follows:

Neutral axis	15.4 in. above girder soffit
Moment of inertia (about N-N)	37,931 $in.^4$
Section modulus – top flange	1,281 $in.^3$
Section modulus – lower flange	2,463 $in.^3$
Cross-sectional area	158.5 sq. in. (at midspan)
Radius of gyration	4.52 in.
Cross-section of ties	30 sq. in.
Weight of girder	27 tons (approx)
Weight of ties	4.80 tons
Weight of ballast	12.5 tons/girder
Weight of timber deck, rails etc	6.5 tons/girder
Applied load of train	1 ton/ft. or 49 tons/girder
Imposed load on lower flange	0.69 tons/ft.
Girder self weight	0.32 tons/ft.

The trussed girder is statically indeterminate and to analyse the loads taken by the chains would have most probably been beyond the capabilities of the engineers of the time. However, the applied pre-stress by the ties can be calculated if the 2.5 in. travel of the adjustment bolts is assumed to be fully utilised. This gives: (Fig 13A)

Vertical pre-stressing force	10.4 tons (at third points)
Inclined force in ties	57.8 tons
Stress in wrought iron	1.9 tons/sq. in. tension

Consider now the bending moment at girder midspan due to pre-stressing by ties alone:

Downwards (sagging) moment = (10.4 x 16.3) + (56.9 x 7.25) = 582 tons-ft.
Upwards (hogging) moment = (10.4 x 51) = 531 tons-ft.

Therefore deficiency in hogging moment = 51 tons-ft.

This shows that the effect of the ties was to <u>add</u> to the bending moment at midspan. But their action was seriously affected, if not altogether lost, by the inward rotation at the anchorages.

Now consider the cast-iron girder acting alone:

Bending effects at midspan:

Maximum bending moment under full dead load and live load:

$M = wL^2/8 = (0.69 + 0.32) \times 98^2/8 = 1213$ tons-ft.
Stress in top flange = 11.4 tons/sq. in. (compression)
Stress in lower flange = 5.9 tons/sq. in. (tension)

This tension stress is very high for cast-iron, and near the ultimate for the material.

In considering the lateral stability of the top flange, recourse has been made to a former well-used Code of Practice, BS 449, for an approximate safe value in bending compression:

Allowable stress in bending compression	=	3.7 tons/sq. in.
Actual stress in top flange	=	11.4 tons/sq. in.

Thus the top flange was stressed far above safety limits from the dead and live loads alone, and no account has been taken of the additional stress due to the horizontal component of the ties' force, nor of the torsion effect due to the deck loads acting on the toe of the lower flange of the girder.

The above must be considered an approximate calculation and no such figures appear in Walker's and Simmons' Report. The figures they quote are based solely on Hodgkinson's formula for static failure load at midspan, but they show the need for assistance from the ties if the bridge was to survive.

The above calculation shows the inadequacy of the cast-iron girder acting alone, and also shows that the ties, far from assisting the girder in resisting bending, actually increased it; that is, if they were effective, which was doubtful owing to the slackening effect of the girder ends' rotation.

In the face of these facts, how did the Dee bridge survive for as long as 7 months before the ballast was placed? The answer seems to be that the rail loading of 1 ton/ft-run was not fully realised over the full length of a girder span, and not even locally under the engine, though hammer-blow effects were no doubt present. If the ballast was not present, and the rail loading is taken as, say, 75% of Stephenson's value, the total live and dead load of the bridge becomes 74.3 tons/girder in place of 99.8 tons. This would reduce the stresses in the flanges by 25% and the buckling tendency of the compression flange would be reduced further by the reduction of the torsion effect from the deck load on the lower flange.

In the face of this analysis it is of interest to see what conclusions Walker and Simmons came to in their Inquest Report, which is now briefly considered.

Inquest Report by James Walker, FICE and Captain J L F Simmons, RE

The Dee bridge disaster occurred on 24 May 1847, and Walker and Simmons were appointed by the Railway Commissioners on 27 May. Captain Simmons was an officer of the Railway Department of the Board of Trade, and James Walker was a past-President of the Institution of Civil Engineers. They attended the Inquest on 2 June and reported on 15 June.

They begin by describing the bridge in detail, then give the actual weight of the engine and train as 60 tons. They point out the inadequacy of the cast-iron girder acting alone, and suggest that when Robert Stephenson said the bridge was sufficiently strong, he must have calculated upon the tension chains also acting. They also point out the short-comings of Hodgkinson's formula applied to such a structure, i.e. its neglect of the contribution of the girder web.

The limitation of the ties' attachment to the girder ends is discussed, and it is obvious that Walker and Simmons were pessimistic about the value of the pre-stressing, and were aware of the ties' tendency to slacken.

They also checked torsion effects, and measured the twist of the girders, and criticised the top flanges narrow width in view of the additional compression from the ties. They noted that on other girders of the bridge which had survived intact, there was a lack of straightness in the top flange which was a maximum of 3 in. at one point. Their observations were obviously pertinent and relevant.

But, Walker and Simmons do not appear to have made any calculation of the effect of pre-stressing by the ties, and this reinforces the impression that such a calculation was beyond the engineers of the time.

The principal conclusions of the Report were as follows:

"That the bridge was of sufficient strength if the cast and wrought iron be supposed to act together, each taking its equal proportion of the strain".

“That there is great difficulty in ensuring the joint action, and that if this is part of the principle of the bridge, we do not approve it”.

“What was the immediate cause of the accident? .....the weakness of the girder”.

Walker and Simmons were unable to calculate the value of the pre-stressing, hence their assumption that the bridge, if completed as designed, was of sufficient strength, which it was not. The geometry of the chains made it impossible. But their main conclusion of the cause of the disaster was true – the girder was undoubtedly weak.

After the disaster, the Government ordered a public inquiry into the use of iron in railway structures and this was initiated on 27 August 1847 and issued on 26 July 1849. Its title was “ The Report of the Commissioners appointed to inquire into the Application of Iron to Railway Structures”.

The Report throws some interesting light on the state of the art in bridge engineering at the time, and has bearing on the development of the girder bridge. It is of interest to consider some excerpts from the Report and examine the state of knowledge in 1848-49.

Report on the Application of Iron to Railway Structures (1849)

The Introduction

This Report was the first of its kind in the world, and it gathered together the opinions of the leading British engineers of the day on various aspects of the use of iron, from the constituents of the material, specification, opinions on design methods and general experience. The only other place where such opinions were hitherto voiced and recorded was in the Minutes of Proceedings of learned societies - engineering societies chiefly, such as the Institution of Civil Engineers, founded in 1818. In the years to come the Report became a useful reference as a guide to good practice.

The authors of the Report were:
John, Lord Wrottesley (Chairman)
Reverend Robert Willis, FRS, Jacksonian Professor, University of Cambridge
Captain Henry James, RE, FRS, Royal Engineers
George Rennie, Esq.
William Cubitt, Esq.
Eaton Hodgkinson, FRS

Moving Loads

The Report began by outlining the problems in assessing the effect of moving loads rather than static ones:

"From the information supplied to us, it appears that the proportions and forms at present employed for iron structures, have been generally derived from numerous and careful experiments, made by subjecting bars of wrought or cast-iron of different forms to the action of weights, and thence determining by theory and calculation such principles and rules as would enable these results to be extended and applied to such larger structures and loads as are required in practice. But the experiments were made by dead pressure, and only apply therefore to the action of weights at rest." (Fig 14).

REPORT

OF

THE COMMISSIONERS APPOINTED TO INQUIRE INTO THE APPLICATION OF IRON TO RAILWAY STRUCTURES.

To the Queen's Most Excellent Majesty.

We, the Commissioners appointed by Your Majesty's Commission, bearing date the 27th day of August, in the 11th year of Your Majesty's reign, to inquire into the conditions to be observed by engineers in the application of iron to structures exposed to violent concussions and vibration, and to endeavour to ascertain such principles and form such rules as may enable the engineer and mechanic in their respective spheres to apply the metal with confidence, and also to illustrate by theory and experiment the action which takes place under varying circumstances in iron railway bridges which have been constructed, beg dutifully to submit that we have called before us such persons as we judged most competent by reason of their situation, knowledge, and experience, to afford us correct information on the subject of this inquiry. We have also obtained from such persons detailed answers in writing to many questions on subjects connected with their peculiar knowledge and experience as to the use and properties of iron. We have instituted and carried on many experiments upon this metal, and have personally examined the construction and action of several existing railway bridges.

We humbly beg to lay before Your Majesty the following Report:—

From the information supplied to us, it appears that the proportions and forms at present employed for iron structures, have been generally derived from numerous and careful experiments, made by subjecting bars of wrought or cast iron of different forms to the action of weights, and thence determining by theory and calculation such principles and rules as would enable these results to be extended and applied to such larger structures and loads as are required in practice. But the experiments were made by dead pressure, and only apply therefore to the action of weights at rest:—On the contrary, from the nature of the railway system the structures employed therein are necessarily exposed to concussions, vibrations, torsions, and momentary pressures of enormous magnitude, produced by the rapid and repeated passage of heavy trains.

These disturbing causes, in a smaller degree, have always occurred in structures connected with mill-work or other mechanism. But the effects upon their stability have not been found greater than could be met by increasing the dimensions of the parts without especially inquiring into the exact principles upon which such increase should be made. Thus, we are informed that the dimensions of cast-iron girders, intended for sustaining stationary loads, such as water-tanks and floors, are usually so proportioned that their breaking-weight shall be three times as great as the load they are expected to carry, or in some cases four or five times as great. But when the girders are intended for railway-bridges, and therefore subject to much concussion and vibration, greater strength is given to them by altering the above proportions, and making the breaking-weight from six to ten times as great as the load, according to the practice of different engineers. On the other hand, some consider that one-third of the breaking-weight is as safe a load in the latter case as in the former.

Appendix No. [illegible] Minutes of Evidence [illegible] 119, 169, 234, [illegible] 360, 431, 535, 70[illegible] 831, 952, 1154, 1362, 1393.

Fig 14 Front page of Report on the Application of Iron to Railway Structures.

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Safety Factors for Bridge Design

The Report quoted the opinion of several eminent engineers on the safety factor they adopted in their bridge designs as follows - the figures represent ratio of breaking weight to greatest applied load adopted in the design. The "Breaking Weight" was the u.d.l. from the Hodgkinson formula.

Static Loads -	Mr C Fox, Mr T Cubitt, three Mr P W Barlow, four Mr J Glynn, five
Railway Bridges -	Mr Brunel, three or 2 ½, "but he considers the rule he adopts for the calculating of the dimensions of his girders gives more than the usual strength". Mr H Grissell, Mr C May, three Mr Rastrick, Mr P W Barlow, Mr R Stephenson and Mr Joseph Cubitt, six Mr John Hawkshaw, seven Mr J Glynn, ten

The names of leading engineers quoted in the Report are mostly unknown to us today after more than 150 years. A few are well remembered by historians and others, such as I K Brunel, J Cubitt, P W Barlow, J U Rastrick, R Stephenson and J Hawkshaw. Some were the engineers of bridges which survive to this day e.g. Brunel, Rastrick and Stephenson. The names quoted in the Report were not necessarily all the engineers consulted for their opinions, but chosen to show the range of opinion existing on major topics.

Brunel had an aversion to the use of cast-iron in tension, and rarely employed it. In his reply quoted above, he was probably thinking of wrought iron rather than cast-iron girders. His bridge girders often had a compression flange of triangular box shape, or were of curved plates less susceptible to buckling than a flat surface. Despite this aversion to the use of cast-iron, Brunel refused to condemn it. He wrote in a letter to the commissioners:

"Who will venture to say, if the direction of improvement is left free, that means may not be found of ensuring sound castings of almost any form, and of twenty or thirty tons weight and of a perfectly homogeneous mixture of the best metal? Who will say that beams of great size of such a material, either in single pieces or built, may not prove stronger, safer, less exposed to change of texture or to injury from vibration, than wrought iron, which in large masses cannot be so homogeneous as a fused mass may be made, and when welded is liable to sudden fracture at the welds?" Brunel thus expressed a far-sighted view in which he declined to limit possible future developments as yet not envisaged. He was supported in this view by others, including Robert Stephenson.

Torsion Effects on Girders

The next point considered in the Report was the question of torsion caused by loading the lower flange of the Hodgkinson or inverted T-girder. It was customary to carry the crossbeams or timber decking bearing the deck loading directly on to one side of the lower flange. Curiously, Rastrick and Locke "did not consider that the strength is diminished by the pressure being so applied" and R Stephenson " did not think the torsion is of sufficient consequence to be noticed". Fairbairn and Hawkshaw "considered it would be advantageous to alter the form of the girders to enable them to withstand the torsion". This reply went to the heart of the matter, accepted there was a problem, and proposed an adequate solution. Later Fairbairn proposed and used hook bolts, which suspended the deck load to the flange from both sides of the girder web. Other engineers expressed other opinions, but none came up with any actual figures giving the compressive stress in the top flange, and the torsion stress, due to the loading. This again seems to indicate the lack of knowledge or ability to perform such calculations.

Railway Loading for Bridge Design

The Report went on to consider the loads for which railway bridges should be designed. This aspect is so important that the observations are quoted in full:

"Mr Hawkshaw states that locomotive engines are the greatest weight which can come on railways, and reckons 1 ¾ tons/ft. as the greatest weight for a single line of way. Mr Fox, Mr Fairbairn and Mr Brunel mention 1 ½ tons. Mr W H Barlow states that on the Midland Railway there are engines on four wheels weighing 32 tons inclusive of the tender, but that the weight is too great for the permanent way, and that the rails are crushed and flattened by it (!). Mr Stephenson and Mr Locke state 1 ton/ft. is the greatest weight which comes on a single line of rail".

In commenting on the above, it is likely that Brunel would be allowing for his Broad Gauge of 7 ft., which generated locomotive and carriage weights generally heavier than the Stephenson gauge of 4 ft. 8 ½ in. Design loadings gradually polarised on 1 ton/ft., which was the loading adopted in the design of the Forth Rail Bridge, opened in 1890, some 40 years after the Report. However, the Report did not make any recommendations on loading.

Comments on Forms of Girder

The forms of bridges in use were listed, i.e. forms of girders in addition to simple cast-iron beams or girders, as follows:

(a) The Built Girder, formed of separate castings fitted closely at the joints and bolted together, and entirely dependent upon the bolts for support. One engineer (Grissell) stated he would have no hesitation in making such girders in spans up to

200 ft, but the majority were not enthusiastic upon this form, considering "that other modes of construction disposed the material more advantageously".

(b) The Arched Girder, or cast-iron used as an arch, which is not strictly a girder. All engineers concurred in approving this form.

(c) The Trussed Girder. Strangely, after the Chester Dee bridge disaster, and the Inquest Report by Walker and Simmons, this girder was not condemned outright. R Stephenson described experiments he had made on a reproduction of the Dee bridge arrangement, and admitted the "the tension rods, though they do not, when acting at the angle they did in the Dee Bridge girders, produce the full effect, yet they add considerably to the strength of the girder". Stephenson seemed to wish to justify his use of the girder, and does not seem to have asked himself, if what he claimed was true, why it fell down. He seemed unwilling to learn from a mistake.

There was discussion on the merits of the arrangement, and objectives to the use of cast-iron and wrought iron acting together because of different amounts of movement in each resulting from temperature effects. No coefficients of expansion were quoted, but Arrol's Bridge Engineer's Handbook (1920) gives the coefficient for cast-iron as 0.6220×10^{-5} and wrought iron as 0.6780×10^{-5} per ^{o}F. The difference is small, and can be compared to the difference between steel and concrete in the use of reinforced concrete.

Brunel objected to the use of cast-iron in long spans, and its combination with wrought iron, "and preferred a framing of wrought iron and timber". He was apparently unconcerned about temperature effects.

(d) The Bowstring Girder. This can hardly be called a girder since it was a tied arch, having an arch of cast-iron and a tie of wrought iron, as in the High Level Bridge of 1849 at Newcastle. The majority of the engineers considered this form free from any objections arising from other modes of combining cast-iron with wrought iron.

(e) Box or Tubular Girders. The first tubular girder bridge at Conwy, North Wales, was in position by 1848 and the second tubular bridge, the Britannia Bridge, was completed in 1850. They were works of immense size and importance, yet the comments noted in the Report were mostly trivial. Brunel alone made two comments worth quoting: "Mr Brunel looks upon the introduction of wrought iron into the construction of girders as the most important step that has been taken for some time in engineering, and he considers that, with ordinary care, and with the improvements which have been introduced in the mode of riveting, the joints made by riveting may be as permanent and in every respect equal to the other parts of the structure, and he does not consider oxidation or vibration can affect them". Brunel evidently looked on wrought iron, with its ability to resist compression and tension in practically equal terms, as a new material. Thus in the past cast-iron had superseded timber as a new material of construction, and steel in its turn would supersede wrought iron as a new material. But wrought iron enjoyed a hey-day of nearly 40 years, and was not superseded by steel for a major bridge until the completion of the Forth Rail Bridge in 1890. So Brunel looked upon the advent of wrought iron as the most important step for some time.

The second comment by Brunel was concerned with riveting: "Mr Brunel considers that two plates could be riveted together so as to ensure their not breaking in any part contiguous to the rivets or joints, because the rivets should not act as pins or bolts, but as clamps, which by pressing the plates together, produce an enormous friction". His insight into the function of the rivet was exactly that of the high-strength friction-grip bolt, introduced in the 1950s as the chosen fastener for all types of railway bridgework. (Berridge, 1969). The mode of tightening the bolt made it possible to calculate the force by which it gripped the plates – somewhat more scientific than the cooling of a red-hot rivet.

(f) The Lattice Bridge. This was a design in wrought iron seemingly adopted from timber lattice bridges in America by Ithiel Town (1784 – 1844) and others. It was being developed in Ireland by John MacNeill (later Sir John), and had a web formed of multiple intersecting members in the form of strips of metal between normal flanges. The web members had little strength in compression because of their slenderness, though several large bridges of the type are still in existence, notably on the Highland Line between Perth and Inverness. It was however, advantageous in places where transport to a remote bridge sites necessitated small members to be carried by mules or the like. The Report noted that "Lattice bridges appear to be of doubtful merit" and did not interview any of their designers.

The Open Web Girder

The first open-web girders in Britain were at Newark Dyke in 1850 by Joseph Cubitt, and at Crumlin in 1853 by T W Kennard. Brunel had also begun foundation work on his celebrated 300 ft. span open-web Chepstow Bridge, which was completed in 1852, a gigantic N-truss of three panels with counterbracing in the centre. The Pratt brothers had also patented their open-web N-truss in America as early as 1844.

Despite this there is no mention of this type of bridge, which was to dominate bridge design from the 1850s right up to the present day, a type which was economical, functional, easy to construct and easy to calculate. Perhaps it came too late for the Report, though it must have been in the mind of Joseph Cubitt, Brunel and others. It is a mystery why it was never mentioned.

Conclusions of the Report

The rambling nature of the Report did not lend itself to brief and precise conclusions, and perhaps that was not the intention. It speaks of general conclusions as follows:

1. That it appears advisable for engineers in contracting for castings to stipulate for iron to bear a certain weight, instead of endeavouring to procure a specified mixture.

2. That to calculate the strength of a particular iron for large castings, the bars used for test-pieces as a unit should be equal in thickness to the thickest part of the prepared casting.

3. That, as it has been shown that to resist the effects of reiterated flexure, iron should scarcely be allowed to suffer a deflection equal to one-third of its ultimate deflection, and since the deflection produced by a given load is measured by the effect of percussion, it is advisable that the greatest load in railway bridges should

in no case exceed one-sixth of the weight which would break the beam when laid on at rest in the centre.

4. That as it has appeared that the effect of velocity communicated to a load is to increase the deflection that it would produce if set at rest upon the bridge.... but it can be diminished by increasing the stiffness of the bridge. It is advisable that for short bridges especially, the increased deflection should be calculated from the greatest load and highest velocity to which the bridge may be liable; and that a weight which would statically produce the same deflection should, in estimating the strength of the structure, be considered as the greatest load to which the bridge is subject.

5. Lastly, the power of a beam to resist impact varies with the mass of the beam, the striking body being the same, and by increasing the inertia of the beam without adding to its strength the power to resist impact is within certain limits also increased. Hence it follows that weight is an important consideration in structures exposed to concussions.

Comment on the Report

The Report made no recommendation on perhaps the most important aspect in any bridge design, i.e. the loading to be adopted. It ended by "Trusting that the facts and opinions which we have been enabled to collect will serve to illustrate the action which takes place under varying circumstances in iron railway bridges, and enable the engineer and mechanic to apply the metal with more confidence than heretofore".

The general impression gained from the Report is that this was the first time there had ever been such a pooling of knowledge on the use of iron for bridge design, and the variety of opinion expressed by the leading engineers must have given many food for thought. The most perceptive observations came from Brunel and Fairbairn. Robert Stephenson, in his observations on torsion and his defence of the Dee bridge seemed to have a much less sound grasp of structural principles. Yet Stephenson at the time of the Report was engaged at Conwy and Menai on the design and construction of the greatest iron bridges yet built.

One important feature which the Report did not touch on was methods of calculation in use at the time. The Hodgkinson formula was in general use for cast-iron beams, but its derivation was never questioned. It is unlikely that an engineer such as Brunel depended on such a rudimentary form of calculation, and he must have had his own methods. R Stephenson never mentioned any method of calculation which might have been employed for his trussed girder, and its basis seems to have been entirely empirical. This omission of discussion of calculation is all the more curious because one of the Commissioners appointed to the Report was Eaton Hodgkinson himself, who might well have raised it.

This brings us finally to a less obvious but interesting feature of the Report – the extent to which the contributors, famous or otherwise, revealed their own abilities and shortcomings. Perhaps they were not aware that their contributions would be recorded for ever in the archives of science for their successors to browse over, but they certainly form a fascinating mosaic of the state of development of the girder bridge at that time and of its designers and builders.

CHAPTER 3

FROM GARDEN TRELLIS TO TUBULAR BRIDGES

The Lattice Girder (or "Garden Trellis")

Other peculiar girders of early days

The development of the tubular girder

The Conwy Tubular Bridge, 1848

The Britannia Bridge, 1850

The Torksey bridge

Chapter 3

From Garden Trellis to Tubular Bridges

The Lattice Girder (or garden trellis)

The Report on the application of Iron to Railway Structures contained the remark "lattice bridges appear to be of doubtful merit" and did not attempt to analyse or describe why this was so. They briefly appeared on the girder bridge stage, but the weakness of the slender webs made them vulnerable to buckling and their use did not last. The wrought iron variety (developed from timber lattice webs in America) were first used on the Dublin and Drogheda Railway by John MacNeill, who had gained experience on railways in Scotland and in the construction of Grangemouth Docks (Horne, 1998). On the Dublin Railway, MacNeill had to span 144 ft. over the Royal Canal with a low-rise requirement that did not allow an arch solution, though a tied-arch was a possibility. Instead, MacNeill designed what is claimed to be the first ever wrought iron railway girders in 1844. There already was a 31 ft. span wrought iron bridge in existence in Glasgow, but it was a road bridge of 1841 carrying the Cathcart Road over the Govan and Pollok Railway south of the Clyde. (Fig 16) (Swailes, 2001).

MacNeill chose a lattice girder some 18 ft. deep and there were three girders carrying the double line of railway. (Fig 15). This had followed a road over-bridge of the same type over the same railway in 1843, but the Royal Canal bridge was the first in wrought iron to carry a railway. It is not clear whether the web design was purely empirical, or whether some attempt was made to calculate shear stresses in the lattice.

MacNeill later became Sir John, and was the first occupant of a chair in the Practice of Engineering in 1848 at Trinity College, Dublin. Another claim to fame was that one of his articled pupils in his railway practice was W J MacQuorn Rankine, later to become one of the greatest of engineering professors at Glasgow University in 1857 – 73, holding the Regius Chair of Civil Engineering. Another MacNeill lattice bridge was on the Dublin to Belfast railway over the river Boyne, a three-span viaduct in iron with flanking masonry arches, which lasted until 1932.

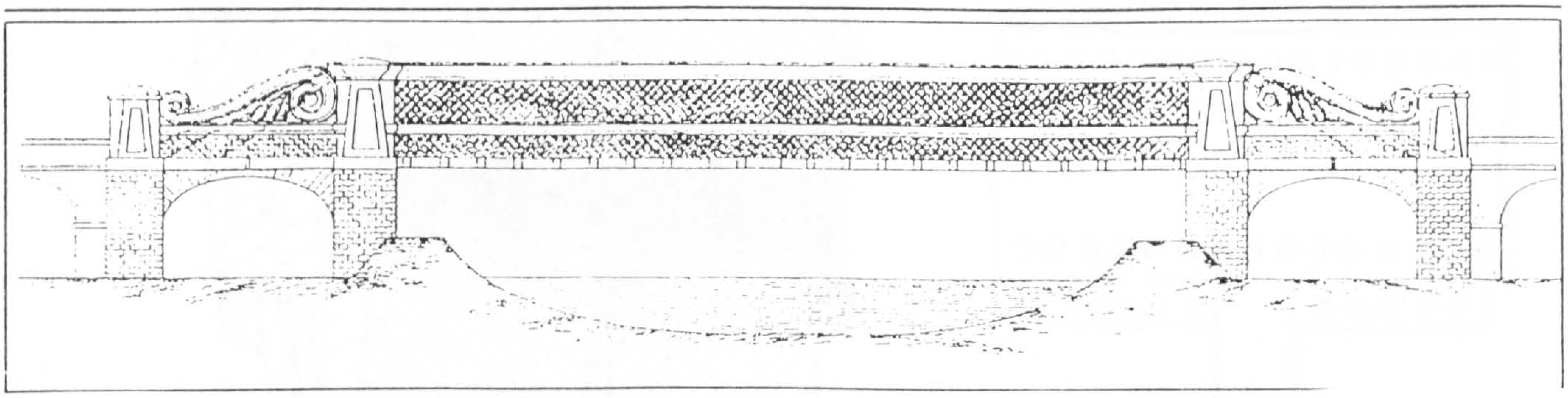


Fig 15 Royal Canal bridge at Dublin - 1844.
Span 135 ft. clear, by John MacNeill.
(Horne, 1998)

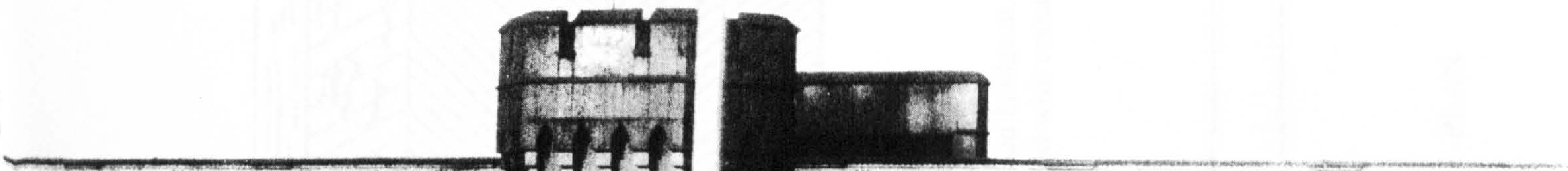


Fig 17 A late lattice girder bridge of 1863-68.
Runcorn bridge.

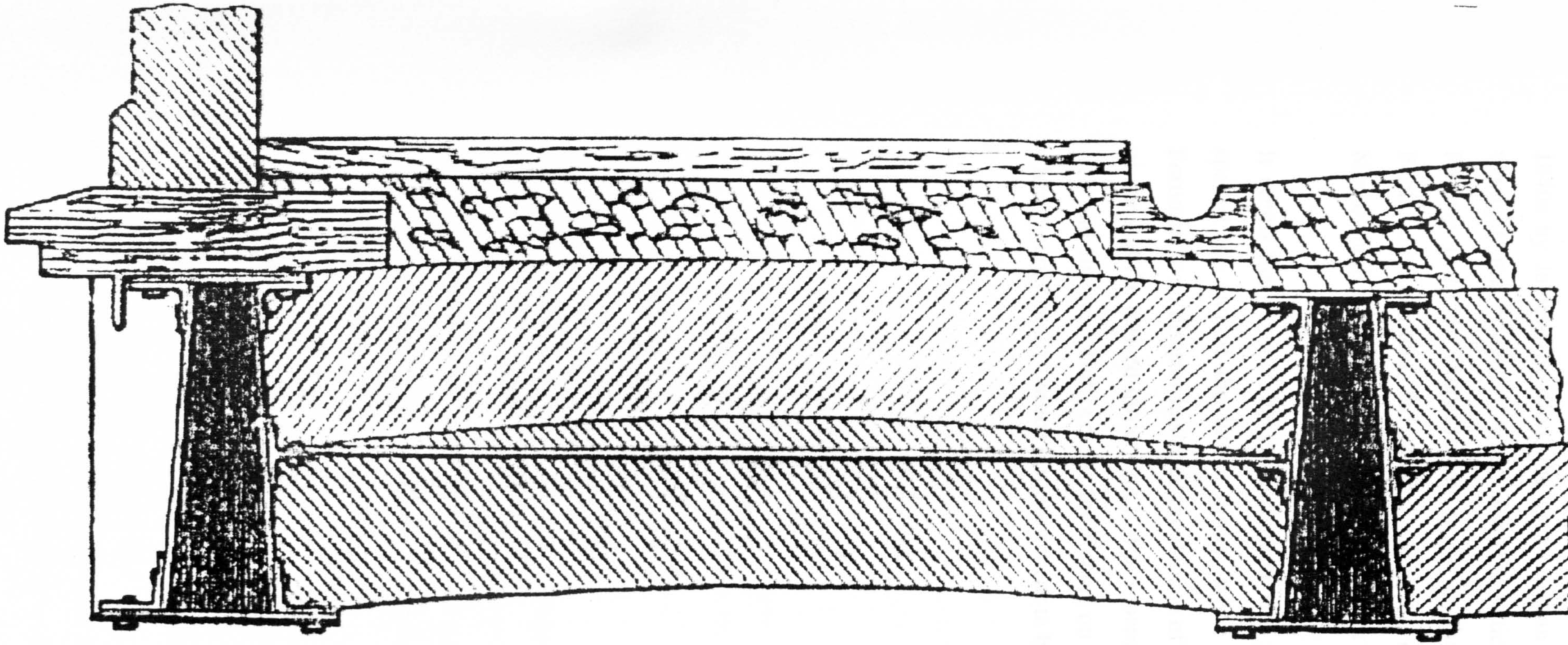
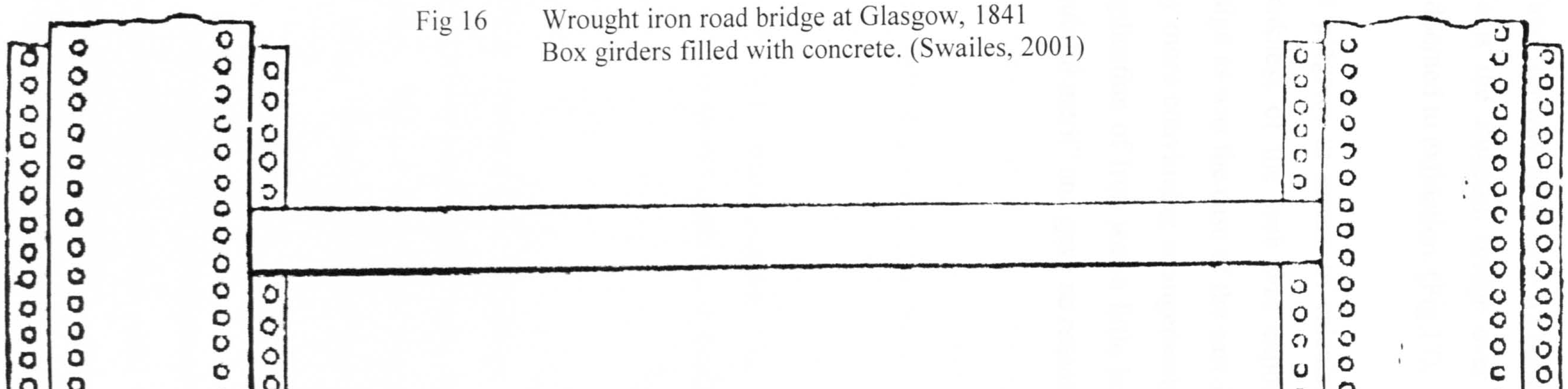


Fig 16 Wrought iron road bridge at Glasgow, 1841
Box girders filled with concrete. (Swailes, 2001)



PLAN

Two other lattice bridges were built in England on the Liverpool and Bury Railway in the 1840s by James Thomson, the greatest span being 84 ft. Curiously, the first wholly wrought iron bridge in America was a lattice girder, built on the New York Central Railway in 1858. Others remain in use on the Highland Railway of 1860 – 62 between Perth and Inverness and another surviving example is the Runcorn bridge over the Mersey built in 1863 – 68, surprisingly late for a type doomed to extinction. (Fig 17).

It seems that complexity in fabrication and perhaps progress in means of calculation spelled the end of the lattice girder, when the weakness of the web was exposed. Fortunately no disaster hastened the demise of the design as was the case of the cast-iron girder and the trussed girder, but it was superseded by more convincing arrangements of web and flanges. But perhaps the Report on the Application of Iron was a little harsh when it curtly said, "Lattice bridges appear to be of doubtful merit" and gave no reason.

Other Peculiar Girders of Early Days

Banbury Bridge

A design which suggests a trussed girder, but which is actually perhaps the first lenticular girder, after Gaunless, was a road bridge carrying the road from Banbury to Lutterworth over the London and Birmingham Railway. This dates the bridge to the 1830s. It had a span of 64ft. (Fig 18).

It had six ribs, cigar-shaped in elevation, of which the upper half was a curved cast-iron arch and the lower half was a curved wrought iron tie. The depth overall at midspan was about 12 ft, and the two halves were braced apart by cast-iron panels acting as struts. The connection between the arch and the tie was by a simple pin-joint. At midspan there was a vertical tubular strut attached to the arch by a screw arrangement. This could be used for forcing the tie downwards, thus adjusting its tension for any lack of fit or slackness. There is no suggestion that this strut was used for pre-stressing the girder, though its presence suggests the possibility.

Complexity of fabrication probably meant that this type did not become popular. The low rise-span ratios of the arch and tie suggests it may also have been vulnerable to deflection.

St Mary's Viaduct

This was a span of 74 ft on the Cheltenham and Great Western Union Railway, by Brunel. (Fig 19). Often he chose to use timber for bridges where possible, and sometimes a combination of timber and wrought iron for larger spans. The use of wrought iron and timber together occasionally solved the problem of using timber in tension and the associated joints.

At St Mary's the structure is exceedingly complex and appears to be a combination of both king-post and queen-post trusses with iron tie members, and timber used for compression. (Berridge, 1969). The construction of the railway was from 1837 - 45, and was delayed by difficulties encountered with Sapperton Tunnel; the bridge must date from this period, probably 1840. Brunel had used a similar design of 66 ft. span on the Bourne viaduct, but probably the St Mary's span of 74 ft. stretched the design to the limit.

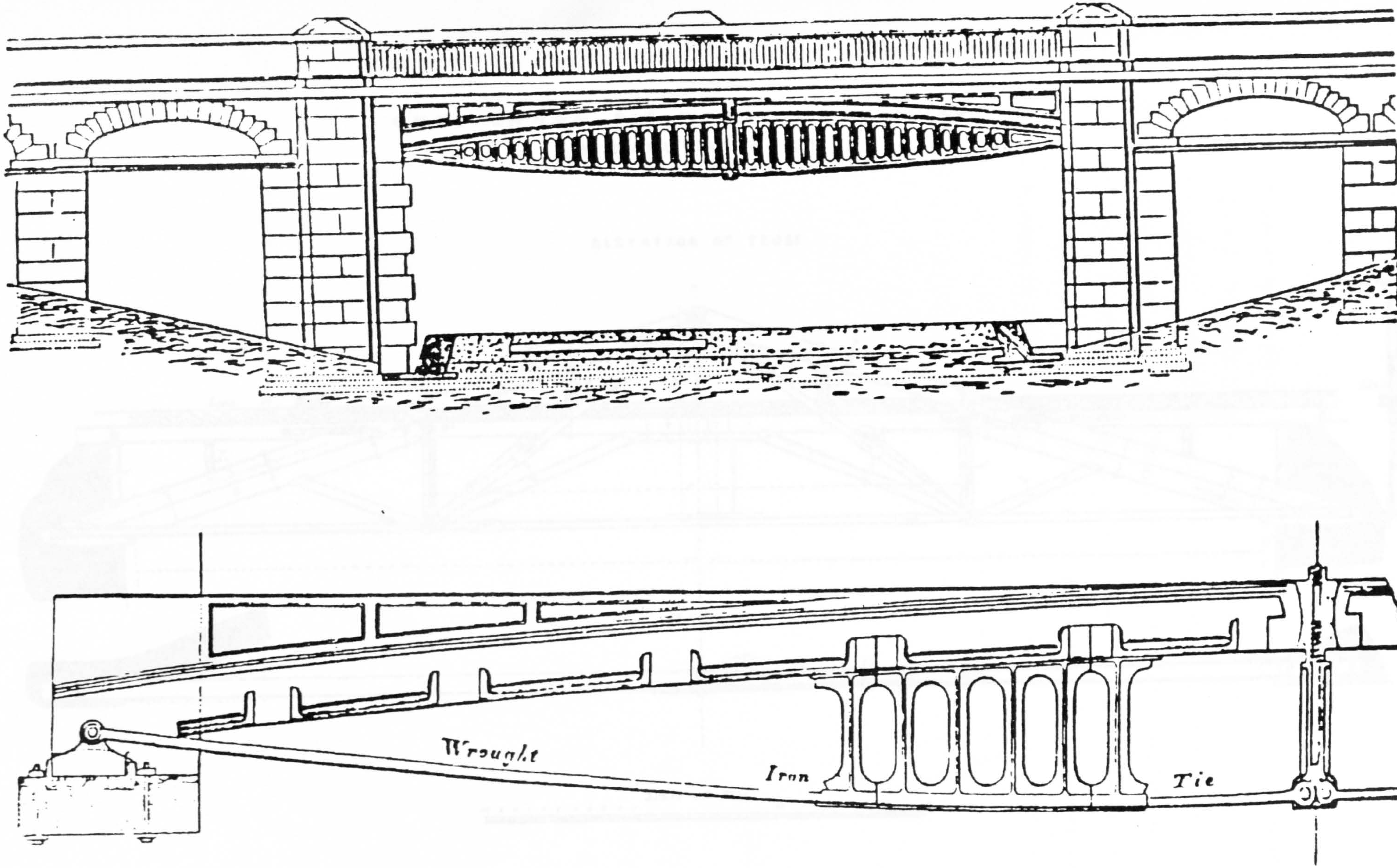


Fig 18 Banbury bridge, 1838. Lenticular truss.
(Stephenson, 1856)

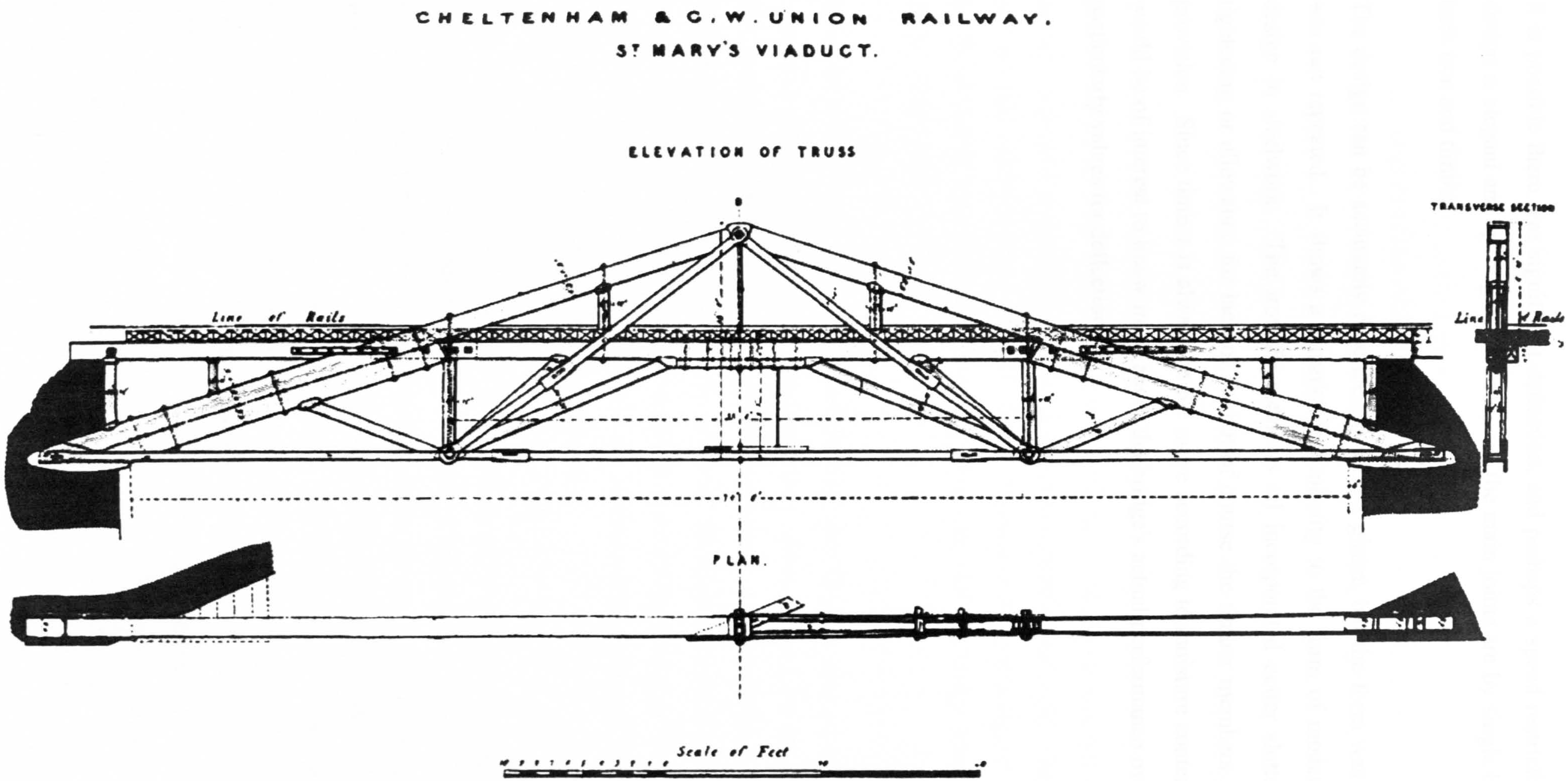


Fig 19 Brunel timber/iron truss, span 75 ft. c. 1840.
St Mary's Viaduct, Cheltenham & G W Union Railway
(Berridge, 1969)

It is possible there was significant deflection, and perhaps a speed restriction. Yet the design is elegant and sparing of material, and the main joints are by single bolts through both iron and timber.

The design can be accurately described as a truss girder, but the form was unique, and was not repeated. It shows a remarkable similarity to the form of modern roof truss design in steelwork. The iron tie members all incorporated cotter slotted holes for tightening or allowance for lack of fit, but of course the timber members had no such provision. Since timber is always on the move according to moisture content changes it would be of interest to know more about the bridge's actual performance over the years, particularly values for deflection.

The Development of the Tubular Girder

In the early 19th Century communication between London and Ireland became increasingly important and resulted in the construction of the Holyhead Road by Thomas Telford in 1815 - 26. The shortest route involved a sea crossing between Holyhead and Howth, Dublin, which is why Holyhead was important. This involved crossing the Menai Straits between Anglesey and the mainland - a wide, tidal and swift-flowing channel of importance to the Admiralty for ships passing between the Welsh coast and the Mersey.

Telford's earliest proposal was for an arch bridge in cast-iron of 500 ft. span at what was known as the Pig Island site. The Admiralty objected to the headroom, although Telford devised ingenious suspended centring for the construction of the arch. They demanded 105 ft. clear headroom for a width of 450 ft. Eventually Telford designed a suspension bridge of 580 ft. span giving the necessary clearance, and this bridge was completed in 1826. (Rolt, 1958).

When the railway route to Holyhead was planned some 20 years later, Robert Stephenson was the engineer, and he too was faced with the problem of bridging the Menai Straits. The implacable Admiralty placed the same restrictions of span and headroom, and left Stephenson with a seemingly impossible problem. (But it is interesting to note that when Stephenson's Menai Bridge had to be replaced in the 1970s, owing to fire damage, steel arches were constructed under the existing structure and apparently allowed by the Admiralty despite the much reduced headroom).

Stephenson was under pressure from the Directors and shareholders of the Chester and Holyhead Railway to complete the line as quickly as possible - but no bridge of span 400/500 ft. had ever been built apart from suspension bridges. Stephenson knew from his experience with the suspension bridge at Stockton that for railway traffic such a design was out of the question; it was far too flexible. His next idea was to gain permission for one carriageway of Telford's suspension bridge to be made available for carriages and wagons to be drawn across by horses, the railway engines being barred from crossing the bridge. This idea, however, was rejected. With hindsight the idea of a high-speed train service being delayed for carriages to be uncoupled and fastened to horses, then towed across and coupled again to an engine appears preposterous, but it shows how the magnitude of the problem exercised the minds of the engineers.

Stephenson however continued to think in terms of a suspension bridge and came up with the idea of a great tube, either cylindrical or elliptical, being supported by chains and stiff enough to avoid excessive deflection. In early 1845 there were no precedents for this type of construction and Stephenson turned to William Fairbairn for advice and assistance. Fairbairn was an industrialist, iron master, shipbuilder and experimental scientist, and probably no man in Britain better understood the use of iron than he.

Fairbairn set up a series of experiments on models of circular, elliptical and rectangular tubes to determine the best shape. He was a practical man, and felt that to test something in practice was much to be preferred to calculation. At the same time he recognised that if a mathematical basis existed for the design of a tube, it would reduce the amount of experimentation required. He sought the permission of Stephenson to employ Professor Eaton Hodgkinson FRS from University College, London as an assistant to develop a general formula to assess the strength of iron tubes. It had become obvious that the rectangular tube was superior to other shapes, though Stephenson himself had favoured the elliptical because of its lesser resistance to wind pressure.

Stephenson's intuition regarding the use of tubes on a massive scale received a boost during the launch of an iron steamship the "Prince of Wales" at Blackwall. During the launch, the ship became stuck on the launching ways above the water level such that the hull was entirely unsupported between the cradles over a length of 110 ft., yet it was not damaged in any way. This was a demonstration of the strength of a wrought iron structure, and greatly encouraged Stephenson. (Sutherland, 1964).

Refining of the details of the tube cross-section now took place as it began to emerge that the top flange tended to fail first due to compressive stress causing buckling. Instead of a flat plate, it was found better to have a series of cells, circular or rectangular, across the full width of the tube. The lower flange was modelled likewise, and the eventual choice adopted was of 8 rectangular cells forming the upper flange and 6 forming the lower. Experiments also showed that the webs joining the two flanges should be of stiffened plating and not an open-web or lattice solution, of which Stephenson was not in favour.

It remained now to determine whether chains were to be used as an additional support. Stephenson's own feeling was that the rectangular tube could be made strong enough to

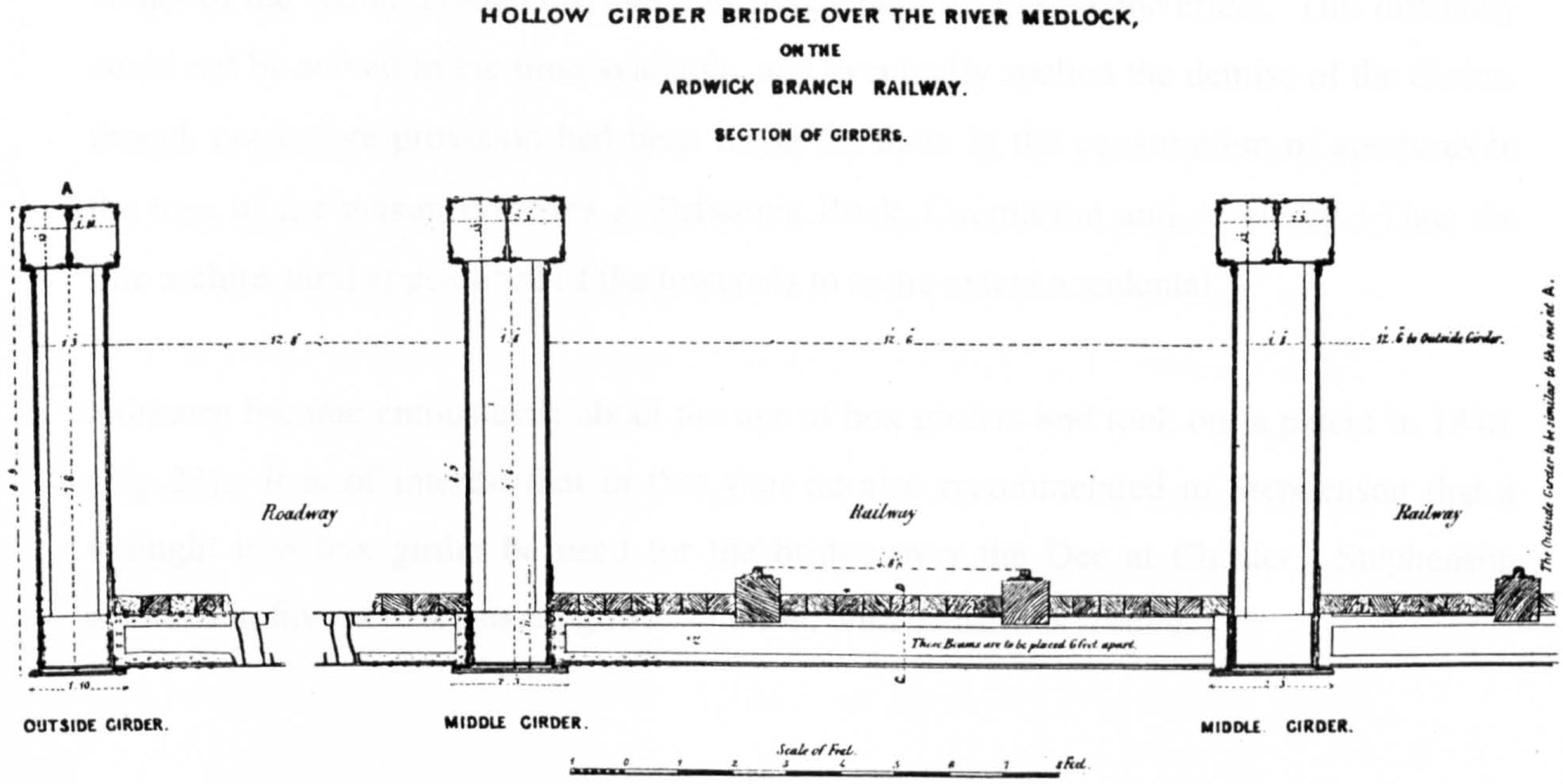


Fig 22 Development of Tubular Girder.
Medlock combined Road and Rail Bridge, with Fairbairn girders.
(Berridge, 1969)

act independently without chains. He sought advice from Hodgkinson, who suggested that chains be retained as auxiliary members. Turning to Fairbairn, he found that he supported his own view that the chains could be dispensed with. A difficulty now rose as to the best point of attachment for chains if they were used and how would they affect the values of the bending moments, and how to calculate accurately the effect. This difficulty could not be solved in the time available, and eventually spelled the demise of the chains, though not before provision had been made for them in the construction of apertures in the tops of the masonry towers at Britannia Rock, Caernarfon and Anglesey. Thus the fine architectural appearance of the towers is to some extent accidental.

Fairbairn became enthusiastic about the use of box girders and took out a patent in 1846. (Fig 22). It is of interest that in that year he also recommended to Stephenson that a wrought iron box girder be used for the bridge over the Dee at Chester. Stephenson declined in favour of a trussed girder solution, with disastrous results.

The Conwy Tubular Bridge

After the foregoing necessary but somewhat lengthy introduction to the conception of the tubular girder bridge, the design of the two bridges at Conwy and Menai will now be considered in some detail.

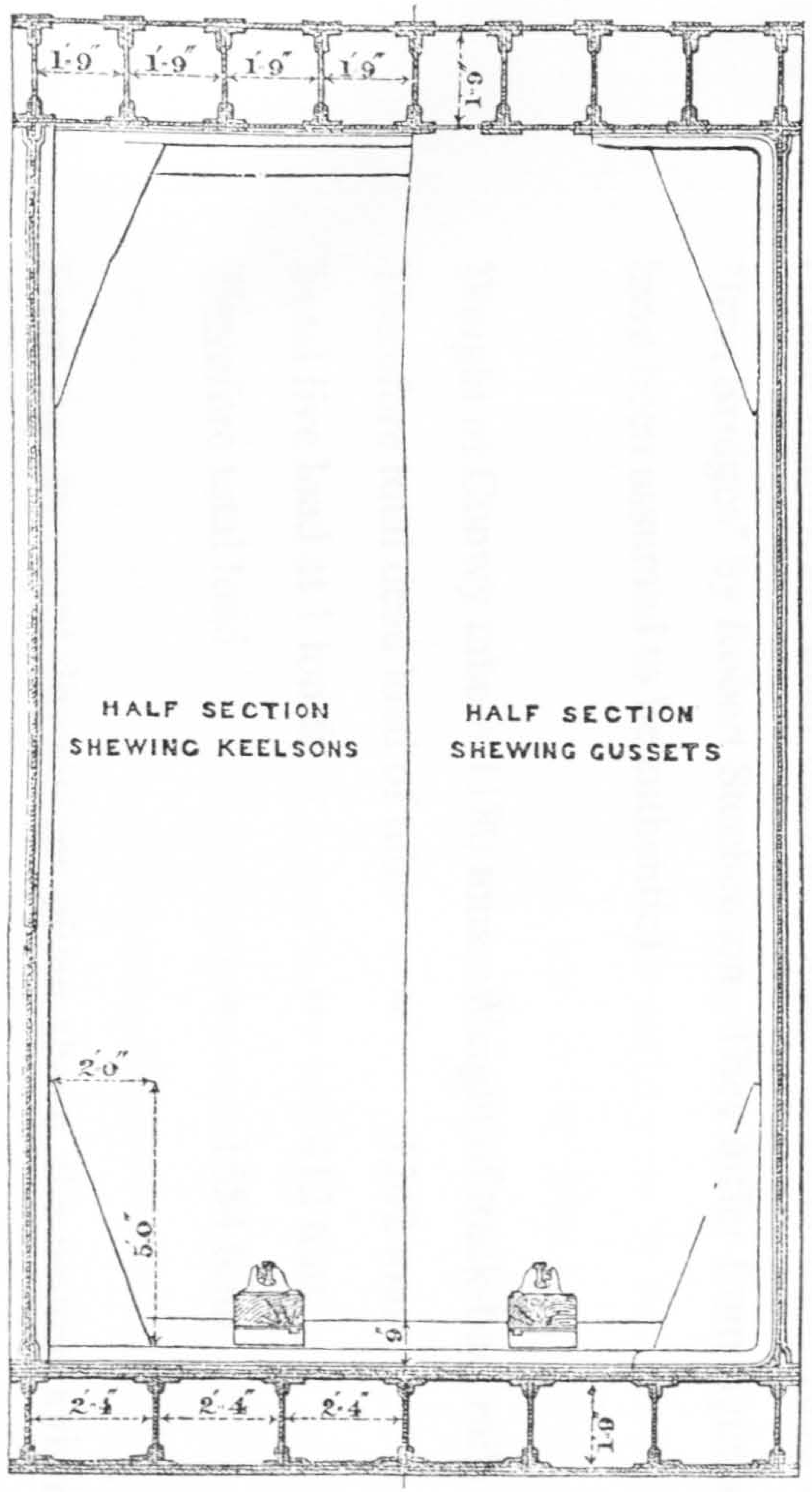
Reference to historical descriptions of Conwy bridge fail to state why a bridge of 400 ft. span giving a headroom of approximately 24 ft. was required at that site. There were no Admiralty restrictions on the use of the waterway and the bridge itself was reduced in span from 400 ft. to 300 ft. by the insertion of extra support piers later in its life.

Thomas Telford had already erected a suspension bridge over the Conwy estuary in 1826. It had a span of 327 ft. Accounts of this bridge likewise do not mention why the single span was adopted, but it must have been due to the depth and rapid flow of the water in the channel making a central support difficult and expensive to construct. (Fig 23).

The construction of the Conwy tube was to provide a useful trial run for the greater bridge at Menai. Fairbairn constructed a new model of Conwy at Millwall, one-sixth full size, and tested and re-tested it until the best form of plates and cells for the flanges emerged. After these experiments Fairbairn had accumulated sufficient data to derive a formula to design a tube for any loading and any span. The box girder continues in use to this day, though somewhat different from the form devised by Fairbairn.

Fairbairn's formula linked the breaking weight, the cross-sectional area of the lower flange, the overall depth, the span and a constant c which was specific to a given form of tube. This sounds suspiciously like Hodgkinson's well-used formula, except that the value for c for the model tube was 76, rather than the value of 26 which was employed for cast-iron beams. As we have seen, the value of 26 was based on the ultimate strength of cast-iron in tension, and it is likely that the value of 76 was based on the much higher ultimate strength of wrought iron, i.e. 19.0 tons/sq. in. compared to 6.5 tons/sq. in., which are conservative values.

The method of calculation for the shear strength of the side plating remained a mystery and for safety the plating at Conwy was made ½ in. thick with tee iron stiffeners at 2 ft. centres for resistance to buckling. This was a remarkably accurate estimate and gave



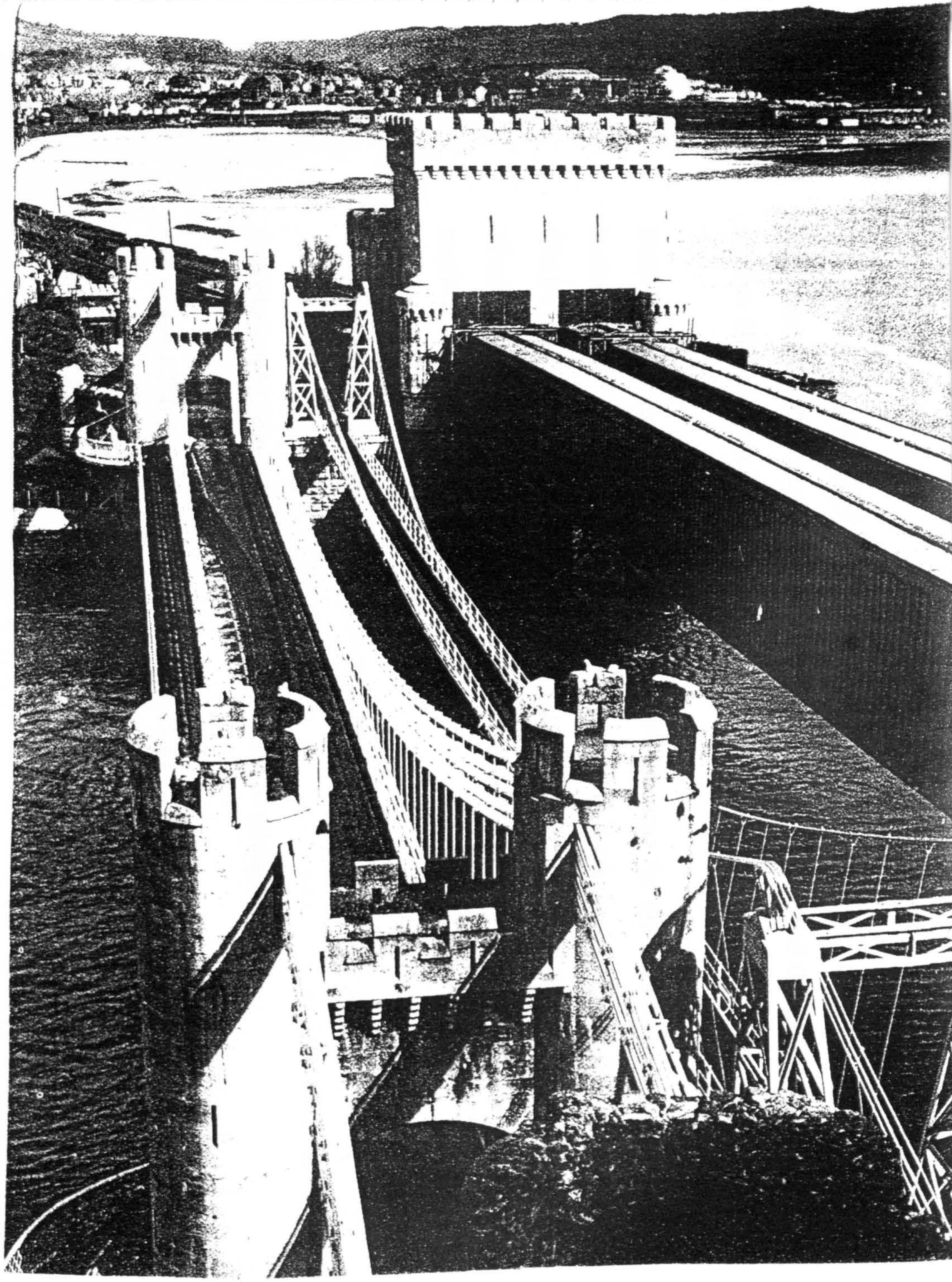


Fig 23 Conwy Bridge and cross-section, 1848
(Telford's Suspension bridge on left)
(Stephenson, 1856)

shear stresses on the high side, but not unduly so. The weight of the plating on the two sides increased the weight of iron in the span by about 20%, and the tee iron stiffeners by another 20% of the total.

Both Fairbairn and Edwin Clark, the resident engineer for the bridge erection, published books on the Conwy and Menai bridges. Fairbairn's book was published first in 1849 and Clark's in 1850. Fairbairn, always the practical man, showed little in the way of calculation. Clark on the other hand has many pages, but the calculations do not tell us much. (Clark, 1850).

It is of interest to derive some figures as follows:

Calculation for one Conwy Tube: (Dimensions at midspan)

Clear span - 400 ft. Span between supports - 412 ft.
Depth overall at midspan = 25 ft. 6 in.
Depth of top flange cells = 2.0 ft. Depth of bottom cells = 2.0 ft.
Net depth of ½ in. thick side plating = 21 ft. 6 in.
Therefore area of side plating = 258 sq. in. (two sides)
Area of top flange cells = 648 sq. in.
Area of bottom flange cells = 585 sq. in.
(These figures are from the Encyclopaedia Britannica, 1856 edition, from an article on "Iron Bridges" by Robert Stephenson. They differ from figures given in Clark's book, but have been assumed to be authentic).

Weight of Conwy tube = 1180 tons. Weight of track-bed, rails etc 62 tons

Therefore total dead load of tube	=	1242 tons
Total live load at 1 ton/ft.	=	412 tons
Therefore total load	=	1654 tons

From the structural dimensions given above, the neutral axis is 159 in. above the lowest fibre, and is 147 in. below the top fibre.

These values give a moment of inertia of 1273 ft^4 about the neutral axis, and a modulus of elasticity of 105.4 $ft.^3$ for the upper flange, and 96.1 $ft.^3$ for the lower flange respectively.

From the dead and live loads given above, for a span of 412 ft. the total bending moment at midspan is 85,181 tons-ft.

From this bending moment, the upper flange stress would be 5.61 tons/sq. in. in compression, and the lower flange stress would be 6.15 tons/sq. in. in tension.

The vertical shear stress at the supports (5/8 in. thick plating) would be 2.56 tons/sq. in. and the corresponding horizontal shear stress at mid-height of the web would be 50% greater at 3.84 tons/sq. in.

In the late 1840s a stress of 5.0 tons/sq. in. was considered by many engineers to be a realistic design figure for tension and compression in wrought iron, and the figures for the bending stresses in the Conwy Bridge can be considered to be acceptable. It can also be said that the live loading from a railway train would be very unlikely to be as high as 1 ton/ft. run over the whole of a 412 ft. span, and might be nearer 50% of that figure.

The shear stress in the web appears high at 2.56 tons/sq. in. It was a rule of thumb as late as the late 1940s, before the general use of BS 449: 1949 (The Use of Structural Steel in Buildings) that the web stress was generally limited to 5/8 of the allowable stress in tension, for steel structures. For wrought iron in the 1840s it is unlikely that any rule existed for limiting web stresses, and in fact stresses in webs were not understood, and not calculated. But the above rule would give an allowable stress of 3.13 tons/sq. in.

Another empirical rule which applied to webs pre-BS 449 was that the web should not be thinner than 1/160 of the clear depth between the flange angles. The webs on the Conwy tubes were each ½ in. thick and the depth between the flange cells was 258 in. This gives a ratio of 1/516! Even if it is allowed that there was two webs, the ratio is still 1/258, which shows the slenderness of the structure. However, the web plating was well stiffened by the T-irons at 2 ft. centres, which no doubt saved the day.

Robert Stephenson and his staff evidently appreciated the slenderness of the web, and the tendency for the 25 ft. high structure to "lozenge" and get out of shape under varying conditions of loading during construction and erection, i.e. when floating out, the ends of the tube overhung the ends of the flotation barges by 70 ft., and when jacking the tube into position the lift was applied to the top of the tube, not the bottom as in the final condition. The webs were therefore strengthened by substantial gusset connections to the flanges at all four corners at frequent intervals along the length of the tube.

It is a mystery why Fairbairn went to the trouble of deriving a formula for the calculation of the structural strength of a tube, when Navier's Lecons of 1826 offered $M/I = f/y$, which contained all that was needed. Although direct tension stress and direct compression stress were well understood (e.g. in suspension bridge and arch bridge design) it may have been that bending stress, resulting in tension or compression, was not understood until some time after 1846.

A clue to the adequacy of a beam or girder can often be found by observing the deflection. If the deflection is excessive, it is undesirable, as in "bouncy" floors or in the Millennium Bridge. But if the deflection returns to zero on the removal of the load, it shows generally that the structure is safe, and not stressed beyond the elastic limit. Calculating deflection involves a knowledge of Young's Modulus for the material. The value for wrought iron varied approximately between 10,000 tons/sq. in. and 12,500 tons/sq. in. (For steel it is 13,400 tons/sq. in). If we assume good quality wrought iron and a Young's Modulus of 12,500 tons/sq. in., then the deflection at midspan under dead load alone, 1242 tons uniformly distributed, is

$$d = \frac{5WL^3}{384EI}$$

where d = deflection, W = total load, L = span, E = Young's modulus and I = moment of inertia of the section.

Here W = 1242 tons, L = 412 ft., E = 12,500 tons/sq. in. and $I = 2641 \times 10^4$ in.^{4}.

Thus d = 5.92 in. for dead load and 1.96 in. for live load. If a value of 10,000 tons/sq. in. is used for Young's modulus, then d = 7.40 in. for dead load and 2.45 in. for live load.

Actual figures for deflection measured at the site were 7.88 in. under dead load alone, and 2.40 in. under a 300 ton ballast train at the centre of the tube. These figures compare well with the theoretical values.

The tube was later floated out and jacked upwards into its final position. The deflection figures quoted from observations at the site may have been influenced by temperature changes, as it was discovered that uplift and sagging amounting to 1.50 in. could result from an overcast or clear sky.

Thus was completed one of the greatest forward steps in bridge girder design. The tubular bridge, with trains running through it, was to have a limited future, but the Conwy tube represented a gigantic achievement, all the more singular because much of it was a leap in the dark, intuition, with little support from the rudimentary calculations of the day.

The first train, driven by Robert Stephenson, passed through the first Conwy tube on 18 April 1848. The previous greatest span for an iron railway bridge in Britain had been John MacNeill's lattice girder of 144 ft. on the Dublin and Drogheda Railway in 1844. Conwy was followed shortly after in 1849 by Brunel's tied-arch of 200 ft. span over the Thames at Windsor, and then by the opening of the Britannia bridge for single-line traffic in March 1850.

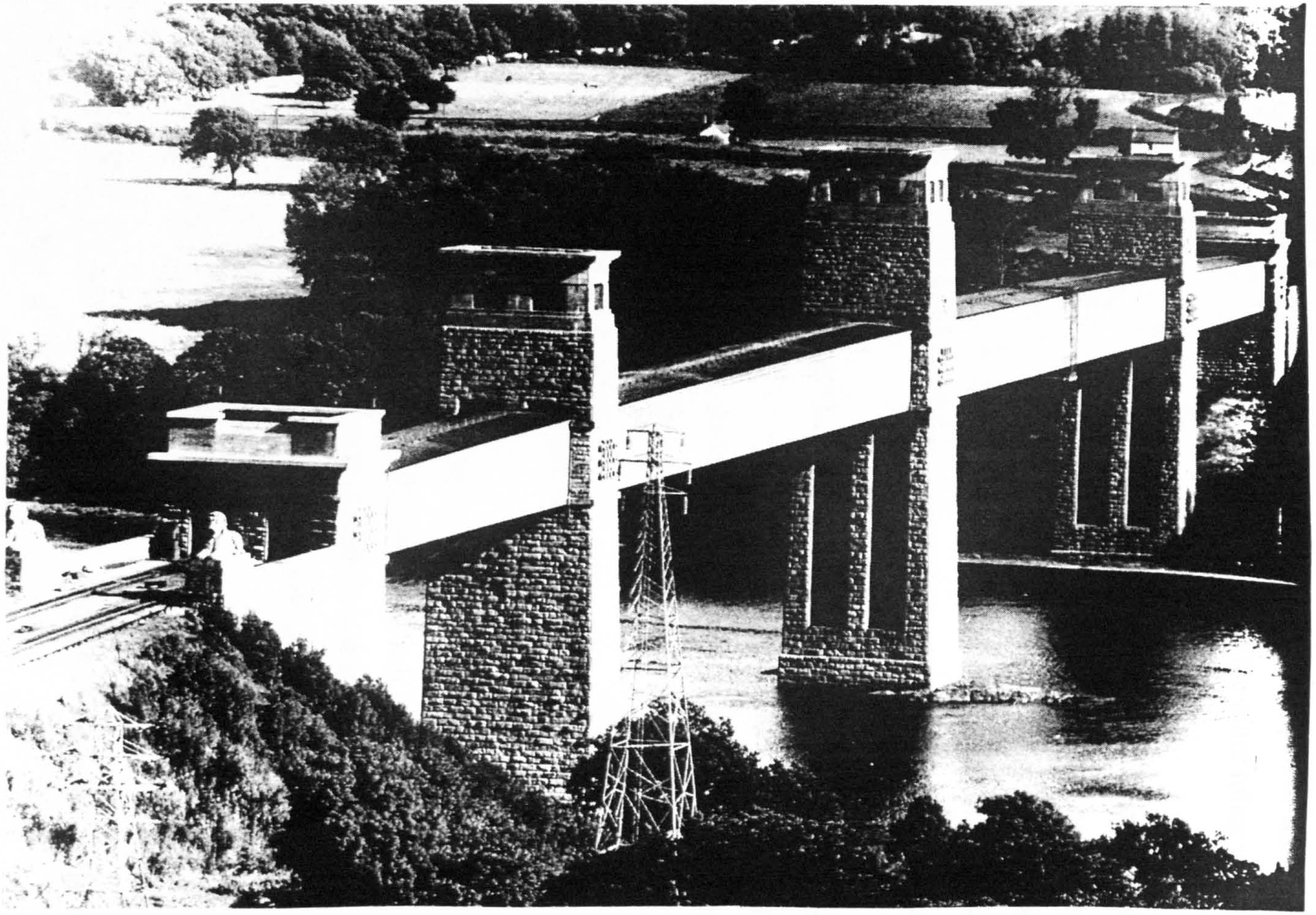


Fig 19A Britannia Bridge over Menai Straits. Completed 1850.
Destroyed by Fire 1970.

The Britannia Bridge

The most interesting feature of the Britannia bridge and its most significant contribution to the advance of bridge design was the attempt to produce continuity between all four spans. The outer spans were of 230 ft. clear, and the inner, 460 ft. For brevity, an inner span only will be analysed in detail. The actual span of the tube was 472 ft. between the support points. The corresponding spans of the outer tubes were 242 ft.

The total length of the tubes is nearly 1500 ft., and their soffit is level between the two ends at Caernarfon and Anglesey. The top of the tubes however is on a parabolic curve, beginning with a depth of 23 ft. overall of the outer ends of the tubes, increasing to 27 ft. at the Caernarfon and Anglesey towers and reaching 30 ft. at the centre at the Britannia tower. This makes the depth of the centre of the 460 ft. span about 29 ft. 4 in., or 352 in.

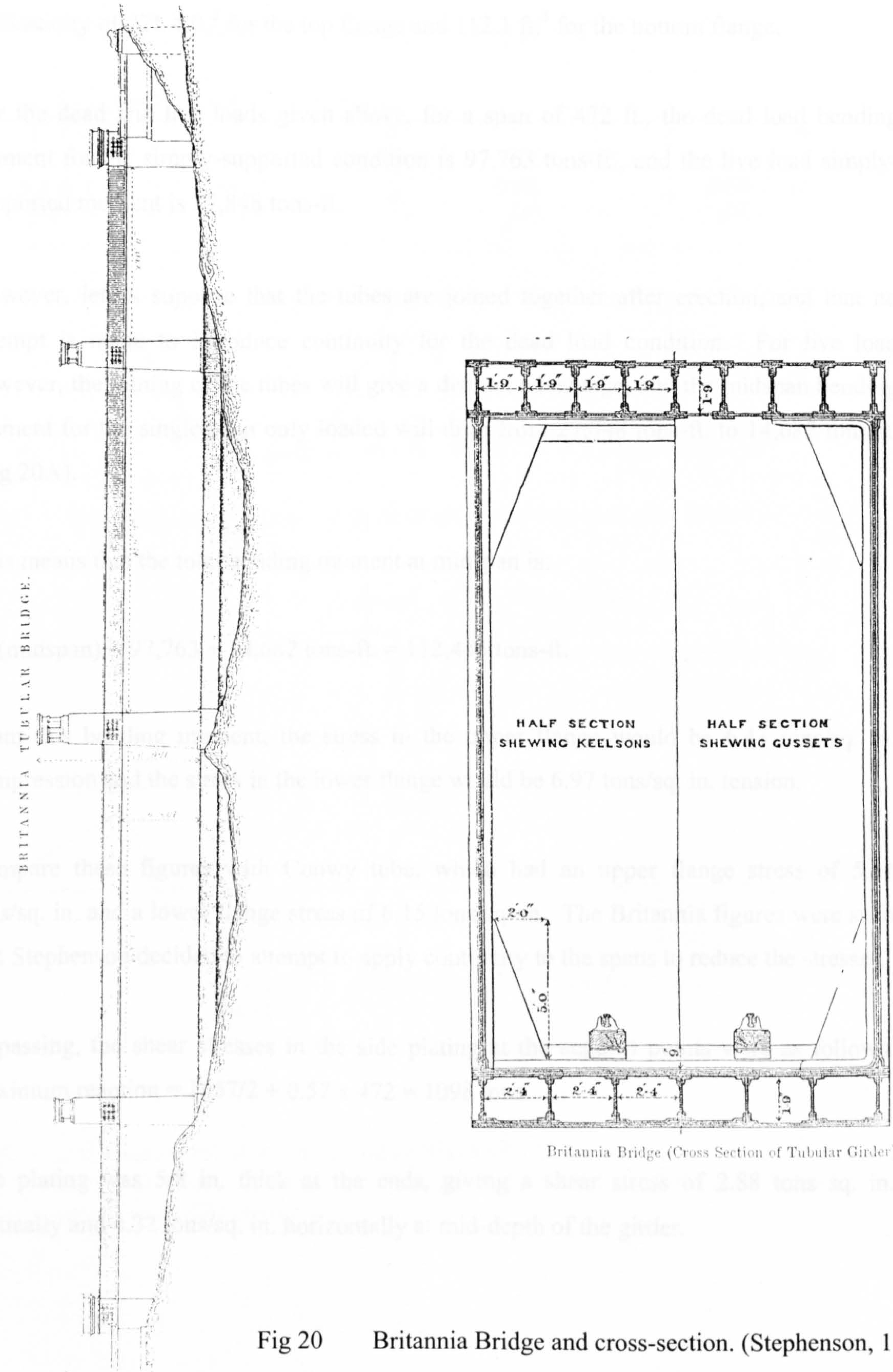
As with the Conwy bridge, the area of the flanges and the thickness of the side plating, etc will be taken from the Encyclopaedia Britannica of 1856 and used in the calculation of the properties of the section. The simply supported condition will be analysed first, followed by the application of continuity to the structure.

Calculation for the Britannia tube:

Clear span - 460 ft. Span between supports = 472 ft.
Depth overall at midspan = 29 ft. 4 in. (352 in.)
Depth of top flange cells = 2 ft. Depth of bottom cells = 2 ft.
Area of side plating = 304 sq. in. (two sides)
Area of top flange cells = 648 sq. in.
Area of bottom flange cells = 585 sq. in.

Weight of Britannia tube = 1587 tons. Weight of track-bed, rails, etc = 70 tons.
Total dead load of tube = 1657 tons.
Total live load @ 1 ton/ft. = 472 tons.
Therefore total load = 2129 tons.

From the structural dimensions given above, the neutral axis is 183 in. above the lowest fibre and is 169 in. below the top fibre.



Britannia Bridge (Cross Section of Tubular Girder).

Fig 20 Britannia Bridge and cross-section. (Stephenson, 1856)

These values give a moment of inertia of 1709 $ft.^4$ about the neutral axis, and a modulus of elasticity of 121.4 $ft.^3$ for the top flange and 112.1 $ft.^3$ for the bottom flange.

For the dead and live loads given above, for a span of 472 ft., the dead load bending moment for the simply-supported condition is 97,763 tons-ft., and the live load simply-supported moment is 27,848 tons-ft.

However, let us suppose that the tubes are joined together after erection, and that no attempt is made to introduce continuity for the dead load condition. For live load however, the joining of the tubes will give a distinct advantage, and the midspan bending moment for the single span only loaded will drop from 27,848 tons-ft. to 14,682 tons-ft. (Fig 20A).

This means that the total bending moment at midspan is:

M (midspan) = 97,763 + 14,682 tons-ft. = 112,445 tons-ft.

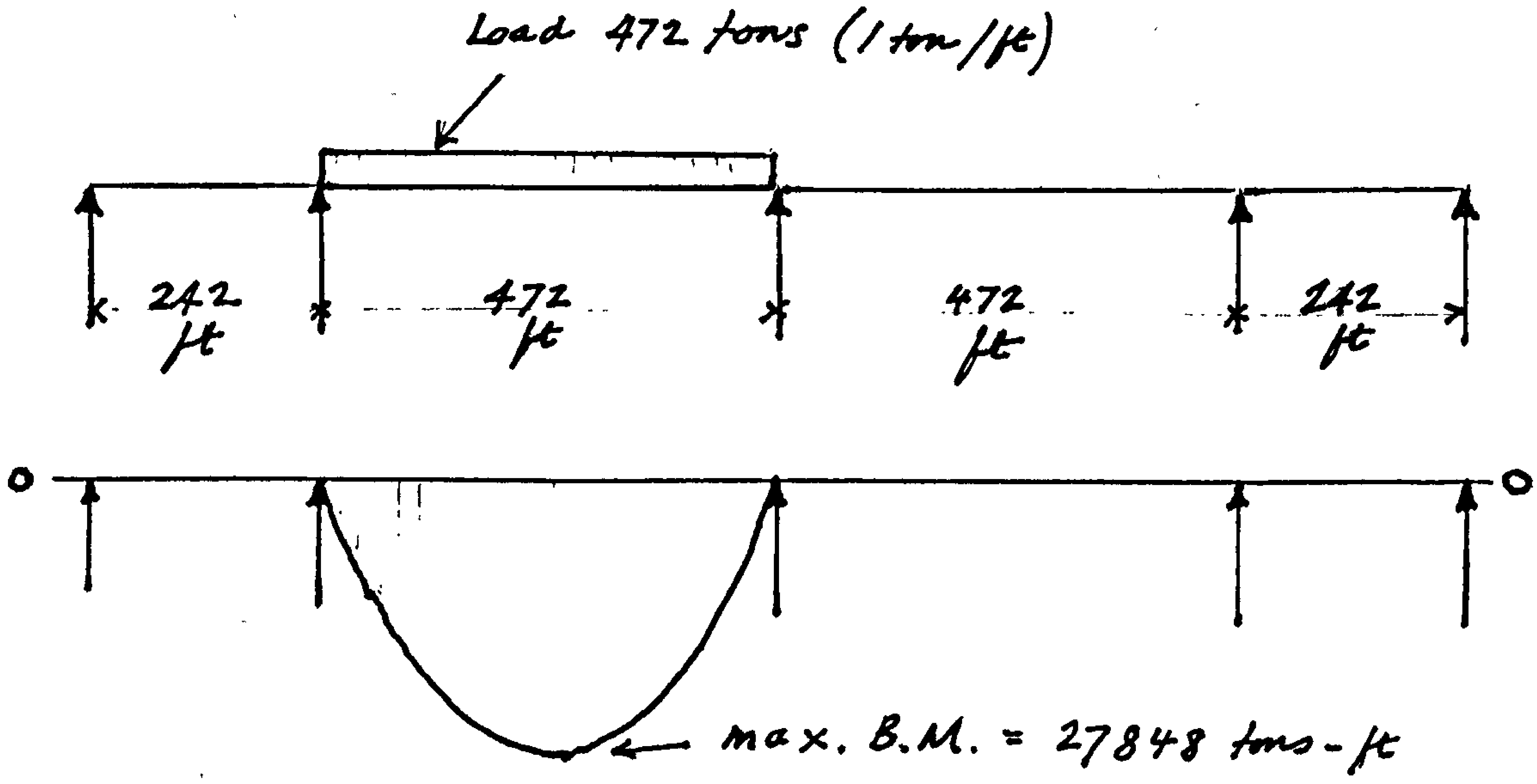
From this bending moment, the stress in the upper flange would be 6.43 tons/sq. in. compression and the stress in the lower flange would be 6.97 tons/sq. in. tension.

Compare these figures with Conwy tube, which had an upper flange stress of 5.61 tons/sq. in. and a lower flange stress of 6.15 tons/sq. in. The Britannia figures were such that Stephenson decided to attempt to apply continuity to the spans to reduce the stresses.

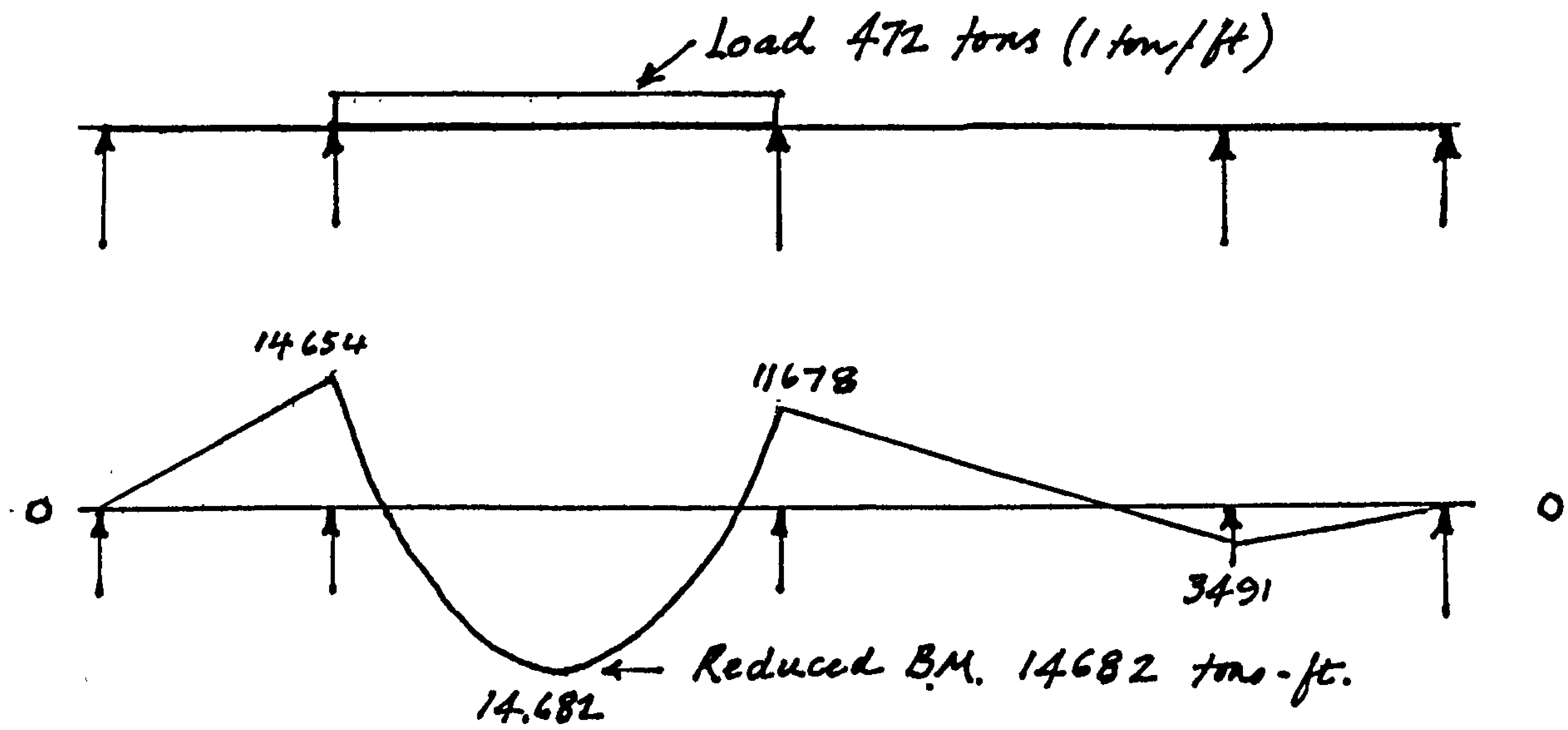
In passing, the shear stresses in the side plating at the support points were as follows: maximum reaction = 1657/2 + 0.57 x 472 = 1098 tons.

The plating was 5/8 in. thick at the ends, giving a shear stress of 2.88 tons sq. in. vertically and 4.32 tons/sq. in. horizontally at mid-depth of the girder.

Fig20A Britannia Bridge Bending Moments



(i) B M diagram for simply-supported condition one span only loaded



(ii) BM diagram for all spans joined after erection one span only loaded as before. Note reduction of midspan BM from 27,848 tons-ft to 14,682 tons-ft.

Application of Continuity to the Tubes

The method of achieving this was to raise one end of one tube by a calculated amount, then make the connection at the far end to the adjoining tube, then lower the raised end to its original position. This could be done for a succession of tubes laid end to end.

At Britannia, the logical sequence would have been to begin at either Caernarfon or Anglesey towers, producing the desired moment at the central Britannia tower, then raising the shore ends of the bridge to produce the desired moment at the Caernarfon and Anglesey towers; a total of three liftings of which two would be identical, to produce a symmetrical arrangement of the bending moments at the support points. There was only one snag, however. There was no engineer living capable of doing the calculations necessary to produce the correct continuity moments over the support. However, there was possibly one mathematician in England, Henry Moseley FRS (1791 - 1872) who had studied the matter and could help. The responsibility for finding a solution to this problem had been given to Stephenson's resident engineer, Edwin Clark, but he is silent regarding to whom he turned for information. He began the operation seemingly not fully understanding the requirements, and instead of making his first modified joint at the Britannia tower, which would have led to symmetry, he began for some reason with the Anglesey tower, making the modified joint between the landspan and main span. Then he proceeded to the Britannia tower, where he intended that the reverse bending moment should be about equal to the midspan moment in the adjacent span. An unknown degree of continuity was obtained, and it was not symmetrical for the whole bridge.

None of the published literature of the time appears to state the final arrangement of the bending moments in the Britannia tubes. Clark sums up the situation somewhat lamely in his concluding remark "The third and last junction made was that at the Caernarfon tower; the calculation of this would be very complicated, but we are justified in assuming that the tube was made perfectly continuous at this point". Well, the tube was made continuous, but what was the value of the bending moment? He does not say.

Yet I K Brunel and his assistant W Bell made available to Clark contemporary work on continuous beams by Brunel for his bridge at Chepstow (Clark, 1850). Bell gave information on his experimental work and referred to Moseley's book published in 1843 which contained "a correct analysis of encastré and continuous beams..." Bell further relates that:

"While examining in the year 1849, the stresses on continuous beams for the later Mr Brunel, in reference to the large bridge at Chepstow, the lower girder of which is virtually a continuous beam of five unequal spans, the Author, using the method of Navier, found the subject one of no inconsiderable difficulty, from the number and complexity of the eliminations required, and he was gratified by the formulae at which he then arrived, for beams up to five spans, being completely verified by the experimental tests to which he subjected them, (Fig 29) and which were devised by Mr Brunel". (Pugsley, 1976).

In the face of this evidence one wonders whether Clark had the necessary structural insight to deal properly with the problem. However, under test load the Britannia bridge performed well, and survived to 1970, when it was damaged beyond repair by a fire, and replaced. Before the fire, an attempt had been made to determine the values of the moments at the supports, and a Paper was given to the I.C.E. in 1975 by Wardle and Lucas summarising these results. The method chosen was to jack the ends of the tubes at all five points of support, raising them very slightly in turn so as to obtain a reading of the load on the jacks, and thus the actual reactions at the supports for the dead load alone.

These readings allowed the moments at the supports to be calculated, and showed that a maximum reduction of approximately 40% had been achieved in the dead load midspan moments for the large spans. This result would have reduced the total compression stress in the top flange at midspan from 6.43 tons/sq. in. to 3.86 tons/sq. in. and reduced the tension stress in the lower flange from 6.97 tons/sq. in. to 4.18 tons/sq. in. These values are acceptable for wrought iron, and had they been known at the time doubtless would have been considered satisfactory.

The Britannia figures refer to a loaded train weighing 472 tons (1 ton/ft.) over the whole span. In his test load for the Board of Trade, their Inspector, Captain Simmons, used a train weighing 228 tons, which was probably realistic. Stephenson had previously personally boldly tested the bridge by driving a train across weighing 503 tons, carrying 700 passengers and 45 wagons of coal. All went well.

Whatever Robert Stephenson's shortcomings, the Conwy and Britannia bridges were a great triumph for him personally. He had come up with the idea of a tube for bridging these immense distances, and his team had made it a reality. Secondly, in the face of the unexpectedly high stresses in the Britannia tubes, he had generated the idea of using continuity, and again his team had made it a reality.

Mike Chrimes, Librarian of the ICE, sums up the influence that the structures had at the time: (Chrimes, 1991)

"Britannia bridge excited the attention of the world engineering community unlike any other structure of its time. While criticism was levelled at its uneconomic use of material compared with the various truss designs, the intellectual effort involved in its design and construction was, and, remains, a source of wonder. This research formed the basis for a variety of structural engineering developments increasingly remotely removed from the tubular girder itself. Although only a handful of tubular girders were ever built, box girders continue to be used to this day. Following on from the Britannia research came a whole variety of bridges, mostly using plate girders......."

Britannia bridge was opened on 19 October 1850, and survived until partially destroyed by fire on 23 May 1970.

The Torksey Bridge (1849)

This bridge is a wrought iron continuous girder bridge of two spans completed in December 1849 which still exists. Since the Britannia bridge was not completed until October 1950, it was Torksey bridge which was the trailblazer for the introduction of continuity into girder bridges, rather than Britannia. But the immense size and siting of the Menai Straits' bridge had captured the world's attention, and Torksey was overlooked. But it and its sister bridge at Gainsborough (constructed at the same time) were the first bridges in Britain to employ continuity as a means of obtaining greater stiffness and strength for girder spans. Torksey will be described rather than Gainsborough since it is rather better documented.

The bridge carried the Manchester, Sheffield and Lincolnshire Railway's double track Retford to Lincoln line across the river Trent in twin 130 ft. spans. Its engineer was John Fowler, later to become Sir John, and the engineer for the Forth Bridge in 1883. The bridge was completed and ready for opening when it was rejected as weak by Captain J L A Simmons, R.E. of the Railway Commissioners. From the debate which ensued, and which was recorded in several meetings of the I.C.E. to discuss it, it appears that the Commissioners did not fully understand the principle of continuity and its function in reducing midspan bending moments and the associated deflection. There was the further difficulty that the means of calculating the adequacy of a continuous girder were largely unknown.

This ignorance of the value of continuity of a beam over a support seems inexcusable when it must have been appreciated by carpenters and other tradesmen that floor joists in house construction were less liable to "bounce" if made continuous over a support wall i.e. deflection was reduced in the neighbouring spans. The Torksey tubular girders were designed by William Fairbairn for Fowler, and this form of girder soon became widely used, and was patented by Fairbairn. Nevertheless if Fairbairn understood continuity it is a mystery why he did not use the box flange at the bottom of his girder as well as the top, since the bending moment at the support would equal the simply-supported midspan bending moment which the box section was designed to resist.

Eventually after a mathematical tour de force by William Pole, and sustained argument by most of the leading engineers of the day, the Commissioners relented and the bridge was

John Fowler's Torksey tubular box girder bridge

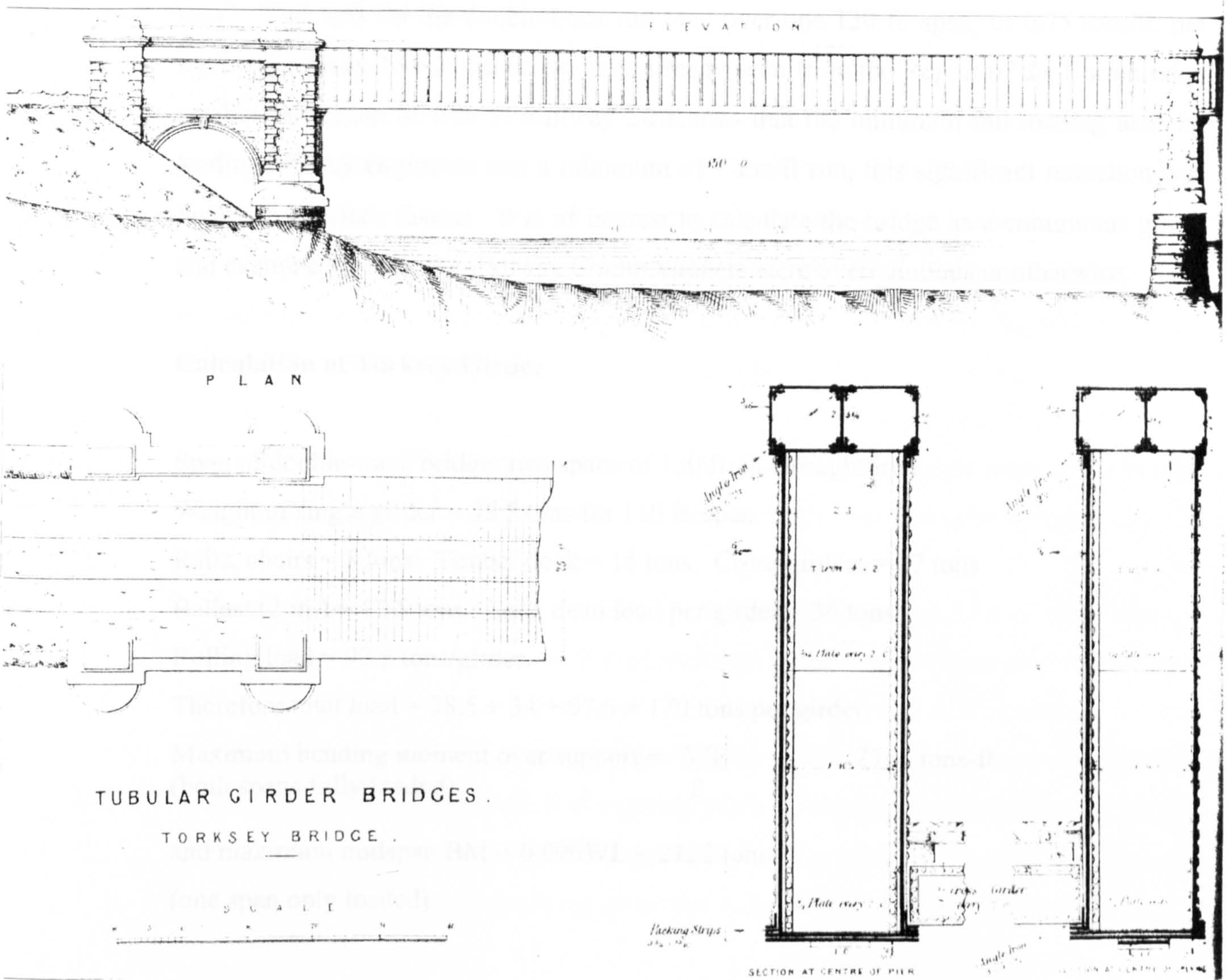


Fig 21 Torksey Bridge. Typical Fairbairn box girder. Two spans of 135 ft. 1850.

opened on 25 April 1850, on condition that the thickness of the track ballast was reduced to 2 inches, a saving in weight of 17.5 tons over the span. There was also the seeming horse-trading by Fowler and Simmons in that "it was agreed that the rolling load be 195 tons". This was for the double-track rail load over one 130 ft. span, or 0.75 tons/ft. per track. (Fairbairn, 1849). Since we have already seen from the Report of the Commission on the application of Iron to Railway Structures that the minimum rail loading used by leading railway engineers was a minimum of 1 ton/ft run, this significant reduction was highly in Fowler's favour. It is of interest to calculate the bridge as a continuous girder and examine whether the Railway Commissioners were over-cautious or otherwise.

Calculation of Torksey Girder

Span of double-track bridge: two spans of 130 ft. in wrought iron.

Weight of single girder = 38.5 tons for 130 ft. span.

Rails, chairs = 8 tons. Timber deck = 15 tons. Cross girders = 27 tons.

Ballast (2 in.) = 17.5 tons. Total dead load per girder = 34 tons

Rolling load = 97.5 tons/girder.

Therefore total load = 38.5 + 34 + 97.5 = 170 tons per girder.

Maximum bending moment over support = $\frac{WL}{8}$ = 2763 tons-ft.
(both spans fully loaded)

and maximum midspan BM = 0.096WL = 2122 tons-ft.
(one span only loaded)

Properties of girder section:

Top flange: 51 sq. in.. Lower flange: 55 sq. in. Web plates: ¼ in. thick

Overall depth = 120 in. Neutral axis 54.6 in. above base and 65.4 in. below top fibre.

Value of moment of inertia about neutral axis = 37.88×10^4 in.^{4}

Modulus of upper flange = 5790 in.^{3} at top fibre.

Modulus of lower flange = 6760 in.^{3} at lowest fibre.

Therefore at support, bending stress = $\frac{M}{Z_u}$ = 4.90 tons/sq. in (compression)

And bending stress = $\frac{M}{Z_l}$ = 5.73 tons/sq. in. (tension)

At midspan, bending stress $= \frac{M}{Z_u} = 4.40$ tons/sq. in (compression)

And bending stress $= \frac{M}{Z_l} = 3.77$ tons/sq. in. (tension)

These stresses are acceptable for wrought iron and only the bending stress in tension over the support could be said to be on the high side. However if the ballast had been the normal 4 in. thickness, and the rail loading had been taken as 1.0 ton/ft. run, this would have resulted in an increase of load per girder of 41 tons or 24%. This would have made the support bending stresses 6.1 tons/sq. (tension) and 7.1 tons/sq. in. (compression) respectively, rather on the high side for wrought iron.

So it seems that the Commissioners were right to be cautious, and no doubt John Fowler (then aged 32) learned not to be too sparing of material in future structural design.

But an important principle on the development of the girder bridge, that of the advantage of continuity over supports, was now tried and established, and has been much used since.

As a footnote, it is said that in erecting the Torksey bridge the two spans were joined on the bank and "rolled out", such that a nosing piece on the girder end was landed on the river pier and the remainder of the bridge hauled into position. The river pier was too narrow to allow of any cranage being employed to assist with lifting the girder end.

If such was the case, the rolling out would entail the minimum of weight, and only the two girders and the deck cross-beams would be riveted up to restrain the compression flanges against buckling.

The girders weighed 38.5 tons each and the deck cross beams 27 tons total.

Therefore weight/girder = 52 tons. Cantilever bending moment = 3380 tons/ft.
Upper flange (tension) elastic modulus = 5790 in.3
Therefore bending stress in upper flange = M/Z = 7.00 tons/sq. in.
Lower flange (compression) elastic modulus = 6760 in.3
Therefore binding stress in lower flange = M/Z = 6.00 tons/sq. in.

These stresses would be temporary and readily acceptable in the circumstances, so erection by rolling-out would be quite feasible. This erection method may also be a "first".

It has often been assumed in the past that Sir William Arrol pioneered the erection method of rolling-out with a girder bridge of seven 120 ft. spans for the railway over the river Clyde at Bothwell in Lanarkshire in 1875. Obviously however, John Fowler was 26 years ahead of Arrol with the erection of Torksey bridge in 1849. Fowler showed himself to be greatly innovative at Torksey with his introduction of continuity and rolling-out.

It has been suggested by some (Horne, 1998) that the continuity obtained in the design of the Torksey bridge was no more than a "happy accident"; that the girders were joined together for the purpose of erection, but left in that condition when the construction was completed and it was realised that there was an advantage in so doing. After all, the bending moment at the joint during erection by rolling-out was greater than the moment at the joint in the permanent condition under railway loading. The writer is disinclined to accept this view, believing that Sir John Fowler did not design his bridges in such a haphazard fashion. The sequence may even have been the other way round - the bridge was designed as continuous, and it was then realised that this feature would lend itself to erection by rolling out. The fact remains that Torksey is the earliest continuous girder in Britain.

Despite these structural advances at Torksey it should be noticed that the bridge was still a sheeted or closed-web bridge in the Conwy tradition. The era of the open-web girder was still to dawn, but it was not far away, and began in Europe with Brunel's remarkable bridge over the Wye at Chepstow in 1852.

CHAPTER 4

PLATE GIRDERS AND BRUNEL'S INFLUENCE

Plate girders

Brunel's plate girders

The Chepstow Bridge, 1852

The Saltash Bridge, 1859

The Pauli and Schwedler Girders

Chapter 4

Plate Girders and Brunel's Influence

Plate Girders

The plate girder made its first appearance with the advent of the first wrought iron bridge. The wrought iron was manufactured mostly in small plates in the early days, 2 ft. wide and perhaps 8 - 10 ft. long. It was riveted into suitable shapes to connect the top and bottom flanges of girders, usually employing angles to join to flange plates, or even large T-sections, with the table of the T forming the flange.

The first wrought iron bridge (1841) was a road bridge, a curious structure carrying a road over the Pollok and Govan railway near the docks at Govan to the south of the Clyde. (Fig 16). It had a span of about 31 ft. and was in reality a box girder bridge, five years ahead of William Fairbairn's patent tubular girders of 1846. The webs of the box were of the "best boiler plate" 3/8 in. thick and 18 in. deep, and the two webs formed a trapezoidal box 6 in. wide at the bottom and 4 in. wide at the top. The 15 ft. wide road was carried on shallow-rise brick jack arches sitting on the lower flanges of four box girders, which were connected by iron ties resisting the outward thrust of the arches.

The box girders were filled with concrete, for an unknown reason. Presumably the concrete filling was placed after the girders had been placed in position, and it must have been a near-impossible job to place the concrete and compact it inside a level box some 35 ft. long. Perhaps the engineer, A Thompson of Govan Iron Works, was nervous about web buckling.

Fairbairn's firm manufactured and erected an immense number of box girder railway bridges before and after 1846. It was said that by 1870 his company had fabricated nearly one-thousand bridges, mainly for railways, with spans from 40 to 300 ft.

Each of those girders needed a web member joining the flanges, and so the plate girder flourished. The need for stiffeners was not appreciated, except at the bearings, and consequently some early plate girders had webs 7 ft. deep, unstiffened.

Brunel's plate girders on his Windsor bridge over the Thames (1849) are typical. The bridge is a tied arch spanning 200 ft. and is still in use today. But Brunel's plate girders had substantial curved-plate compression flanges which exerted a restraining effect on any tendency to buckling.

The plate girder developed into a maid-of-all work widely used down the years to the present day. It was simple to build, economical, and had a reserve of strength, provided it was properly stiffened at intervals along its length, dividing the web into near-square panel sizes. It served the railways well.

Its usefulness began to be more fully appreciated with the completion of the Conwy bridge in 1848 and the Britannia bridge in 1850. Both these bridges had tremendous web members, 25 ft. or more in depth and a mere ½ in. thick. They were stiffened with T-members at 2 ft. intervals, and formed 40% of the weight of the tubular girders.

The big drawback in using the plate girder in the 1840s was that the forces and stresses in web members were unknown, and there was no known means of calculation. The problem was not solved until 1856, when a Russian engineer, D J Jourawski, who was also a practising railway engineer, established a method of calculating the shear stresses in the webs of beams. He pointed out that the most effective position for the stiffeners for the tubular bridges would have been at 45° to the vertical, when half the amount of iron would have sufficed.

However, conventional plate girders down the years have continued to be designed with the stiffeners vertical, because they formed convenient attachment points for the cross-girders of the decking, and could be easily used to transmit the forces evenly to the remainder of the web. The forces in the web can also be easily visualised, but approximately, if the panels between stiffeners are thought of as N-truss panels, with the compression force resisted by the stiffener acting as a strut, and the remainder of the web plating acting in tension diagonally between the top and bottom flanges.

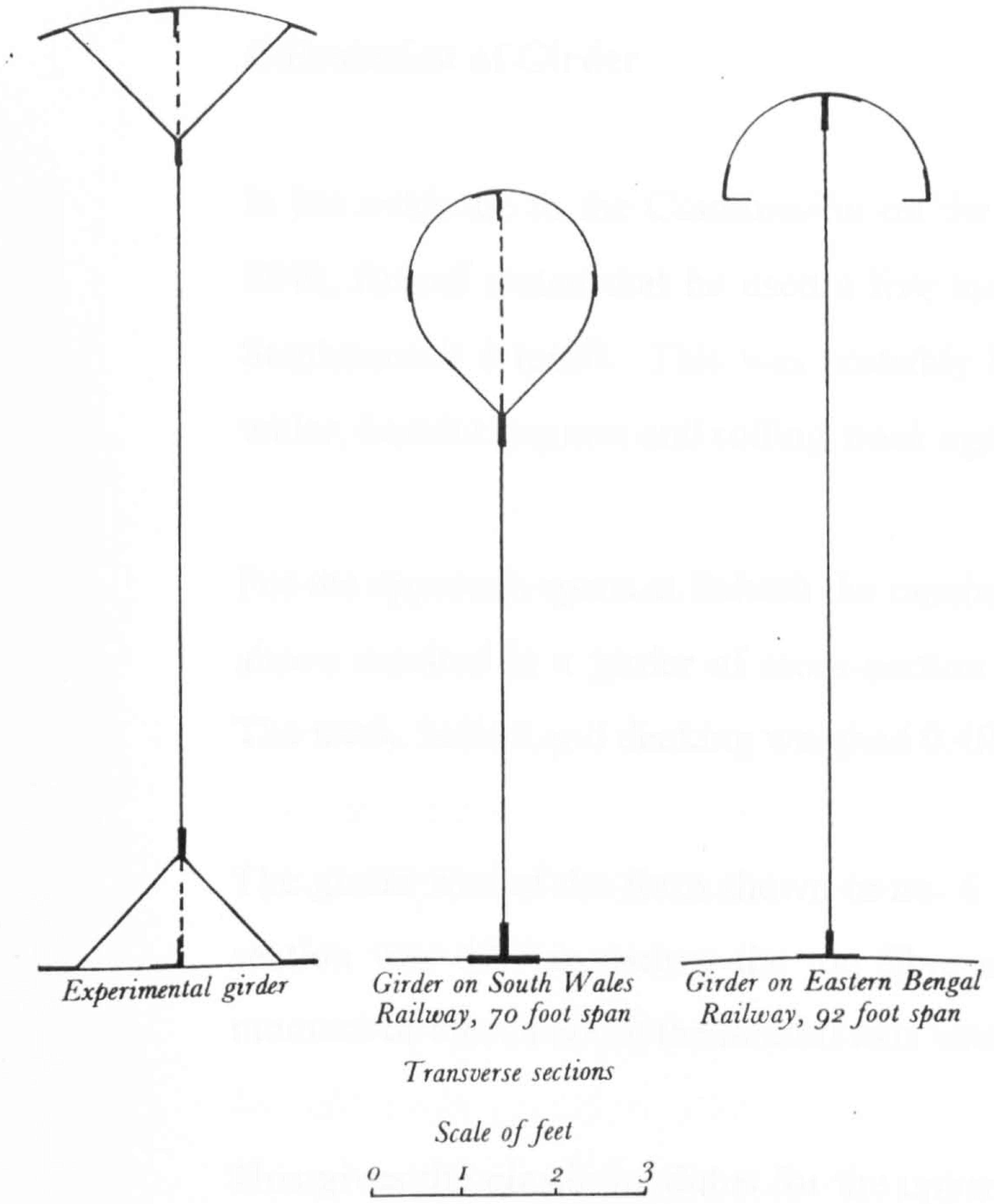
Brunel's Plate Girders

For years a section of an iron plate girder could be seen in the basement area of the Institution of Civil Engineers' headquarters at Great George Street, London, in the open space between the building and the street. The section was of large size, about 6 ft. long and 7 ft. 6 in. high, but there was no inscription of any kind to indicate where it came from. Aficionados of bridge history, however, had no difficulty in identifying the plate girder as a section from the approach spans of Brunel's Chepstow bridge (1852) which were replaced in 1944 - 45. Heavy wartime rail traffic at Chepstow caused damage to the web of one girder, and the approach spans were replaced. The girder was recognisable because it had a triangular box section for its upper flange, with a curved top plate, and also a slightly curved lower flange plate. These girders were continuous over three 100 ft. spans at Chepstow.

The approach spans at Saltash were also replaced in 1928 - 29. The increase in weight from modern locomotives and rolling stock meant that they had become unable to bear the increase in axle loads. The original plate girders differed from Chepstow in that the triangular box section had given way to a simpler design of the top flange, incorporating a single semi-circular plate fastened to the web by angles. The lower flange again was a slightly curved plate, fastened by angles.

John Binding in his book on Saltash bridge gives the scantling sizes for the approach spans, and a calculation has been made to assess the adequacy of the girders. The approach spans were on a curve and were all of different lengths, the largest span being 93 ft. and the smallest 69 ft. 6 in. (Binding, 1997).

The girder was 8 ft. deep overall, and the web thickness was ¾ in. The semi-circular top flange was of ½ in. thick plate, on a 12 in. radius, and was attached to the web plate by two 3 ½ x 3 ½ x 3/8 in. angles. The lower flange was 3 ft. wide, ½ in. thick and similarly attached. It is known that Brunel carried out experiments on plate girders in refining his designs. (Fig 24).



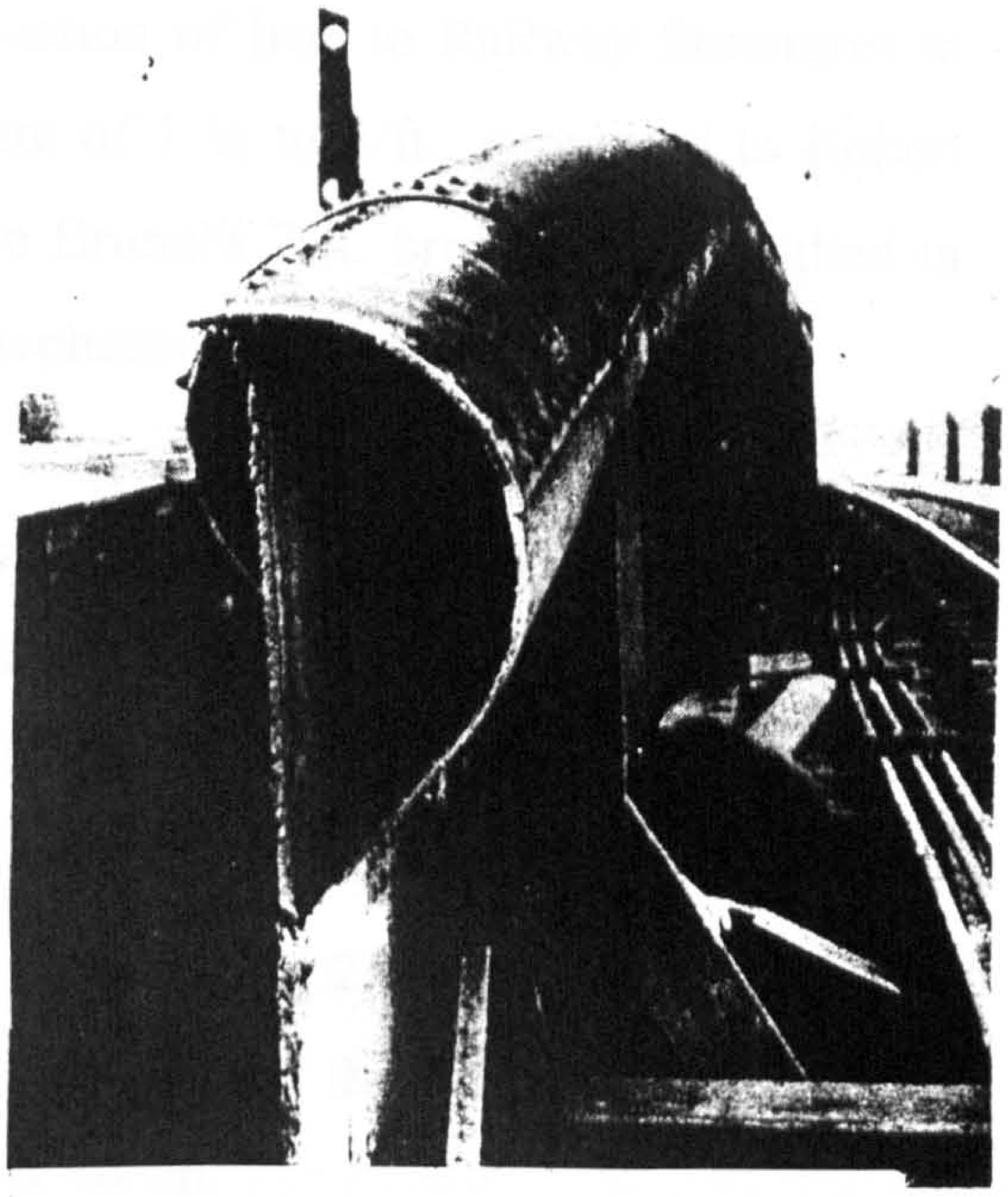


Fig 25 Balloon Flange of 1848 At Over

Fig. 1 Types of wrought-iron girder used by Brunel. Cross Sections not drawn to scale: dotted lines represent cross diaphragms.

1 Experimental
2. Cumberland Basin Bridge, Bristol
3 and 4. Two types used on the South Wales Railway
5. Windsor railway bridge
6. East Bengal Railway bridge design
7. Balmoral Bridge

Fig 24 Brunel's Plate girders
Note No 7 - Balmoral bridge.
(Buchanan & Jones, 1980)

Calculation of Girder

In his evidence to the Commission on the Application of Iron to Railway Structures in 1849, Brunel stated that he used a live load figure of 1 ½ tons/ft., compared to Robert Stephenson's 1 ton/ft. This was probably because Brunel's 7 ft. broad gauge resulted in wider, heavier engines and rolling stock against Stephenson's 4 ft. 8 ½ in. gauge.

For the approach spans at Saltash the maximum span was 93 ft., and the dimensions given above resulted in a girder of cross-section of 119 sq. in. of iron, weighing 0.18 tons/ft. The track, ballast and decking weighed 0.40 tons/ft.

The girder was of the form shown in no. 6 of the group in Fig 24, the neutral axis of the section was 47.7 in. below the top fibre and 48.3 in. above the bottom fibre, and the moment of inertia about the neutral axis was 15.43×10^4 $in.^4$.

This gives the elastic modulus for the upper flange as 3235 $in.^3$ and for the lower flange, 3195 $in.^3$. The remarkable similarity was due to the neutral axis being practically at the midheight of the girder, and Brunel must have planned it that way to give near-equal stresses in bending tension and compression.

The bending moment at midspan for the loadings given above was 1513 tons-ft. giving a bending stress in the upper flange of 5.61 tons/sq. in. compression and a bending stress in the lower flange of 5.68 tons/sq. in. tension.

The shear stress in the web was low at 0.90 tons/sq. in. Photographs show that the webs of the girders were well stiffened throughout their length. Although the bending stresses were on the high side, the train loading (which formed more than 50% of the total) was probably high at 1 ½ tons/ft. It would be reduced somewhat when the broad gauge reverted to the Stephenson gauge in 1892.

Brunel also designed a remarkable plate girder road bridge at Balmoral for Queen Victoria in 1857. (Fig 24). The bridge is unmistakably Brunel with its curved top flange, and has decorative diamond-shaped perforations in the web, which is reinforced with iron straps around the perforations. The bridge was 3 years in gestation, which did not please the Queen. She and Prince Albert considered it too plain. Brunel replied that he favoured a design "of perfect simplicity" making sure that there was no offensive or unsightly ornament. The bridge spans 129 ft. and carries a road 13 ft. wide, and the girders have a mere 4 stiffeners giving a panel length of 26 ft. On the whole it is a handsome bridge. (Buchanan & Jones, 1980).

As well as the designs described above, Brunel in his early days also produced plate girders having a circular, tube-like top flange. These seemed to belong to a family of early railway bridges with "balloon" top flanges and were often part of hog-backed plate girder designs. (Fig 25).

The last of his plate girders were for the Saltash bridge of 1859, the year of his death, and were all of the type with the semi-circular top flange calculated above.

It is interesting to consider how Brunel may have calculated the girders. The Hodgkinson formula, widely used for cast-iron beams, would seem too rudimentary and approximate. Brunel was educated in France and could speak fluent French, and it seems possible that he used Navier's method from his "Lecons" published in 1826. In 1826 Brunel was aged 20 and at the start of his engineering career, and Navier's work would be well known to him.

The Chepstow Bridge

It would not be going too far to describe this bridge as the first open-webbed truss girder in Europe. It was a landmark in the development of bridge design, providing a link from the past to the open webbed girder of the future which were to dominate all bridgework for larger spans down to the present day. (Fig 26).

Some engineering writers, notably Berridge, describe the Chepstow bridge as a suspension bridge, claiming it was one of only two suspension bridges surviving on Britain's railways. (The other is Saltash). However, in a suspension bridge the roadway is suspended from chains or cables which are free to move or oscillate vertically or horizontally. There is no other means of support for the roadway apart from the chains.

At Chepstow the chains form tension members of a truss structure, and the bridge deck is attached to A-frames which are rigidly connected to the plate girders (lower chord) supporting the deck, and to the 9 ft. diameter wrought iron tube which forms the compression member or top chord. The tube is a rigid member, and the plate girders are rigid members between the abutments and the A-frames. The truss bridge at Chepstow spans 300 ft. and the plate girder deck is formed from three members made continuous at their joints.

The Approach Spans

The three approach spans are plate girders each of 100 ft. and they too are continuous over the supports. The proportion of the live load to dead load was just over 50% and there was considerable advantage to be had in making the girders continuous for live load simply by joining their ends, without attempting to produce continuity for dead load conditions by raising their ends and then joining up the girders, as at Britannia. At Britannia bridge the proportion of live load to dead load was much less, at about 28%, and the second operation was necessary. There is no account in the contemporary records of any attempt to produce continuity for dead loads at Chepstow, so it has been assumed to apply only to the live loading.

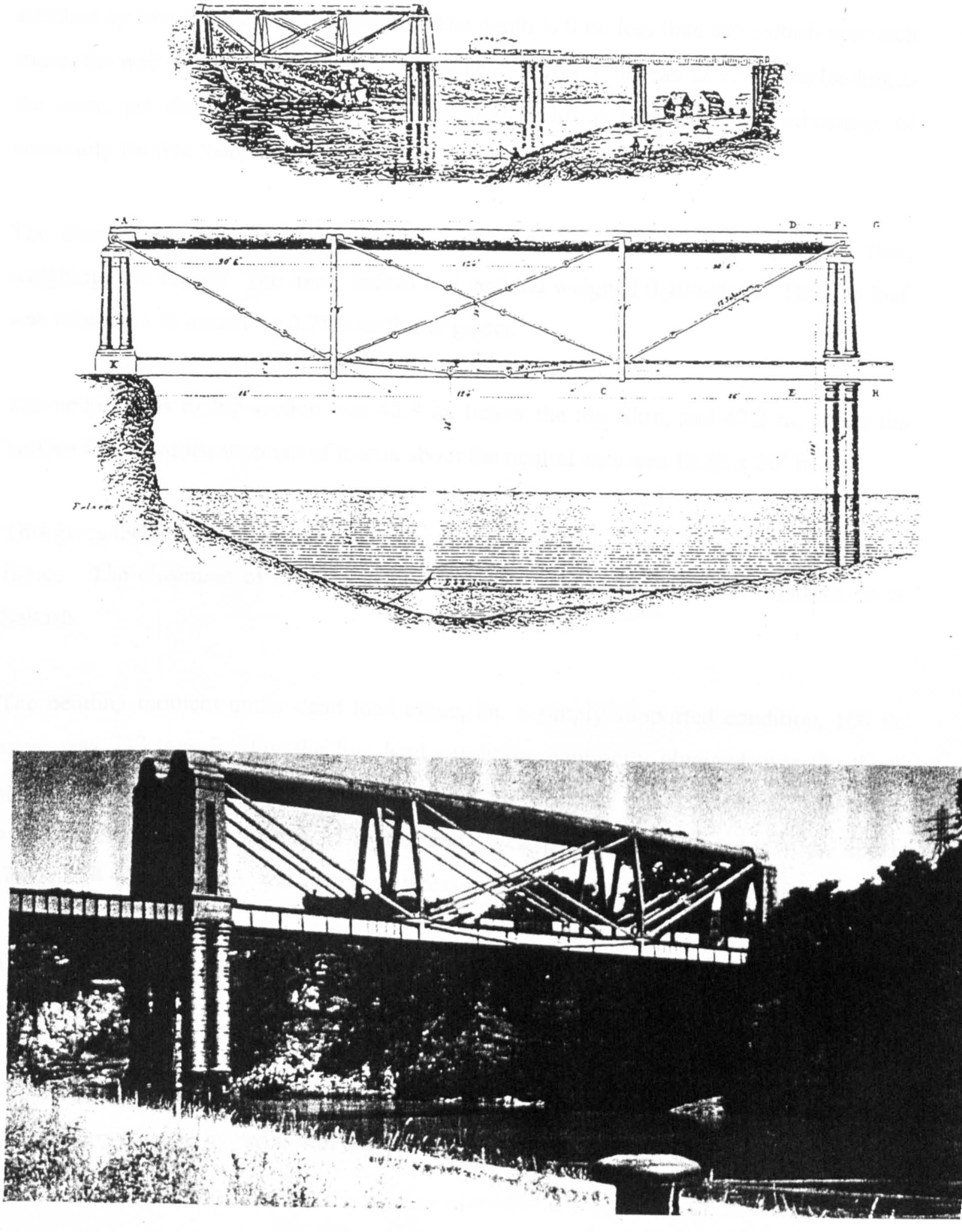


Fig 26 Chepstow Bridge - 300 ft. main span.

Approach Spans – Calculation

The girder was 7 ft. 6 in. deep overall, and the web thickness was 5/8 in. The curved top flange was of ½ in. plate, 3 ft. wide, and was attached to the web by two sloping flat plates about 17 in. long and ¼ in. thick. The lower flange was 2 ft. 6 in. wide and attached by two 3 ½ in. x 3/8 in. angles. The depth is 6 in. less than the Saltash approach spans, the web is 5/8 in. instead of ¾ in., the span is 100 ft. instead of 93 ft., the loading is the same, yet the girder is less highly stressed. This is because of the advantage of continuity for live load conditions. (Fig 27).

The dimensions given above result in a girder of cross-section 103 sq. in. of iron, weighing 0.16 tons/ft. The track, ballast and decking weighed 0.40 tons/ft. The live load was taken as 1 ½ tons/ft. or 0.75 tons/ft. per girder.

The neutral axis of the section was 42.8 in. below the top fibre, and 47.2 in. above the bottom fibre, and the moment of inertia about the neutral axis was 15.82 x 10^4 $in.^4$.

This gives the elastic modulus of the upper flange as 3696 $in.^3$, and 3352 $in.^3$ for the lower flange. The closeness of the two values gives near equal stresses in the flanges, as at Saltash.

The bending moment under dead load alone, i.e. a simply supported condition, 100 ft. span, was 737 tons-ft. For the live load condition, continuity obtained over the three spans, and the maximum moment, at midspan in an end span, was 631 tons/ft. This gives a total moment of 1368 tons-ft. This moment gives rise to a bending stress in the upper flange of 4.44 tons/sq. in. and a bending stress in the lower flange of 4.90 tons/sq. in.

The shear stress in the web at an intermediate support was 1.30 tons/sq. in., slightly higher than it would have been without continuity, but not a high stress.

Those stresses show Brunel's choice of sections resulted in moderate stresses for these plate girders assisted by the skilful use of continuity.

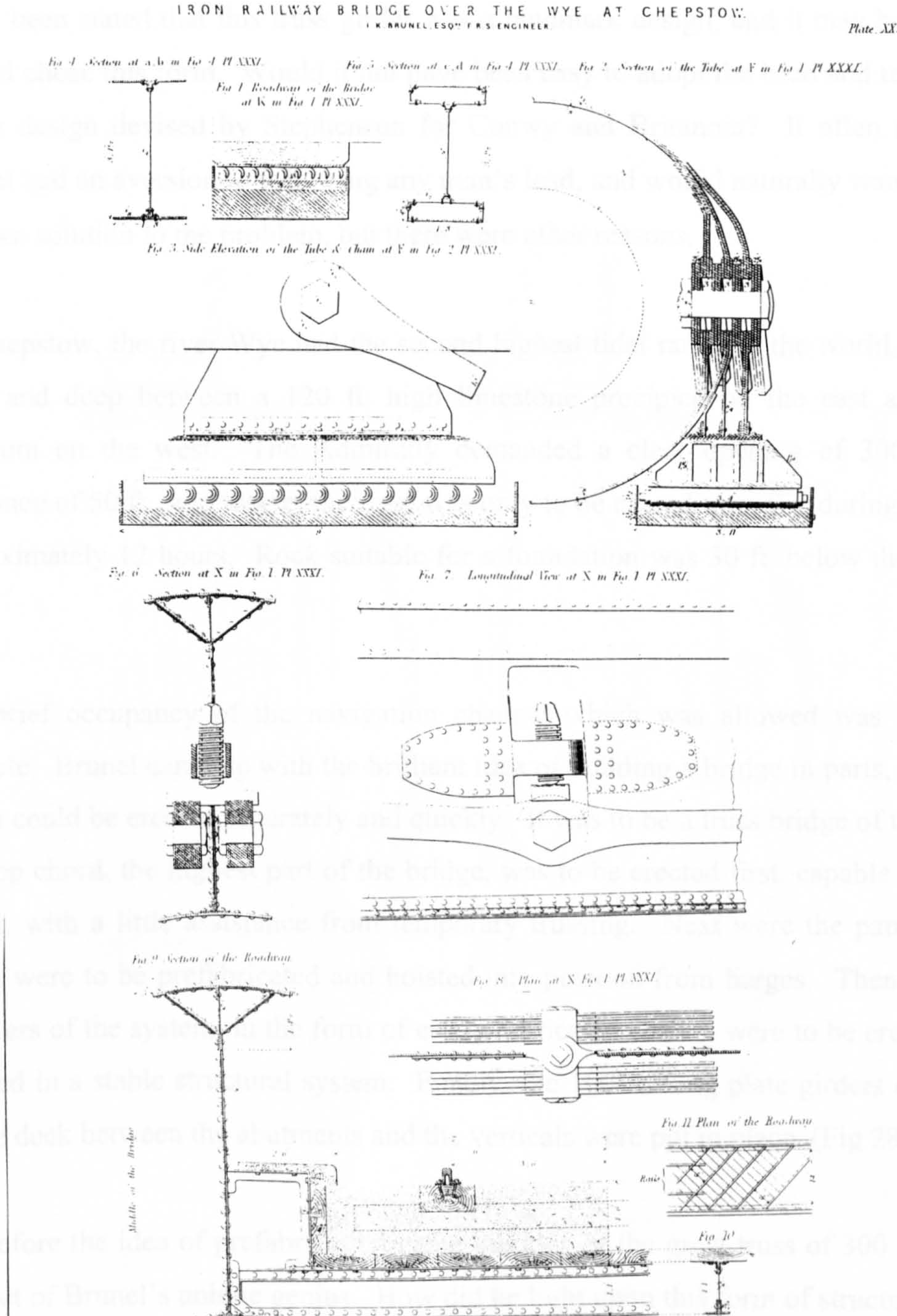


Fig 27 Chepstow Bridge - details.
(Binding, 1997)

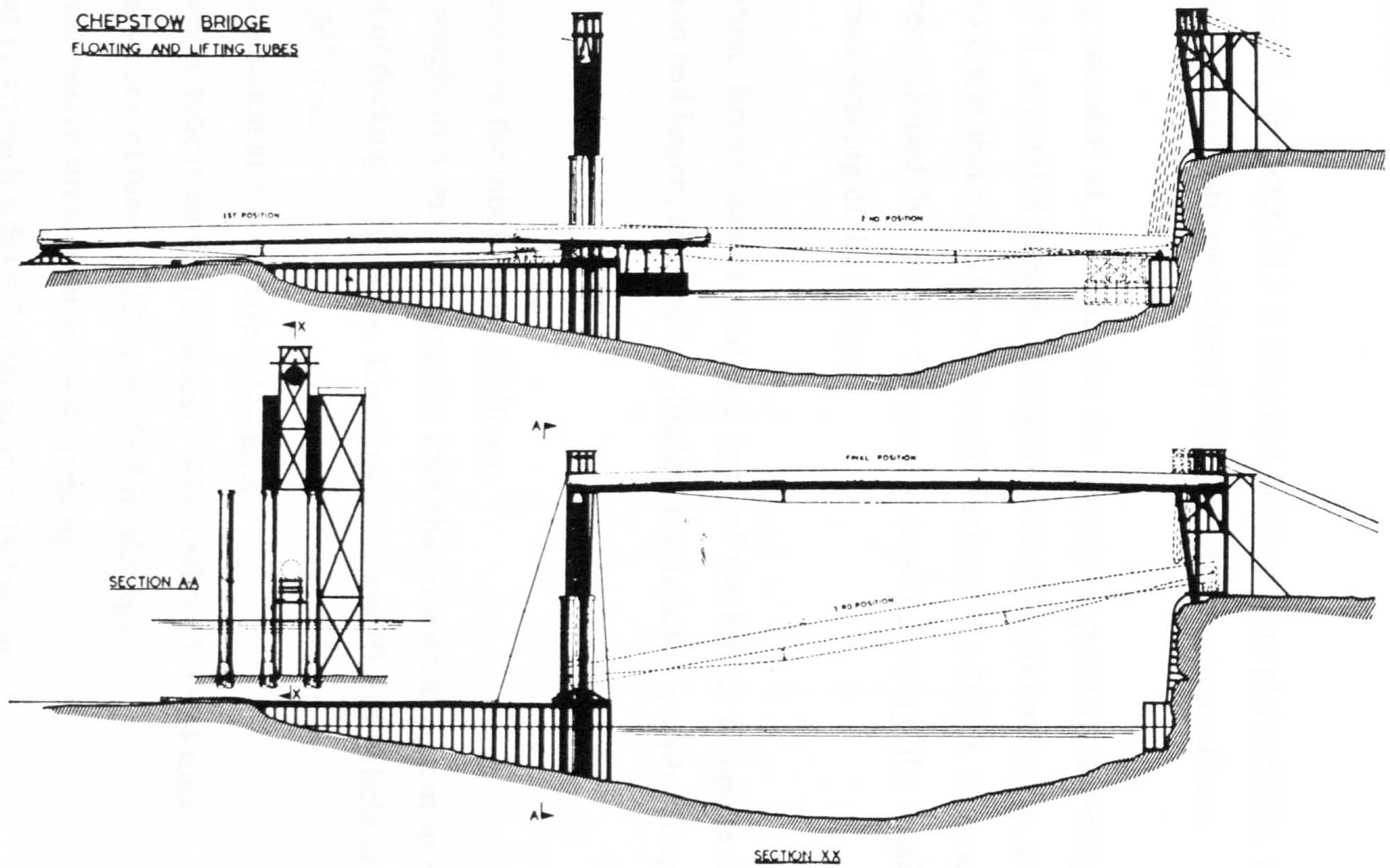
The Main Span of 300 ft.

It has been stated that this truss girder was a landmark design, and it may be asked why Brunel chose this form. Would it not have been easy to adopt the tried and tested tubular girder design devised by Stephenson for Conwy and Britannia? It often appears that Brunel had an aversion to following any man's lead, and would naturally want to develop his own solution to the problem, but there were other reasons.

At Chepstow, the river Wye had the second highest tidal range in the world, and flowed wide and deep between a 120 ft. high limestone precipice on the east and shelving alluvium on the west. The Admiralty demanded a clear opening of 300 ft. with a clearance of 50 ft., and worse, the river was only to be closed to traffic during one tide, or approximately 12 hours. Rock suitable for a foundation was 30 ft. below the bed of the river.

The brief occupancy of the navigation channel which was allowed was the greatest obstacle. Brunel came up with the brilliant idea of building a bridge in parts, each part of which could be erected separately and quickly. It was to be a truss bridge of three panels. The top chord, the highest part of the bridge, was to be erected first, capable of spanning 300 ft. with a little assistance from temporary trussing. Next were the panel verticals, which were to be prefabricated and hoisted into position from barges. Then the tension members of the system, in the form of easily fabricated chains, were to be erected, which resulted in a stable structural system. Finally, the 100 ft. long plate girders carrying the bridge deck between the abutments and the verticals were put in place. (Fig 28).

But before the idea of prefabrication came the idea of the great truss of 300 ft. span, the product of Brunel's unique genius. How did he light upon this form of structure? Had he seen or heard of the Pratt brothers' patent of 1844, five years before, a fraction of the size required at Chepstow? Or was it a flash of light, the culmination of months, even years, of reflective thought? Certainly, like the Stephenson tubes, nothing like it had been seen before, and it was another step forward in the development of girder bridge design.



Brunel's method of erecting the 300-ft long tubes used in the "river" spans at Chepstow.

Fig 28 Chepstow Bridge - Method of erection.
(Berridge, 1969)

Calculation of Main Span

The bridge was to carry a double line of railway, and to minimise the weight of bridge parts Brunel decided on a separate bridge for each line of track. The first bridge, the Down line, was completed first. The 9 ft. tubular top chord was launched across the river and raised clear of shipping in a single day on 8 April 1852, and the line was opened on 14 July. (Fig 28).

The tube was 9 ft. diameter, 312 ft. long and built up from plates 5/8 in. thick. It weighed 161 tons, and had an elastic modulus of 5616 $in.^3$ at the extreme fibres.

The bending moment at midspan for the simply supported self-weight of the tube, spanning 300 ft., was 6038 tons-ft., giving stresses of 12.90 tons/sq. in. at the outer fibres. There was no doubt that the tube was insufficient to span 300 ft. without assistance, and this was accomplished by minor temporary trussing using the chains forming the permanent cross-bracing of the bridge.

As a precaution, Brunel had erected one span of the bridge complete on the riverbank before erection had begun, and had test-loaded it satisfactorily with 770 tons, or 2 ½ tons-ft. run.

The final stresses in the tubes were as follows:
Weight of wrought iron, etc, allowing for load shared with abutment in end spans = 427 tons. Weight of decking = 160 tons. Live load (1 ½ tons/ft.) for whole bridge = 300 tons. Total load = 887 tons.
Total vertical reaction at end of tube = 444 tons.
Therefore force in tube (triangle of forces) = 444 x 90.5/43 = 934 tons.
Area of cross-section of tube = 212 sq. in. (5/8 in plating)
Therefore compressive stress in tube = 4.41 tons/sq. in.
Area of chains (1 set each side) = 2 x 96 sq. in. = 192 sq. in.
Force in chains = 444 x 100.2/43 = 1034 tons.
Therefore tensile stress in chains = 5.39 tons/sq. in.

These stresses, which are based on a Brunel live load of 1 ½ tons/ft. are compatible for wrought iron. The tubes were well restrained by being linked together and braced by the

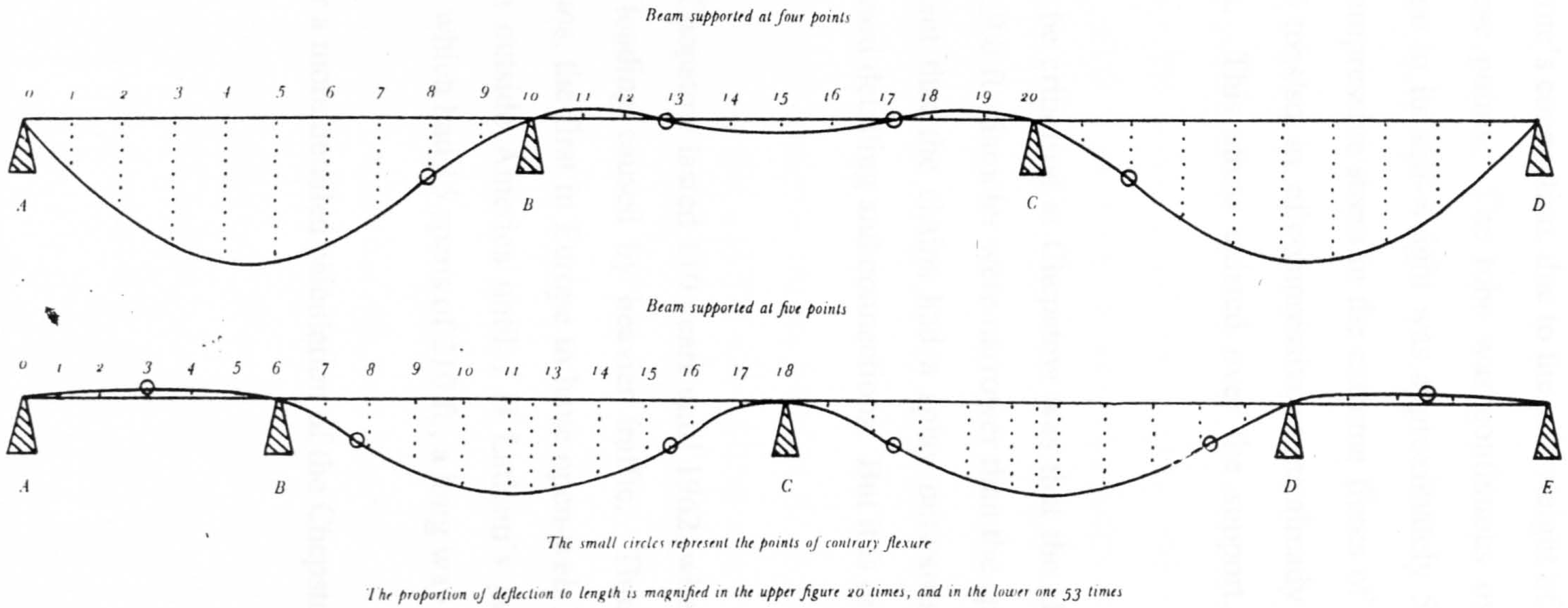


Fig 29 Brunel's experimental work on continuous girders.
(Pugsley, 1976)

vertical A-frames, and the stress in compression is not high at 4.41 tons/sq. in. The l/k ratio was approximately 23.

In addition to the stresses in the tubes calculated above, there was also a small local stress at the vertical A-frame's connection, due to the self-weight of the tube causing a tendency to sag between these points. The tube was continuous over the connection and the bending moment due to its self-weight was approximately 540 tons-ft. This caused an equal tensile and compressive stress in the extreme fibres of 1.15 tons/sq. in. This must be added to the 4.44 tons/sq. in. of compressive stress already calculated, and gives a total of 5.59 tons/sq. in. This stress existed over the support, where the tube was well restrained.

A detail that could be criticised at Chepstow was that the chains were not in a vertical plane. The tubes at 9.0 ft. diameter were narrower than the spacing of the track girders at 15.0 ft., which meant that the chains had a splay outwards from the top downwards, causing some awkward detailing and connections. But it is difficult to see how this could have been avoided.

The main spans at Chepstow lasted 110 years until 1962, when they were replaced owing to the increase in loading caused by heavier traffic. Thus passed into history these remarkable iron spans, the first in Europe to have open-web N-truss panels. There were none others in iron outside America until J H Latham's railway bridge at Allahabad, India, in 1859 – 65, which had 15 spans of 210 ft., a long way short of Chepstow.

See Appendix A for a more detailed calculation of the Chepstow main span.

The Saltash Bridge of 1859

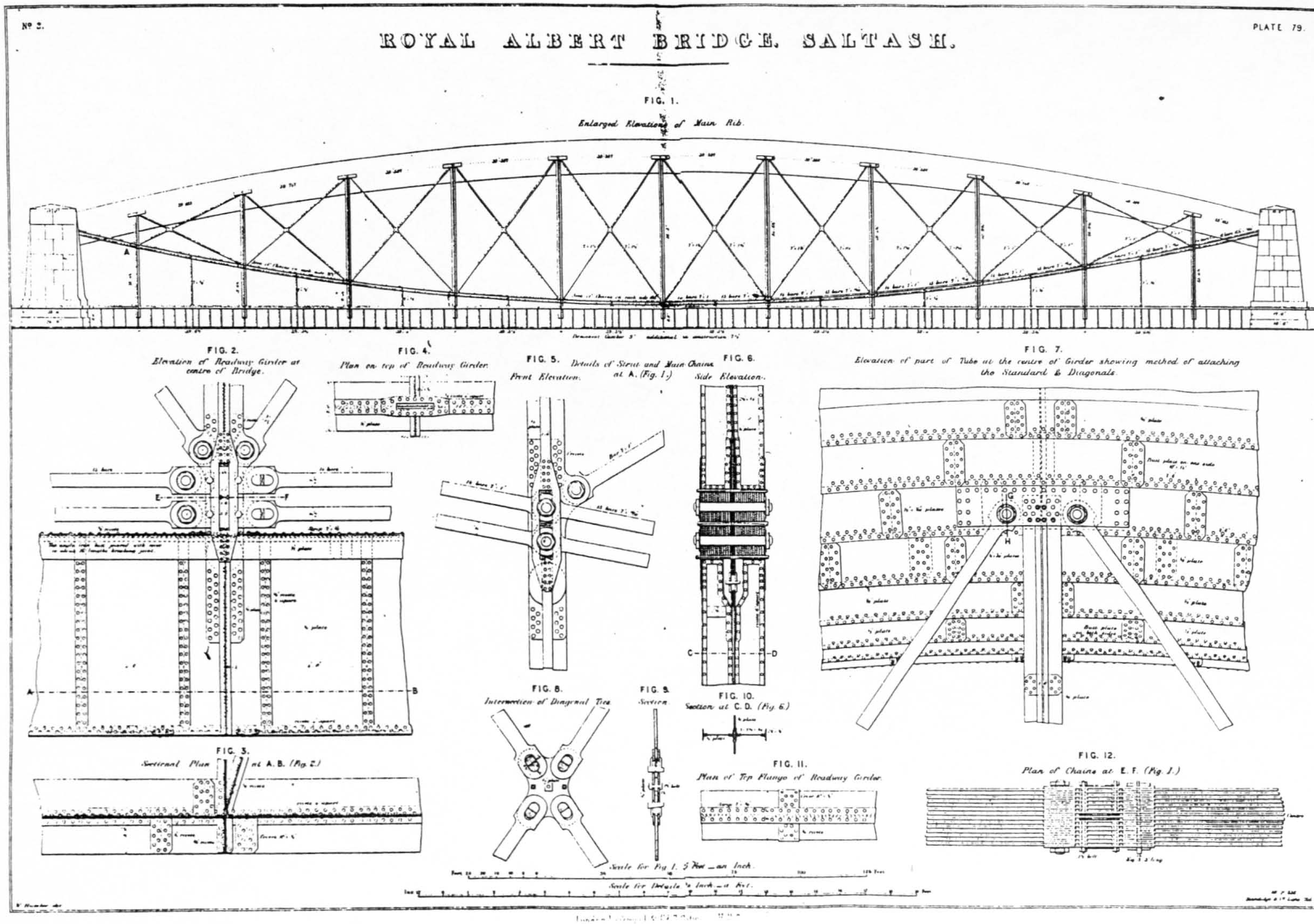
Brunel's last and greatest work was his wrought iron railway bridge across the river Tamar at Saltash for the Cornwall Railway. Since the history and construction of this epic work are well known, it is proposed only to assess whether it advanced the development of the girder bridge, and to calculate in a rudimentary way the basic structure of the main spans, each of 455 ft. (Fig 30).

Almost contemporary with Brunel was the work of F A Pauli (1802-83) who developed the lenticular truss, mainly to be found bridging the rivers Rhine and Danube in north Germany. This truss had curved chords, symmetrical about the neutral axis, which ran horizontally at the centre of the girder between the points of support. The chords were strongly braced apart in the web system, which was generally of Pratt-truss or Warren-truss form. The most important of the Pauli lenticular trusses was that over the Rhine at Mayence, built 1860-62, and much smaller in scale than Saltash.

Saltash differed from the lenticular trusses because its chords (the arched tube and the chains) resisted the shear forces, and the web system was non-existent apart from light bracing to assist in distributing unequal loading between panel points.

The structural system at Saltash consisted of the arch acting in compression and the chains in tension, the latter resisting the outward thrust of the former. They were braced apart by eleven vertical members at approximately 39 ft. centres, which Brunel called "standards". The standards transmitted the load from the bridge deck to both chains and tube, hopefully 50% to each. Since it was inescapable that the thrust of the arch had to be equalled by the pull of the chains, any difference between the two (arising, say, from non-symmetrical loading) had to be evened out by the action of the standards, which were of cruciform cross-section, designed to act as struts.

The space between the standards was bridged at deck level by plate girders 8 ft. deep, and of the same form as the approach spans. Each standard was fixed to the plate girders at its base and to the tube at its top, and to the chains where it intersected them between these points.



5.7 *Details of construction from drawings by William Humber c1864.*

Fig 30 Saltash Bridge - Details of Main Spans

(Binding 1997)

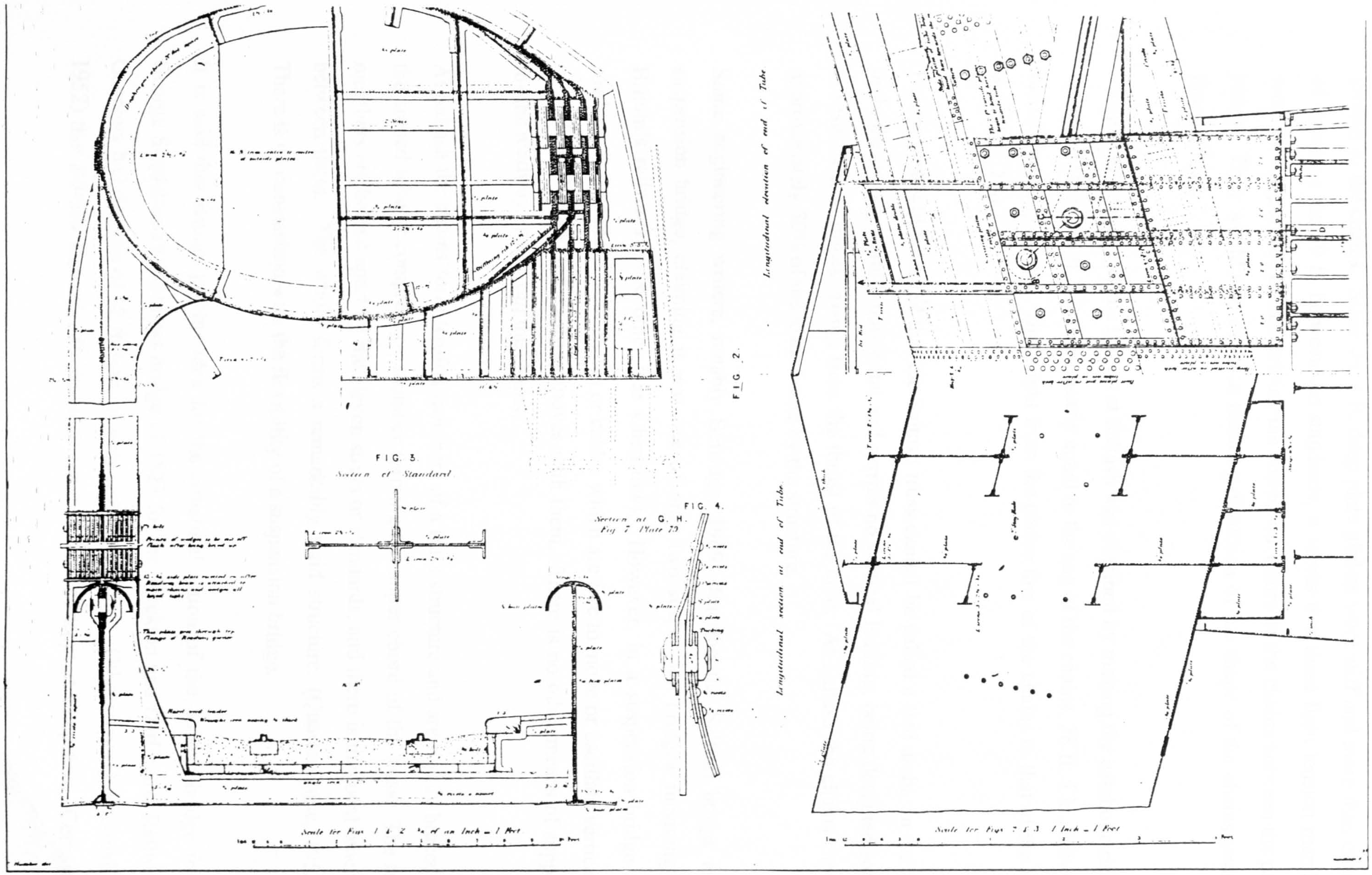


Fig 31 Saltash Bridge - Main span cross-section
(Binding, 1997)

Between the standards, equidistant from each, light tension members connected the plate girders to the chains. Since the 8 ft. deep plate girders were stiff and more than capable of spanning the 39 ft. between the standards, it seems that these light tension members were for the purpose of maintaining the correct curvature of the chains between the panel points. They would also assist in resisting distortion of the shape of the chains under a moving load.

The graceful outline of the trusses at Saltash was obtained by making the parabolic rise of the arched tube, 28 ft. 9 in. very nearly equal to the sag of the chains, 28 ft. 5 in., between their connecting points. The height from the centre line of the chains to that of the arch was 57 ft. 2 in.

Lastly, it should be noted that the Saltash truss cannot be called a tied arch. In the tied arch, the tie is generally horizontal and carries no vertical loading, being designed solely to resist tensile forces arising from the thrust of the arch. At Saltash the chains carried approximately 50% of the loads borne by the structure.

Some engineering writers, notably Berridge, 1969, describe the Saltash bridge as a suspension bridge, claiming it was one of only two suspension bridges surviving on Britain's railways. (The other was Chepstow). However, in a suspension bridge the roadway is suspended from chains or cables which are free to move or oscillate vertically or horizontally, and the roadway moves with them. There is no other means of support for the roadway apart from the chains.

At Saltash the chains form tension members of a truss structure and are rigidly braced to the arched tubular compression member forming the upper chord of the truss. The two members are braced apart by the eleven struts or standards, and there is diagonal bracing between them. The whole forms a remarkably rigid structure. (Quartermaine, 1952). There is no comparison with the flexibility of a suspension bridge.

It is said that Brunel got his idea for the structural action of the Saltash bridge from George Stephenson's Gaunless bridge of 1823 for the Stockton and Darlington Railway. Gaunless had 4 spans of 12 ft. 6 in. of very similar outline. Others say (Quartermaine, 1952) that Saltash was a development of the structural concept at Chepstow. Certainly

Fig 33 Auchindrean Bridge - Nr. Ullapool, Wester Ross.

Saltash is less advanced structurally than Brunel's 300 ft. span at Chepstow, but it was still an open-webbed girder and a truss rather than a tied arch.

Similar bridges to Saltash are difficult to find, and there is only one in existence known to the writer. It is at Auchindrean near Ullapool, Wester Ross, on the northwest coast of Scotland, over the river Broom, near its entry to Loch Broom. (Fig 33). A design of the same type was planned for the Metropolitan Railway at Farringdon Road, London, but never built.

Achindrean bridge carries a farm road and has a span of 102 ft. 6 in., and a width of 9 ft. Its timber deck is suitable only for light vehicular traffic, and it was probably built between 1867 and 1875. The ironwork bears no manufacturer's name, and the contractor is unknown. The engineer was probably Sir John Fowler, who had extensive estates at Braemore, nearby, and was also the engineer to the Metropolitan Railway. The tension member in the bridge is reduced to twin flat plates, curved but straight between panel points, and riveted at the joints. Thus the disadvantages of a chain are avoided.

Calculation of Saltash Main Span

Sir Alfred Pugsley's book on Brunel has a chapter on the Saltash bridge written by Sir H Shirley-Smith, a past-president of the Institution of Civil Engineers (1968). No calculations are given for the structure but Smith's account includes the questionable statement:

"As soon as the fabrication of the first span was completed, the span was freely supported at the ends and test-loaded with 1190 tons uniformly distributed throughout its length (2.62 tons/ft). The deflection at the centre of the span, due to this load in addition to the weight of the truss, amounted to 5 inches, and the maximum stress in the tube and chains was about 10 tons per square inch. As those results were considered very satisfactory, preparation went ahead at once for floating out the span...."

Smith must have guessed the figure of 10 tons/sq. in., as the calculated stresses shown on the following pages for the given loading are 6.11 tons/sq. in. compressive stress for the tube, and a corresponding tension stress in the chains of 5.83 tons/sq. in. These figures were checked using two different methods of calculation. The corresponding deflection at midspan, using a value of Young's modulus for the wrought iron of 11,000 tons/sq. in. is 5.13 in., which agrees with Smith's figure of 5 inches quoted above.

Precise information on the Saltash arched tube is difficult to trace, but Binding's book is helpful. The tube was elliptical in cross-section, 16 ft. 9 in. wide x 12 ft. 3 in. high, and mainly of 5/8 in. thickness, though there were places where ½ in. and ¾ in. thick plates were also used. The chains were in two tiers, of 14 links 7 in. deep x 1 in. thick, and 15 links 7 in. deep and 15/16 in. thick. (Fig 31) (Binding, 1997).

Area of cross-section of tube (annular ellipse) = 340 sq. in.
Add 10% for longitudinal laps in the platings = 374 sq. in. Weight = 0.57 ton/ft.
The tube was parabolic and 468 ft. long, strengthened by both longitudinal and annular stiffeners and by extra plating at the ends where the chains were anchored. The weight of the tube alone was therefore approximately 300 tons.

The chains were of similar length and had a cross-section of 196 sq. in. There were two sets, totalling 392 sq. in. weighing approximately 310 tons.

The plate girders carrying the deck weighed 0.18 tons/ft., or a total of 170 tons. The deck beams spanning 16 ft. 6 in. weighed 60 tons. The decking without the track weighed 170 tons, the standards 45 tons and the light bracing 19 tons.

The above weights total 1069 tons which agrees with the figure given of 1060 tons of wrought iron to be compared with the 1587 tons of a Britannia tube of 460 ft. span.

If we consider Brunel's live loading to be 1.50 tons/ft. (the broad gauge of 7 ft. gave rise to heavier loading than Stephenson's 4 ft. 8 ½ in. gauge, taken as 1.0 ton/ft.) then the total live load was 690 tons.

Thus the combined dead and live loading was (1060 + 690) = 1750 tons

Thus the combined load bending moment:

$$M = \frac{WL}{8} = \frac{1750x460}{8} = 100{,}625 \text{ tons-ft.}$$

Consider this to be resisted by a couple formed by the forces in the tube and chains, 57 ft. apart:

Therefore forces in tube and chains = $\frac{M}{d}$ = 1797 tons.

Therefore stress in tube = $\frac{F}{A} = \frac{1797}{374}$ = 4.80 tons/sq. in. compression

Stress in chains = $\frac{F}{A} = \frac{1797}{392}$ = 4.58 tons/sq. in. tension.

If the live load was reduced to 1.0 ton/ft. run, then the total loading would be (1060 + 460) = 1520 tons, and the midspan bending moment would be 87,400 tons-ft.

Therefore this would give a corresponding stress of 4.17 tons/sq. in. compression in the tube, and 3.98 tons/sq. in. tension in the chains.

When the bridge was tested by the Board of Trade, the presiding officer, Colonel Yolland estimated the stress on the tube as "would not exceed 4.20 tons per square inch", which is remarkably close to the above figure of 4.17 tons/sq. in.

If a more refined method of calculation is employed, using the value of the moment of inertia of the section about the neutral axis, the following figures are appropriate:

Taking the effective area of the tube cross-section as 374 sq. in., the neutral axis is located 328 in. above the centre line of the chains, and 344 in. below the centre line of the tube at midspan.

Therefore moment of inertia about the neutral axis = 8740 x 10^4 in.^{4}
Modulus of elasticity of tube = 25.40 x 10^4 in.^{3} at centre.
Modulus of elasticity of chains = 26.65 x 10^4 in.^{3} at centre.

Therefore compression stress in tube = $\frac{M}{Z_u}$ = 4.13 tons/sq. in.

Tensile stress in chain = $\frac{M}{Z_L}$ = 3.94 tons/sq. in.

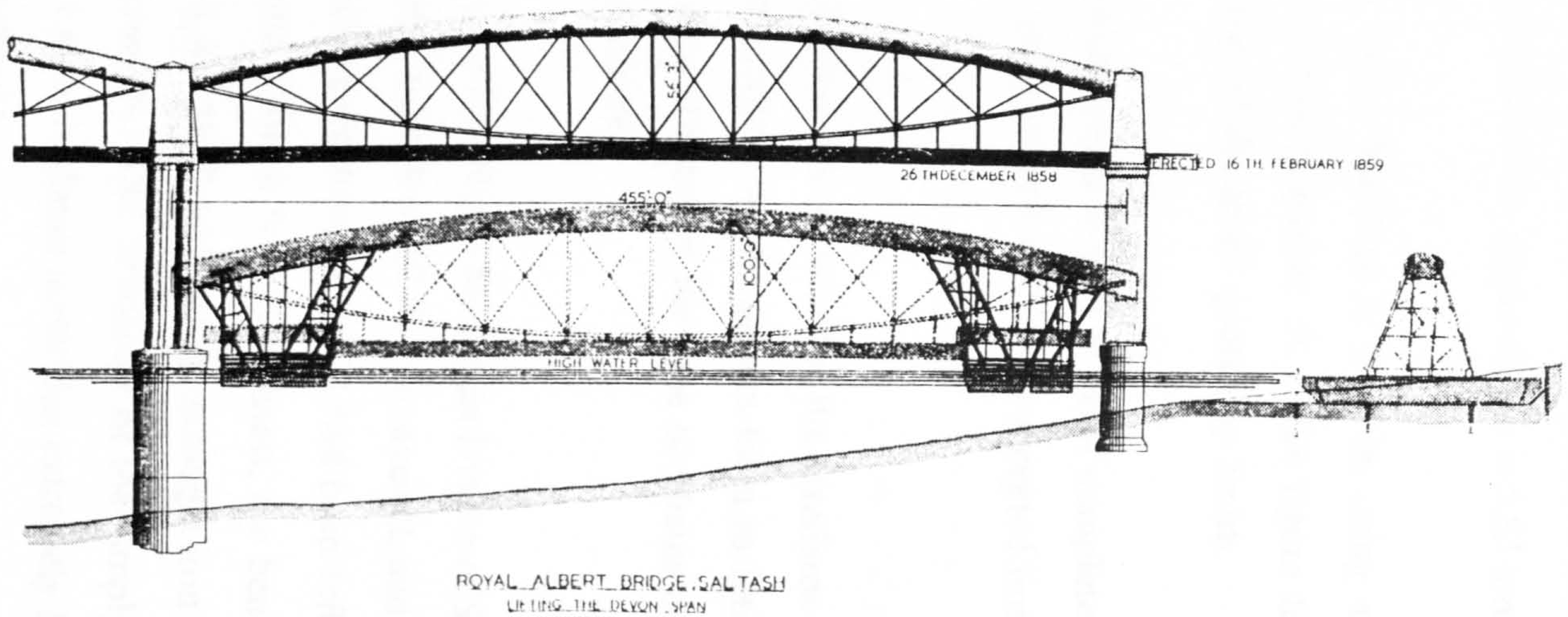


Fig 32A Saltash Bridge erection method. The prefabricated spans were floated out and jacked up into their final position. (Berridge, 1969)

The difference between these stresses and the same stresses calculated by the simpler method above is negligible; the two methods are equally exact (or equally approximate).

Lastly, under the test load of 1190 tons quoted by Smith, the total bending moment at midspan is 129,375 tons-ft, and the compressive stress in the tube is 6.11 tons/sq. in. The corresponding tension stress in the chains is 5.83 tons/sq. in.

The deflection under this load is 5.13 in. using a value for the Young's modulus of wrought iron as 11,000 tons/sq. in. This figure for the deflection agrees with the "5 inches at the centre of the span" quoted by Smith.

Thus this remarkable truss is seen to be of completely adequate and indeed conservative design, and the stresses well within the accepted limits of 5 tons/sq. in. for wrought iron at the time.

It is an interesting point as to whether the standards were ever subjected to tension, and whether alternating stress would subject them to fatigue problems. Also, whether there were any locked-up stresses in the tube or chains arising from construction, before the bridge went into service.

The tube was an annular ellipse having an I-value of 98×10^4 in.^{4} about the neutral axis, a weight of 300 tons, a depth of 12 ft. 3 in. overall, and a span of 460 ft. between supports. It was constructed on centring, but if it had been left to span 460 ft on its own, simply supported, without restraint from the chains, the bending stress at midspan would have been 15.5 tons/sq. in., much too high for wrought iron whose elastic limit was of the order of 18 to 19 tons/sq. in. The deflection of the simply supported tube would have been approximately 61 in. at midspan under this extremely high stress - in other words the tube would have been flexible.

Thus, as the centring was removed, the weight of the tube would be absorbed directly by the standards before generating a thrust to be resisted by the chains. This would load the standards in compression and also load the chains.

The chains would be stressed by their own weight and also by load from the tube. This would mean that the pull from the chains would exceed the thrust from the tube at this point. This would tend to arch the tube upwards, relieving the load in the standards until equilibrium was achieved.

The plate girders supporting the bridge deck were attached to the chains and tube by the standards, and probably the deck loads were shared equally between them. However there were additional secondary tension members connecting the plate girders to the chains at regular spacings between the standards. These were of light construction and may have been designed to assist in maintaining the exact curvature of the chains. But they may also have transmitted deck loading to the chains. If this was so there were double the connections from the plate girders to the chains than from the girders to the tube. This may have meant a division of load sharing between chains and tube of two to one, both for the deck dead load and for the live load. In this case the standards would be loaded very obviously in tension, more than sufficient to overcome the initial compression described above, and the possibility of alternating stresses exists.

To sum up, the internal stresses in the structure were indeterminate and difficult to quantify. But Brunel's design was amply sound, and he had obviously kept to one of his own maxims:

"Always allow rather an excess of material in quantity" for on 30 December, 1854, Brunel had written a letter to one of his assistants (who was abroad) which included the following technical advice:

"Let me give you one general piece of advice - that while in all works you endeavour to employ the materials used in the most economical manner, and to avoid waste, yet always put rather an excess of material in quantity. You cannot take too much pains in making everything in equilibrium; that is to say, that all forces should pass exactly through the points of greater resistance, or through the centres of any surface of resistance. Thus in anything resembling a column or strut, whether of iron, wood or masonry, take care that the surface of the base should be proportioned that the strain should pass through the centre of it.....so in trussed framework of wood or iron, experience shows that you cannot refine too much upon the perfection of the designing of every little detail by which all strains are carried exactly through the centres of the rods or struts and the centres of bearing surfaces..... this is, I assure you, valuable advice, to be followed literally and strictly, and not to be considered as a mere theoretical refinement, to be neglected in practice." (Life, 1870).

In 1953 the writer had the experience of crossing the Saltash bridge on foot, and it was noticeable in the detailing of the original bracing construction especially, how carefully the above advice had been followed. They all bore the impression of care and neatness, and exactness. Later additions to the bracing however, were not detailed with such care, though no doubt they fulfilled their purpose. (Fig 32).

The Saltash bridge design was unique, and a masterpiece, but it did not advance the development of the girder bridge. It remains as a tribute to Brunel's outstanding genius.

It may be asked why the Saltash design did not advance the development of the girder bridge. It should be remembered that Brunel's design was the answer to a particular problem – how to span 455 ft. with railway loading in a way that was both practical and economic. This feat had only been accomplished once before, at the Menai Straits, in a way which Brunel considered had its shortcomings and which he had no desire to emulate. He instead evolved the Saltash design as his solution to this particular problem, of which the chief feature was the very long span to be bridged. At the time of its erection the basic Saltash truss weighed 1060 tons compared to the 1587 tons of the corresponding Menai span, and was designed for Brunel's broad gauge loading of 1.50 tons/ft. against Stephenson's narrower gauge loading of 1.0 tons/ft. Structurally therefore the design was a significant advance on Stephenson's tubular bridge solution.

The workmanship in the construction of the Saltash truss, however, was complex and demanding. The great tube was elliptical in cross-section and parabolic in elevation, and had to be built up with wrought-iron plates no longer than 2 ft. x 10 ft. The chains had to be manufactured, and each link tested individually, though some were second-hand from Hungerford. The details of the attachments of the standards tube and chains involved extremely careful workmanship for lack of fit to be avoided. The temporary works i.e. scaffolding and centring for the arch and chains was also complex, and had to be substantial. The curve of the chains and their connection to the walls of the tube demanded more than ordinary skills in construction. Curved work is always more expensive than straight line fabrications, and the Saltash trusses had more than their

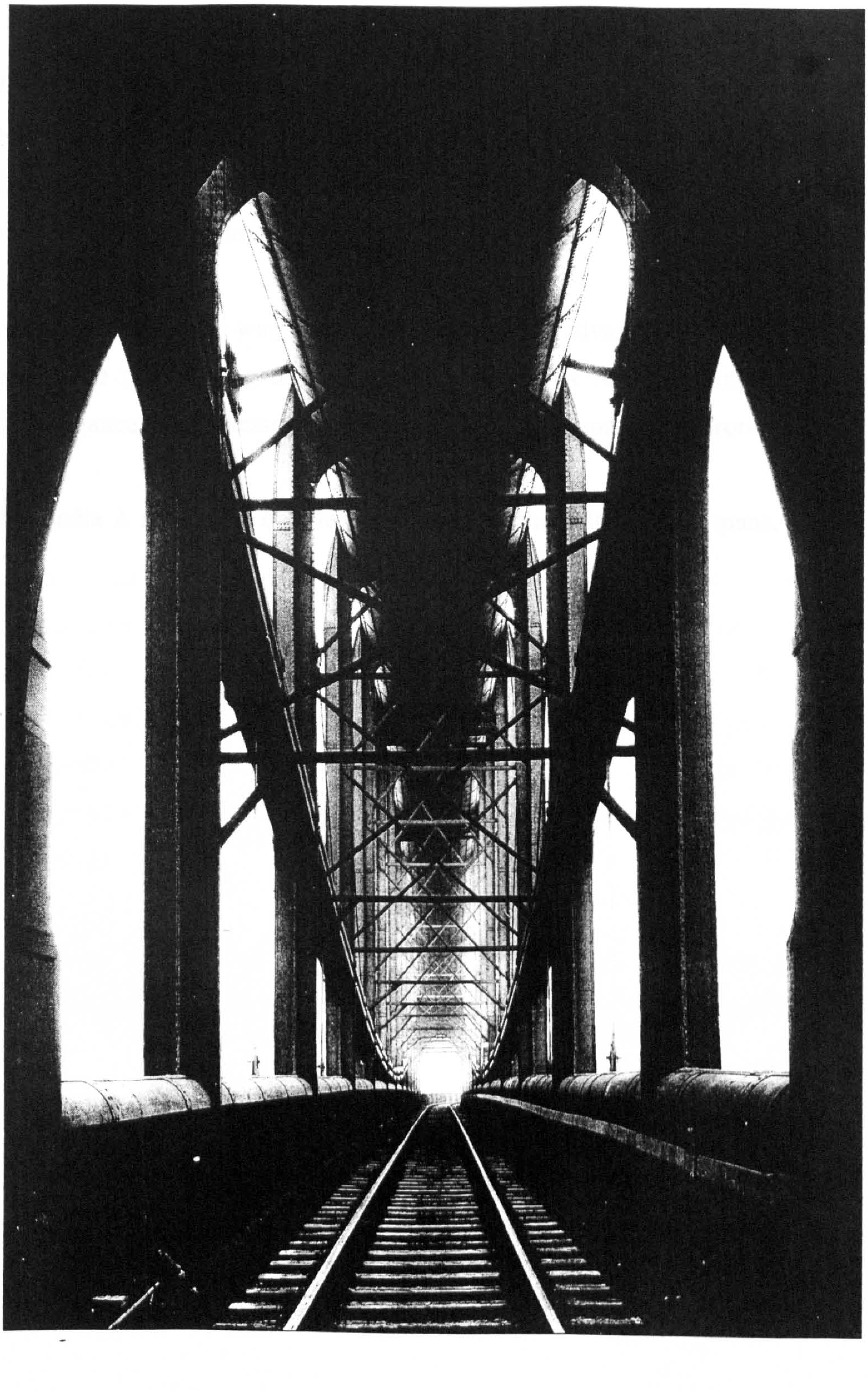


Fig 32 Saltash Bridge - Internal View (1927)

share. The panels between the standards were all of non-rectangular shape and offered little opportunity for repetition or prefabrication. Finally, the truss did not lend itself to easy erection methods such as rolling-out or cantilevering.

All these factors added to time and cost of construction. The panel truss girder had already appeared over the horizon with the arrival of Newark Dyke and the Crumlin Viaduct, and these Warren types, with others such as the Pratt and Howe, offered greater simplicity and economy in building.

Nevertheless something was lost in the inexorable advance of these more utilitarian designs. The picture (Fig 33) of the Saltash type at Auchindrean shows a beautiful curving elegance not possessed by any of the later developments in iron trusses.

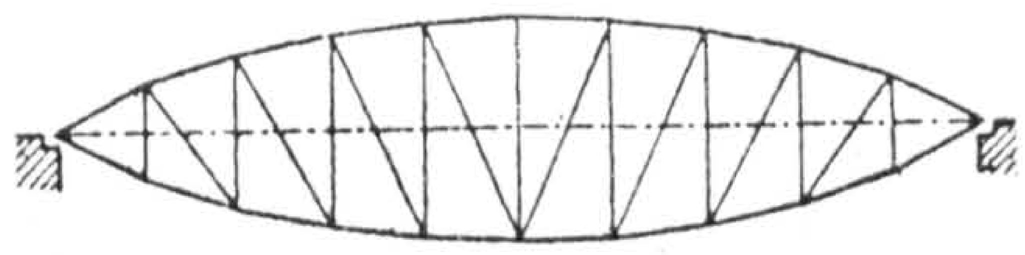
See Appendix A for a more detailed calculation of the Saltash main spans.

The Pauli and Schwedler Girders

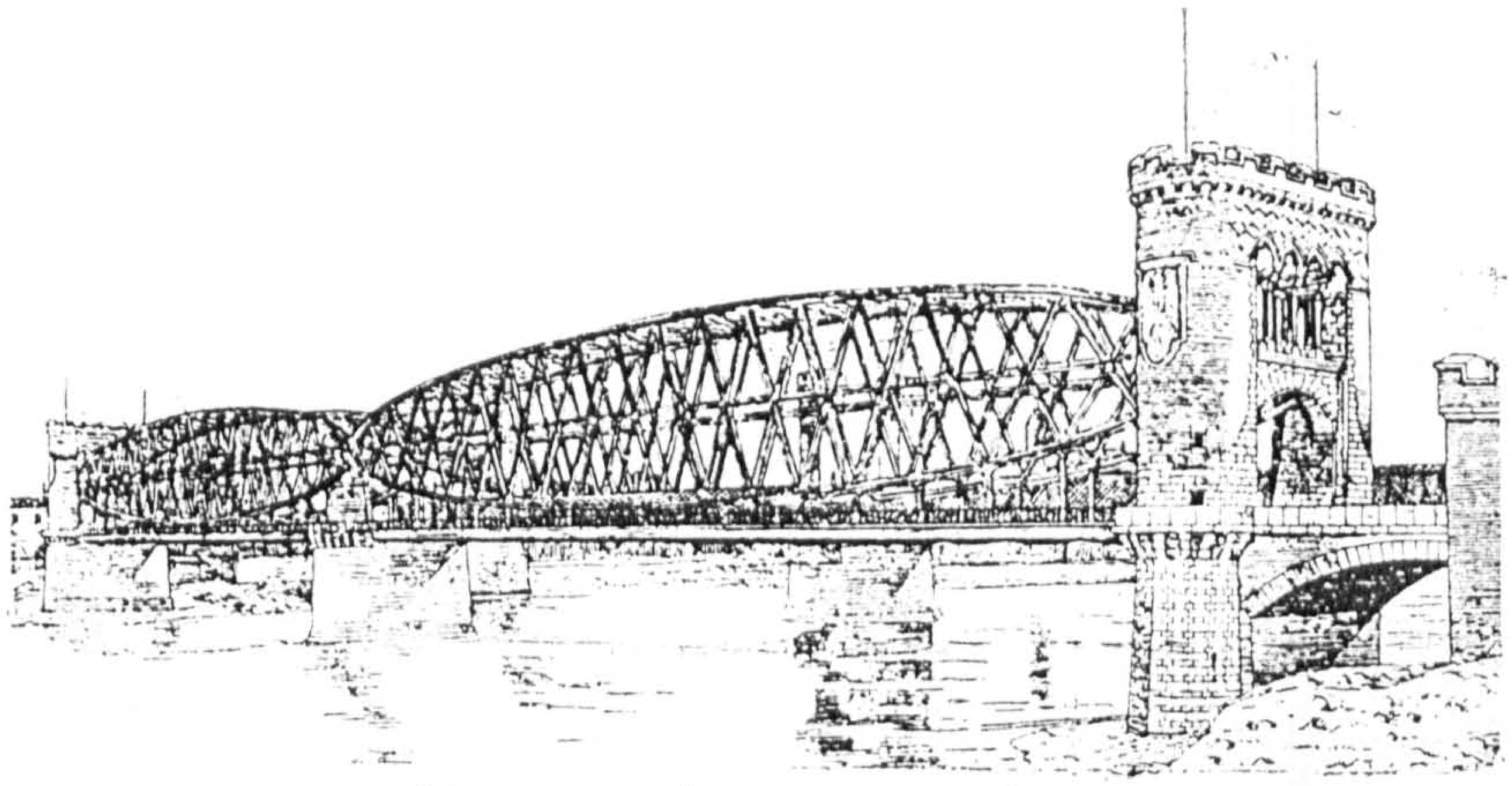
During the 1850s and 1860s two other girder systems, named after their inventors, were developed in Germany. One was the Pauli girder (Fig 33A), a lens-shaped truss with curved flanges, invented by F A v Pauli (1802 – 1883). His idea was to keep axial forces constant both in the top and bottom chords. Pauli was of the opinion that the harmful vibrations caused by passing trains could be reduced by suspending the girder on the neutral axis, and by making this axis a straight line (Straub, 1952). It is not recorded whether this idea was successful, and it seems unlikely, for vibration is linked to deflection of the structure and deflection depends on stiffness, which is related to the depth of a girder perhaps more than any other factor. Nevertheless several Pauli bridges were built in the valleys of the Rhine and Danube, the largest of which was that at Mayence, built in 1860-62.

A few years later (1869) J W Schwedler (1823 – 94) developed a web system in which all the diagonals were acting as ties. This leads to a slight change of curvature at the centre of the top chord which is characteristic of the Schwedler girder (Fig 33A.) This feature is unattractive and Schnedler suggested that the top chord might be given the form of a "basket handle" or semi elliptical arch, more pleasing to the eye. The first bridge of this type was erected over the Elbe at Hamerten in 1867. It was followed by several others (Straub, 1952).

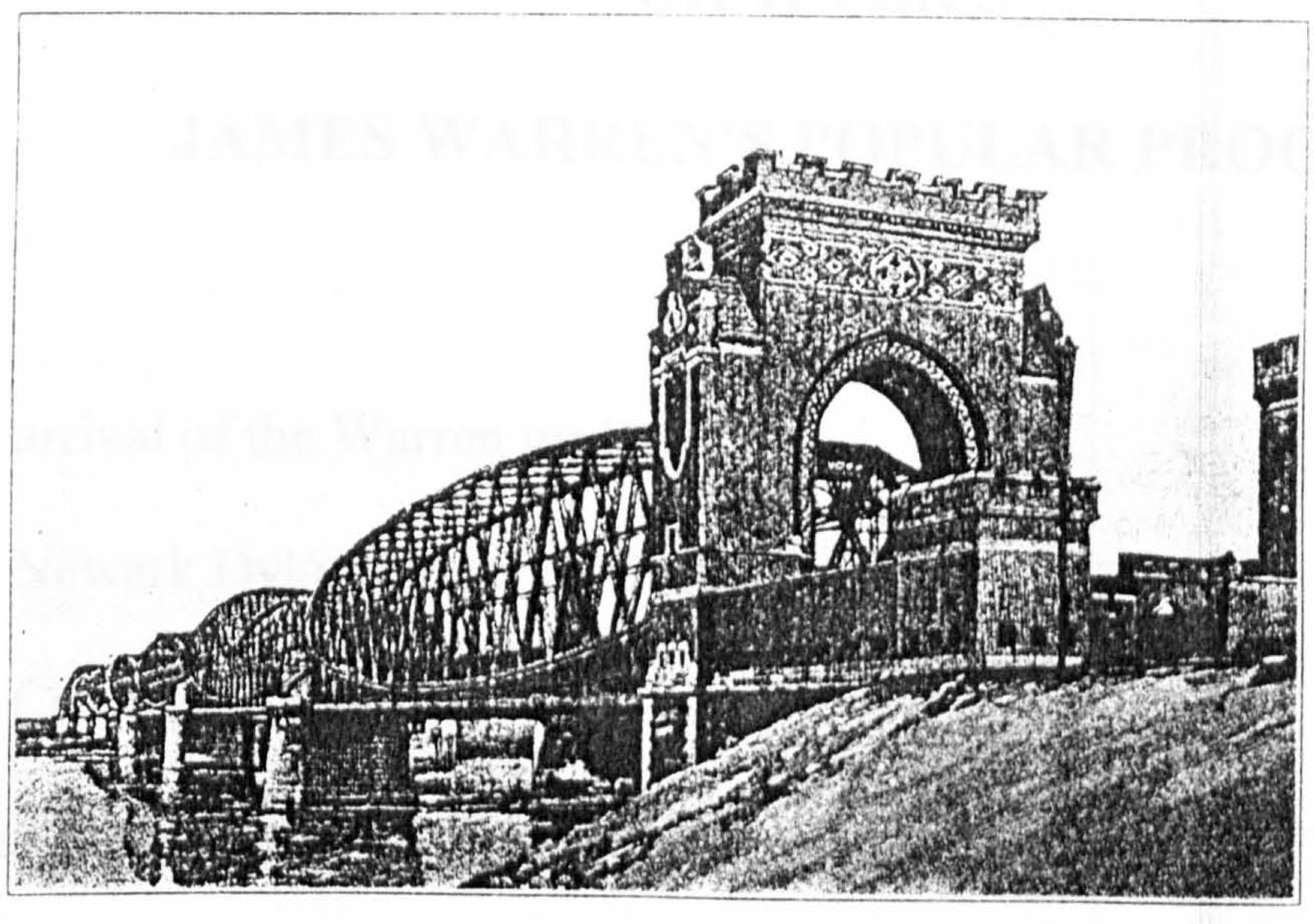
Both the Pauli and Schwedler girder types vanished after a few years, probably for the same reasons as the Saltash type described earlier – costly to build and difficult to erect.



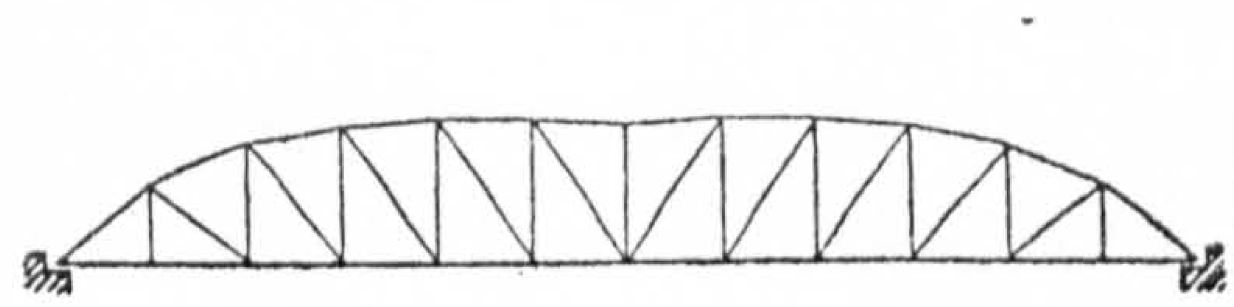
Pauli Girder.



Marienburg Bridge over the Nogat



Weichsel Bridge Germany



Schwedler Girder.

Fig 33A Pauli and Schwedler girders (Fowler, 1929 and Straub 1952)

CHAPTER 5

JAMES WARREN'S POPULAR PROGENY

The arrival of the Warren girder

The Newark Dyke bridge, 1852

The Crumlin Viaduct, 1853-57

Other forms of the Warren girder

Chapter 5

James Warren's Popular Progeny

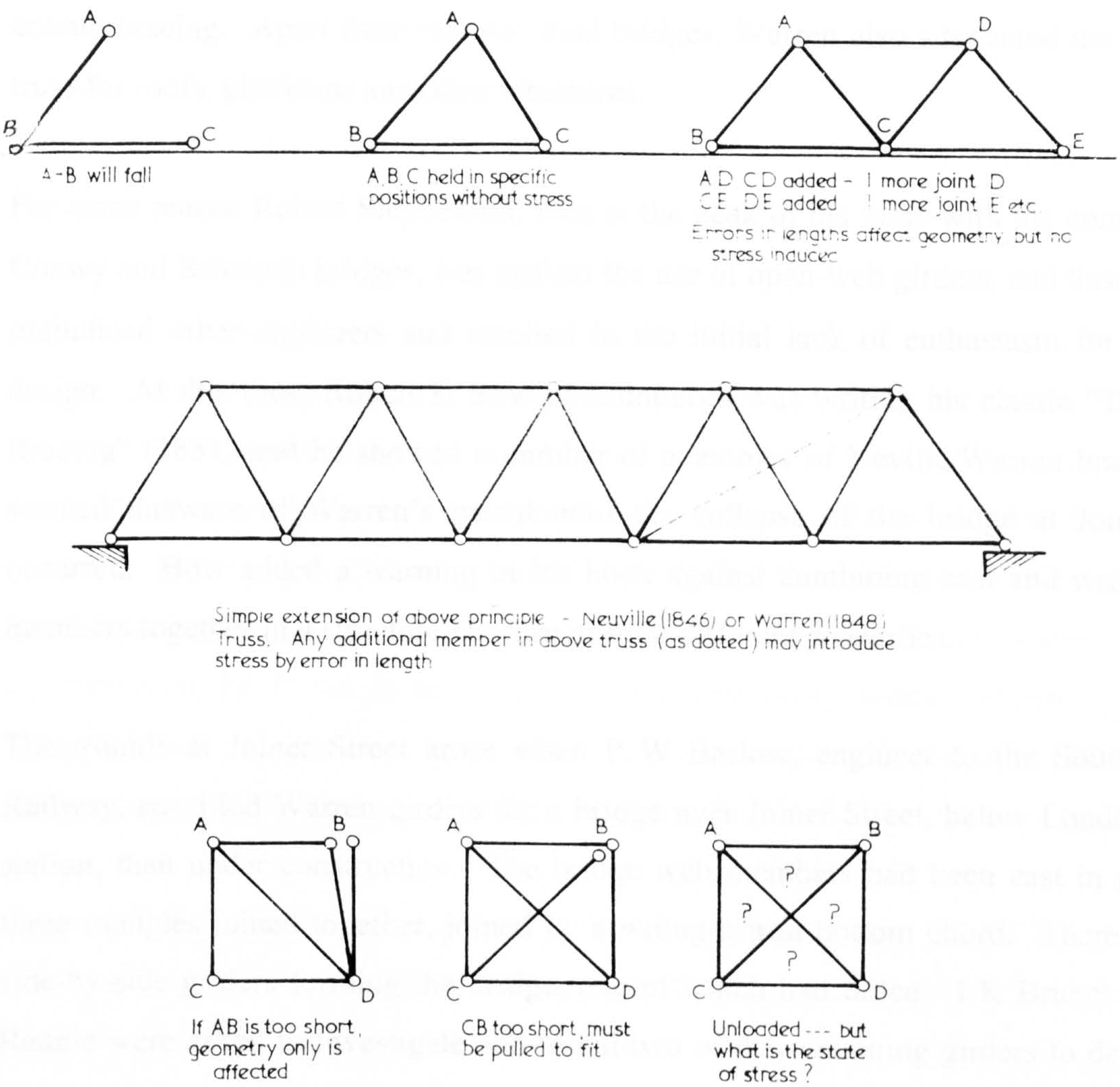
The Arrival of the Warren Girder c. 1850

This form of girder is seen all around us at the present day and is easily recognised in its triangular panels. It was first introduced in Britain c.1850, and was eventually to sweep all before it in its simplicity and easily constructed form. But it was slow to get started, and its first example was the Newark Dyke bridge of 1852 followed closely by the Crumlin Viaduct of 1853. Some of the delay in its use arose from the failure of a partially completed Warren truss over Joiner Street, at London Bridge Station, the reasons for which were not clearly reported and remain obscure.

The use of triangulated structures for girders was hinted at as far back as the time of Palladio, and earlier. The triangle was a stable shape which could not be altered without altering the length of any of its sides, and was of great simplicity. Used in the Warren truss it also lent itself to easy calculation, and had aesthetic appeal. (Fig 34).

It has been seen that the era of the open-web girder pioneered in Britain by Brunel at Chepstow in 1852 was becoming a reality, and the Warren girder was perhaps the most fundamental of all open-web truss designs. James Warren took out his patent in 1849, but was preceded by another entrepreneur and engineer, A H Neville, in 1838. Neville built several triangular truss bridges in Europe up to his death in 1861, and his bridges were noted for having the deck supported somewhere between the top and bottom chords, and having additional intermediate chords. As a result, the structure was complex and not easily analysed. Neville's designs were soon overtaken by Warren's, whose trusses were capable of carrying a deck either on the top or bottom chords or even on both.

Warren specified no particular angles for his triangular design, though they were generally 60° and equilateral, or isosceles, in the early days, and can be chosen to suit the spacing required by the designer. Because the geometry of the triangular joints was more difficult to make in timber than a T-joint or 90° joint, the design lent itself more to construction in iron than timber, and few timber Warrens were made. Warren claimed that the diagonal members of his girder could be either cast-iron or wrought iron,



The triangulation of trusses.

Fig 34 Typical Warren Girder.
(Hopkins, 1970)

depending on the nature of the forces in each. However, at midspan a reversal of stress can occur under a moving load and requires care in the design, and possibly the use of counterbracing. Apart from rail and road bridges, Warren also advocated the use of his truss for roofs, platforms and other structures.

For some reason Robert Stephenson, then at the peak of his fame with the completion of Conwy and Britannia bridges, was against the use of open-web girders, and this may have prejudiced other engineers and resulted in the initial lack of enthusiasm for Warren's design. At this time, Robert H Bow of Edinburgh was writing his classic "Treatise on Bracing" (1851) and he showed a number of examples of Neville/Warren bracings, but seemed unaware of Warren's patent until the collapse of the bridge at Joiner Street occurred. Bow added a warning in his book against combining cast and wrought iron members together in a structure because of their different properties.

The trouble at Joiner Street arose when P W Barlow, engineer to the South Eastern Railway, specified Warren girders for a bridge over Joiner Street, below London Bridge station, then under construction. The bridge web members had been cast in groups of three triangles joined together, joined by a wrought iron bottom chord. There were six side-by-side girders forming the bridge, one of which had failed. I K Brunel and John Rennie were asked to investigate and tested two of the remaining girders to destruction. They said the strength of the bridge was insufficient. Barlow however, maintained that the bridge had been overloaded by the contractor piling 120 tons of bricks on it during construction. The Government inspector agreed with Barlow and the bridge was rebuilt with slight modifications, and happily exists to this day. The failure however was bad publicity for Warren, and may explain some of the delay in the widespread adoption of his design.

A difficulty with the Warren truss was that the triangular design led to a wide spacing of panel points suitable for the positioning of cross-girders. In the equilateral triangle, this spacing was 1.15 x the depth of the truss. In an effort to reduce the spacing, the truss depth had to be reduced, and this led to high span: depth ratios (Newark Dyke 16:1) and to consequent problems of deflection. An alternative was to use an isosceles triangle with a base equal to the desired cross-girder spacing, but this was wasteful of material.

Other solutions were to divide the Warren panels by verticals from the lower chord to the apex of the triangle (which was done at Newark Dyke) or to superimpose a second Warren system on the first, separated by half a panel length. This latter system was much loved by the Victorians, and used by Thomas Bouch (1822-80) in numerous bridges, including Belah, Deepdale, and the first Tay Bridge. It was known as the double-triangular truss.

Another difficulty with the Warren truss was the method of calculation, though nowadays it seems obvious that a careful application of the triangle of forces would solve the problem. But apparently in these far-off days in the early 1850s engineers had to wait for the publication of a book by W Fairbairn in which he stated that in 1850 he obtained a method of analysis of a Warren girder from W Bindon Blood, Professor of Engineering at Queen's College, Galway. Blood experimented with a scale model which gave very good correlation of the forces in the members with calculated values. In America, Squire Whipple also published a book ("An Essay in Bridge Building", 1847) dealing competently with the analysis of trusses, including Pratt and Howe types.

The real impetus to the development of the Warren truss came from a rising engineer, Charles Heard Wild (1819-57), who independently seems to have found a solution to the problems of analysis, and was keen to apply it.

The Newark Dyke Bridge, 1852

C H Wild had worked with Edwin Clark on the Britannia bridge and apparently had acquired some rights to use Warren's patent. He prepared a design for a Warren span of 250 ft., wholly in wrought iron. At this time Sir William Cubitt and his son Joseph, consultant and engineer respectively to the Great Northern Railway were faced with a major problem in crossing the river Trent at Newark. The Cubitts learned of Wild's design and its possibility of easy fabrication and quick erection, and were then approached by Fox, Henderson & Co of Birmingham, ironwork contractors, with a design for a span of 259 ft. based on Wild's proposals, the clear span being 240 ft. 6 in. (Fig 35).

Each line of track was carried on two girders, and involved trusses 277 ft. long and 16 ft. deep. The upper chord of each truss was a cast-iron tube, and the lower chord was of flat wrought iron tie bars. The diagonal struts were of cast-iron with a cruciform section, and the diagonal ties were of wrought iron flat bars. The panels were equilateral triangles of 18 ft. 6 in. sides. This rather large panel spacing of 18 ft. 6 in. was divided into two by verticals from the apex of the triangles, and the connections were made with 5.5 in. diameter wrought iron pins. The trusses were spaced 14 ft. apart. However, because of its slenderness the bridge suffered from lack of rigidity and deflection, worsened by wear at the pin joints. It experienced an ever-increasing weight of traffic, and began to suffer from excessive maintenance problems. It was replaced in 1889.

It is not known how Wild developed his method of calculation but it seems possible that he was in touch with Professor Blood in Ireland, and learned of his work with force triangles and his experimental work. In 1852 Blood, in association with W T Doyne, published a Paper on his findings in the Proceedings of the ICE (Blood & Doyne, 1852), and this seems to be the first mathematical study of the forces acting in the webs of girders, though Moseley may have been working on the same problem for some time. Wild came close to establishing an understanding of the shear stresses in webs of girders.

The Newark Dyke bridge is instantly recognisable in any illustration though of the common Warren form. (Fig 35). The struts and ties of the diagonals betray their purpose immediately, and the joints are neat and formed exactly for their purpose – there are no unsightly gusset – plates, or rivet groups. It is almost as if its engineer recognised that it was a landmark design, and gave it special treatment aesthetically. Even the handrail is

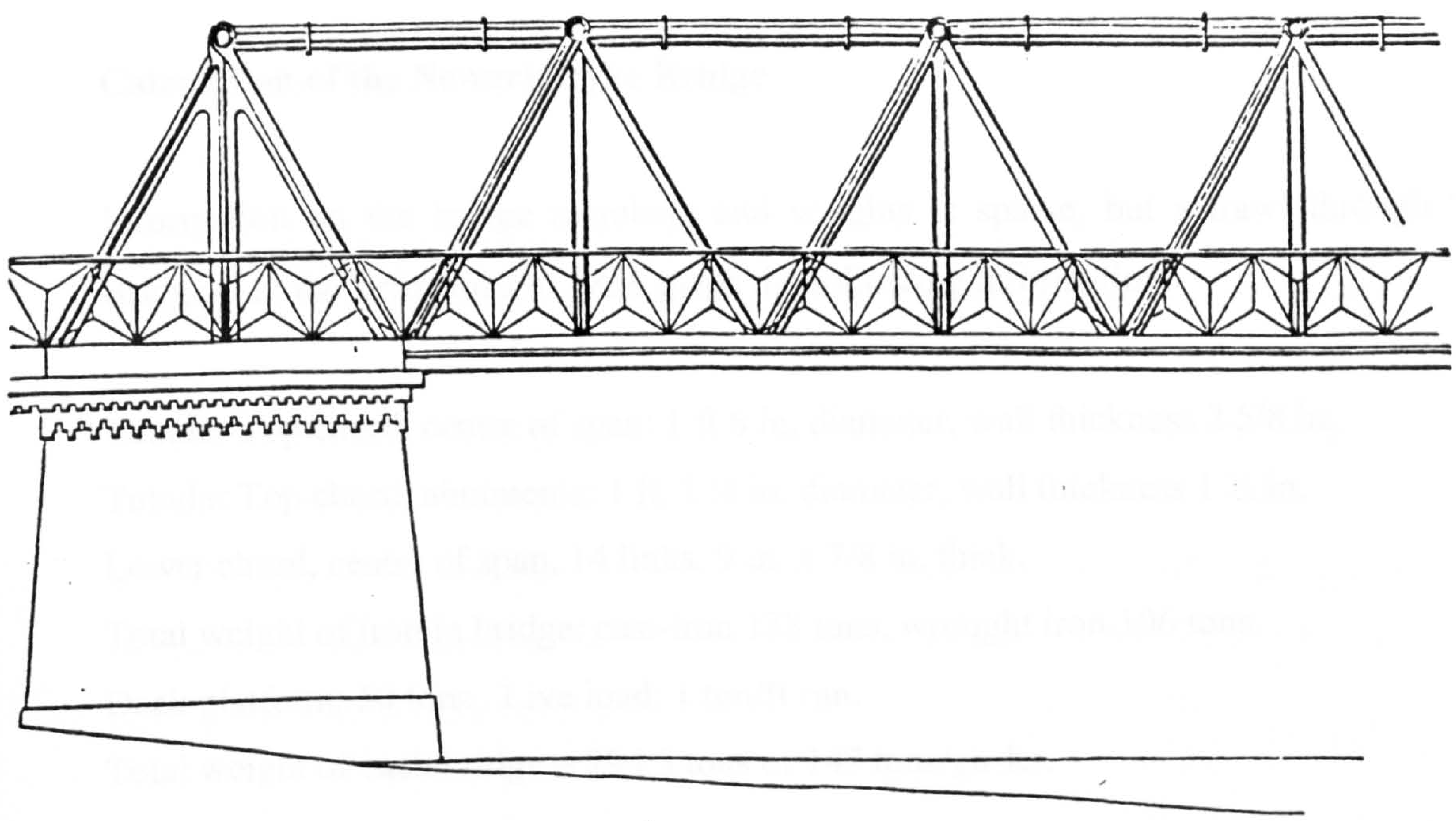


Fig. 33.—Newark Dyke Bridge.

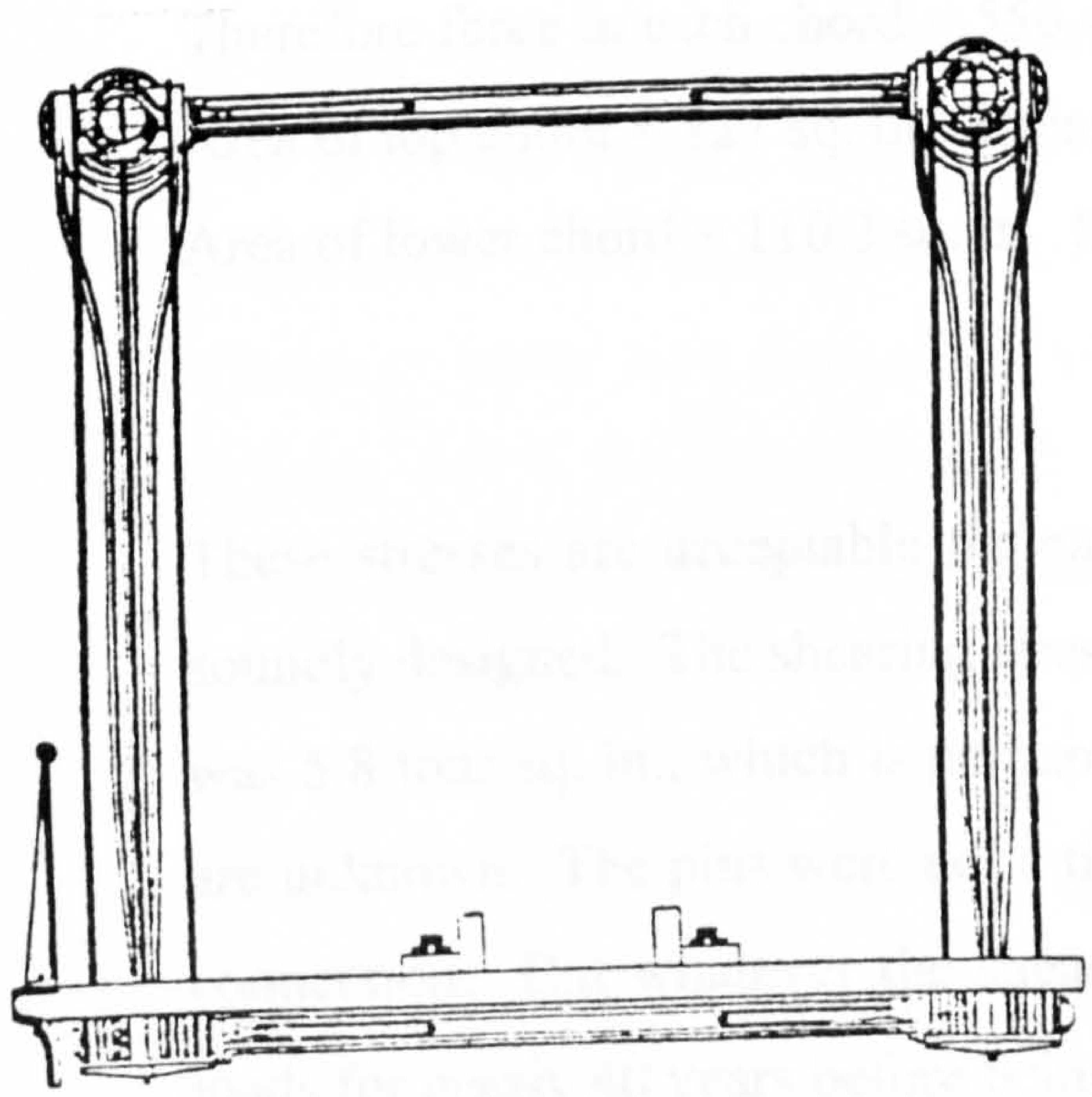


Fig. 34.—(Section).

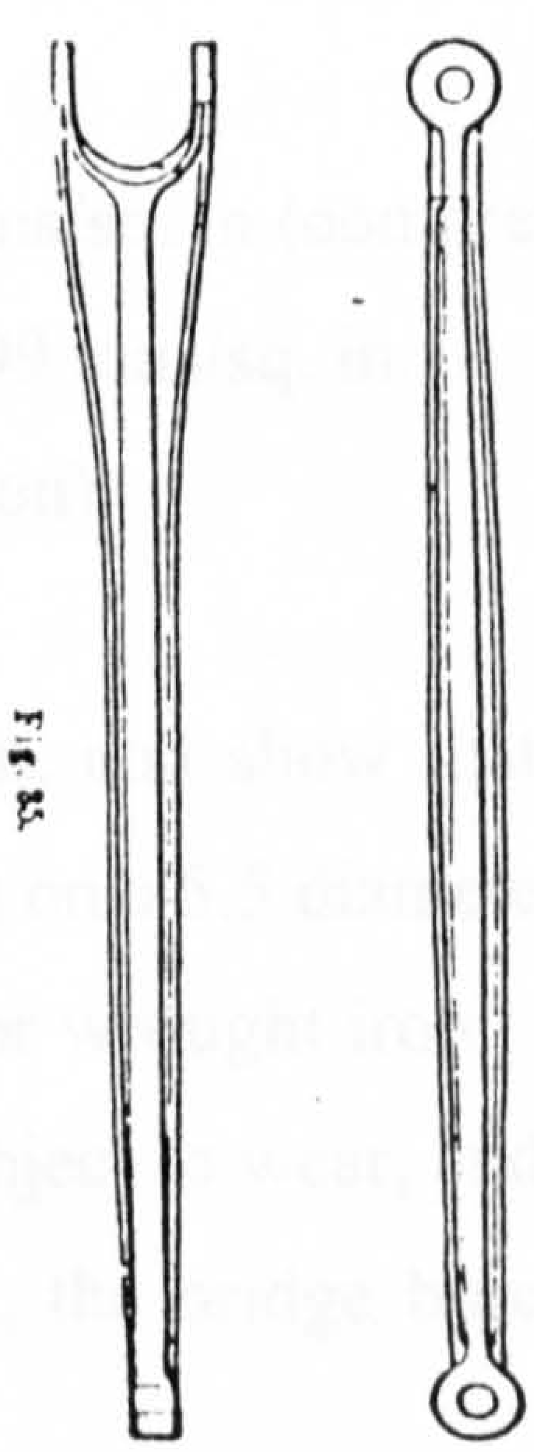


Fig. 35.

Fig 35 Newark Dyke Bridge (1852)
(Stephenson, 1856)

decorative without being ornamental and the entrance to the bridge is a semi-elliptical arch in iron rather than the goal posts seen on many another bridge.

Calculation of the Newark Dyke Bridge

Information on the bridge members and weights is sparse, but a trawl through "Iron Bridges" in the Britannica of 1856 gives the following information:

Tubular Top chord, centre of span: 1 ft 6 in. diameter, wall thickness 2 5/8 in.
Tubular Top chord, abutments: 1 ft. 1 ½ in. diameter, wall thickness 1 ½ in.
Lower chord, centre of span, 14 links, 9 in. x 7/8 in. thick.
Total weight of iron in bridge: cast-iron 138 tons, wrought iron 106 tons.
Deck platform: 50 tons. Live load: 1 ton/ft run.
Total weight of each bridge = 294.5 tons or 147 tons/girder.

Therefore maximum bending moment at midspan = $\frac{WL}{8}$ = 8806 tons-ft

Lever arm between chords = 16 ft.
Therefore force in each chord = 550.4 tons at centre.
Area of top chord = 127 sq. in. Stress at centre = 4.32 tons/sq. in (compressive, cast-iron)
Area of lower chord = 110.3 sq. in. Stress at centre = 4.99 tons/sq. in.
(tensile, wrought iron)

These stresses are acceptable for cast and wrought iron, and show that the bridge was soundly designed. The shearing stress (quadruple-shear) on a 5.5 diameter pin connection was 5.8 tons/sq. in., which is perhaps somewhat high for wrought iron. Bearing stresses are unknown. The pins were not a tight fit and were subject to wear, and to "play" in the connection. But whatever the shear stress on the pins, the bridge bore ever-increasing loads for nearly 40 years before being replaced.

It is appreciated that the above calculation is somewhat rudimentary, but it does give an indication of the adequacy of the structure. Calculation of the web members is not possible because no section sizes are available.

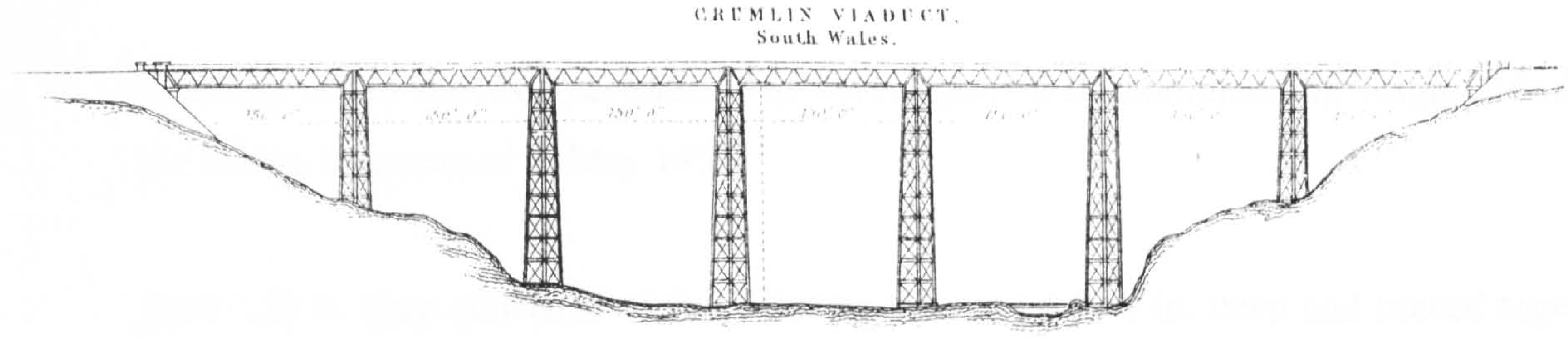
The Crumlin Viaduct, 1853-57

Apart from the pioneering Warren bridge at Newark Dyke, only two other major Warren girder bridges were to be built in Britain for several years. There seemed to be a long-standing and groundless prejudice against large open-web girders because of their apparent lightness and seeming lack of strength compared to the solidity of the plate girder. But they enjoyed great popularity abroad, particularly when exported to India.

After Newark Dyke came the Crumlin Viaduct of 1853, then there was a gap of 20 years before the Meldon Viaduct, Devon, of 1874. This is surprising in view of the boldness and simplicity of the Crumlin design, and its low cost. But the Warren girder was making its name overseas whatever the aversion to it in Britain. Even the Americans used it, calling it a "triangular girder" and avoiding any reference to Warren.

The Crumlin viaduct crossed the steep-sided valley of the river Ebbw in Monmouthshire, carrying the double line of the Taff Vale extension of the Newport, Abergavenny and Hereford Railway. The company's engineer was Charles Liddell, with Professor L Gordon of Glasgow University as consultant. Tenders were invited for a design and build contract, which resulted in a competitive design in which material was pared to the limit and there was nothing extra to limit corrosion or provide for any higher loading that might occur in the future. This however was the fault of the specification prepared by the Company rather than the parties bidding for the contract. A design was prepared by Warren and T W Kennard which was accepted, and it is Kennard's name which is generally associated with the structure. He may have been assisted by C H Wild. (Berridge, 1969).

By any standards the Crumlin viaduct was innovative. (Fig 36). It crossed the valley by way of a small hill which divided the structure into one group of seven spans and another of three. The spans were relatively modest at 150 ft., but were a maximum of 200 ft. above the valley, supported on hexagonal groups of cast-iron tubes rather than masonry piers. The apparent lightness of the construction attracted much interest among engineers and the public, particularly since iron trestles on that scale had never been seen before. The method of erection of the spans appears to have been by hoisting them, prefabricated, from ground level. The Kennard family were manufacturers of wrought iron at nearby Blaenavon which supplied the wrought ironwork. They also had an Ironworks at Falkirk,



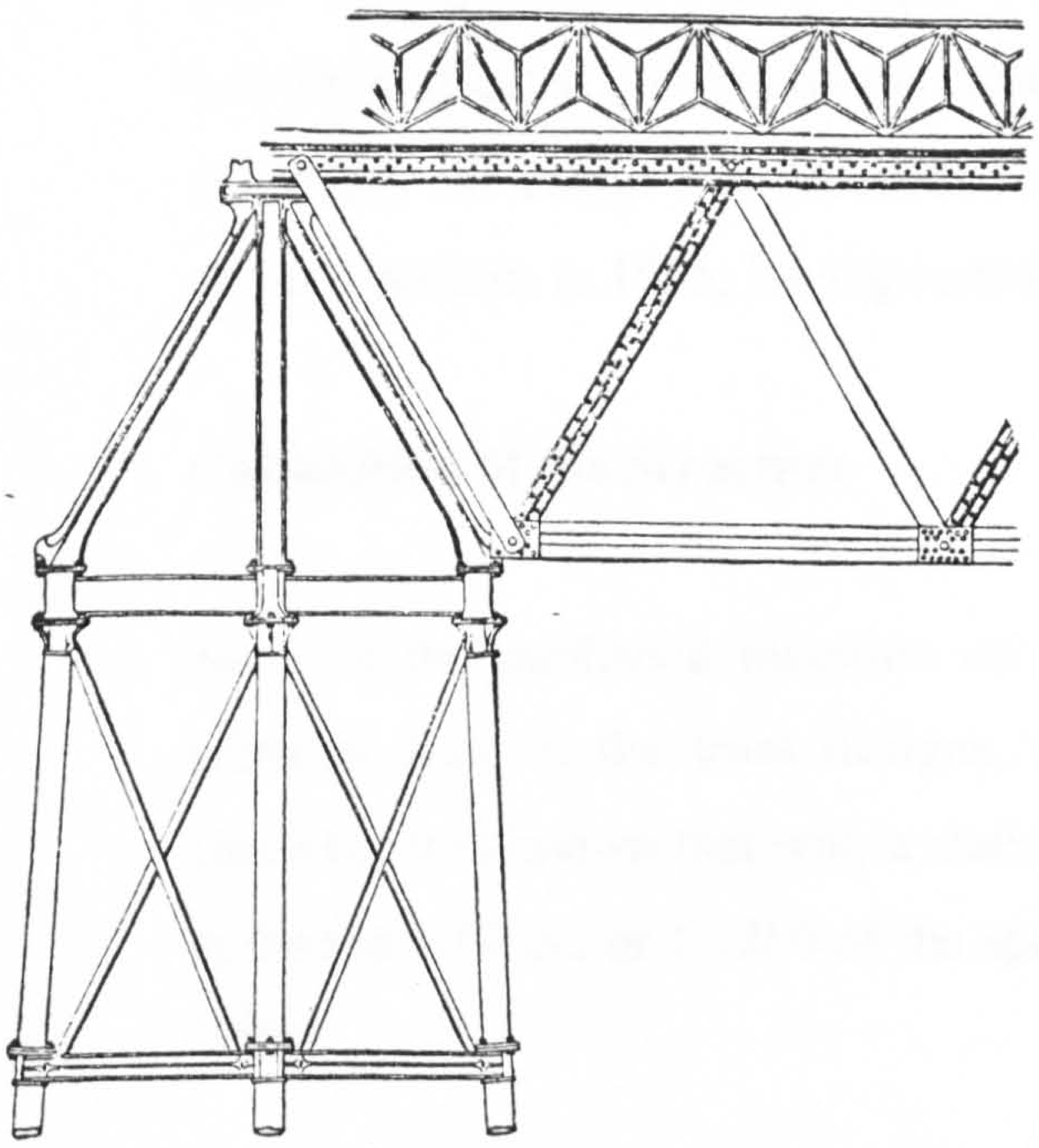
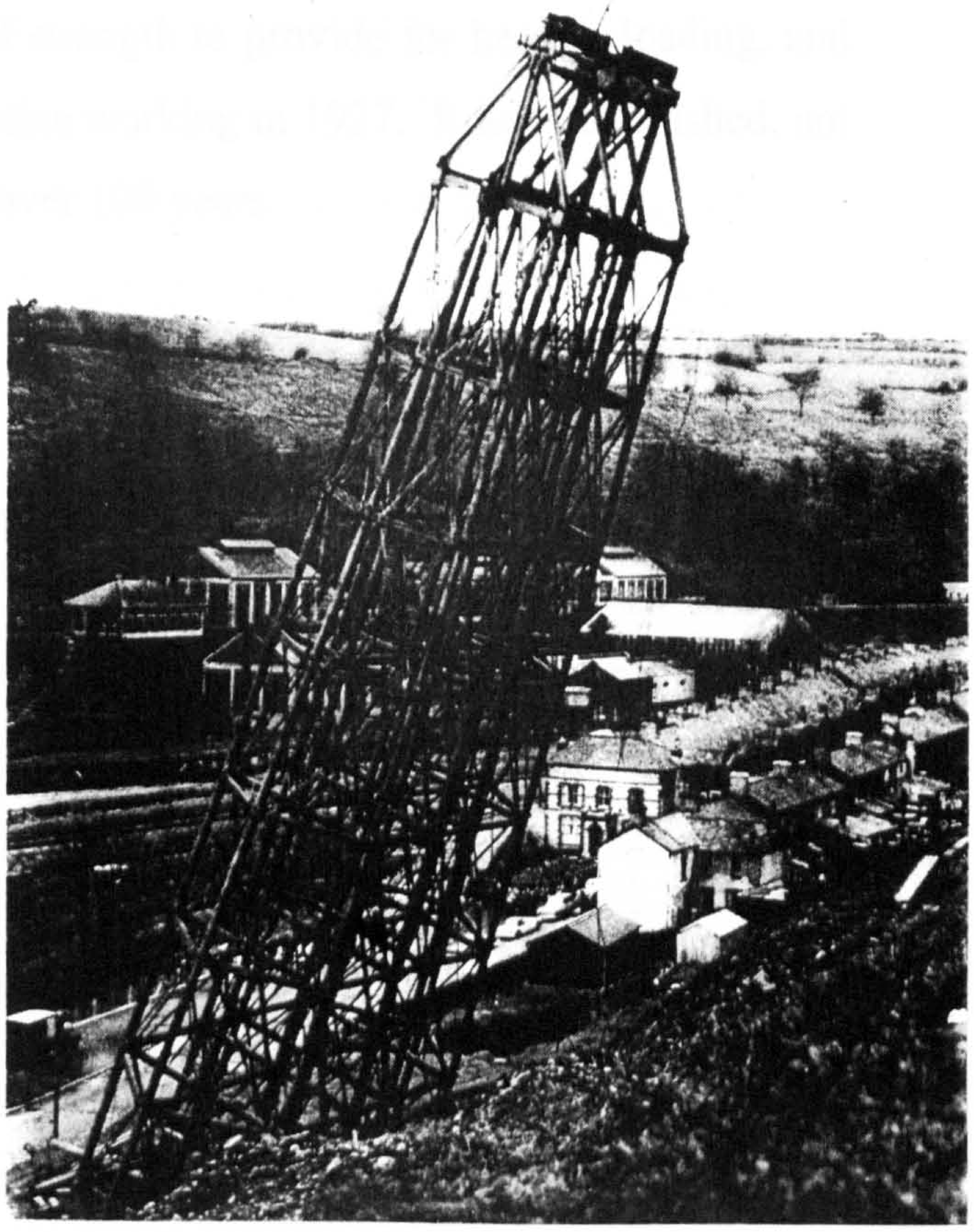
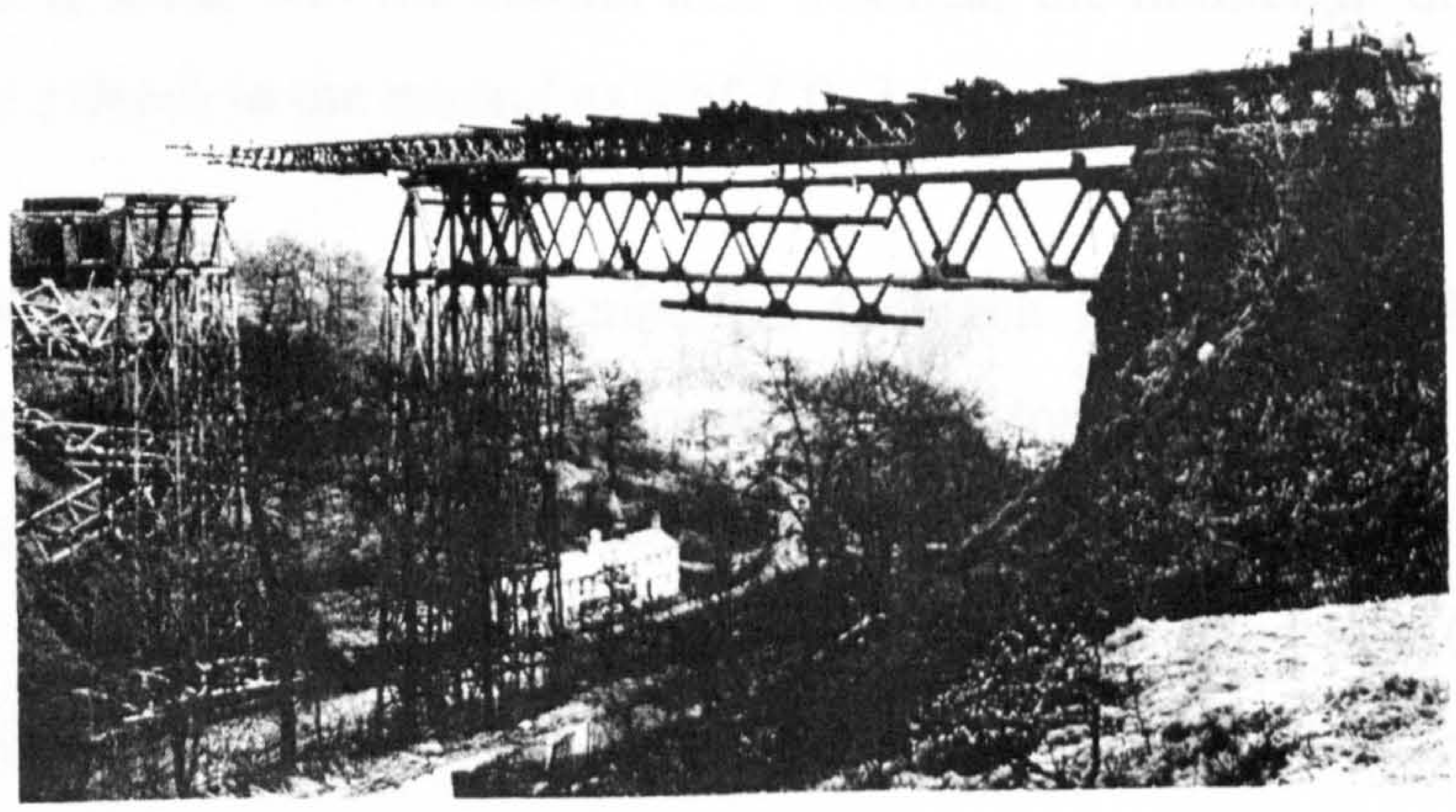


FIG. 128.—Part of Crumlin Viaduct.



One of the Crumlin Viaduct piers being felled by winching over. (Photo: B.R.



Bailey bridging being used to dismantle the trusses at the Crumlin Viaduct. (Photo: B.R.)

Fig 36 Crumlin Viaduct (1853-57).
(Stephenson, 1856)

Scotland which supplied castings. The seven spans were completed in August 1855 and the bridge was opened in May 1857.

Each 150 ft. span consisted of four Warren trusses 14 ft. 6 in. deep and braced together. The connections were by pin-joints as at Newark Dyke, but they again suffered much from wear and tear. The wrought iron work corroded, though the cast-iron of the piers was little affected. There was no reserve of strength to provide for heavier loading, and eventually the bridge was reduced to single-line working in 1927. It was demolished, not without protest, in 1965, having lasted well over 100 years.

Calculation of the Structure

None of the published literature on the Crumlin bridge gives any information on the sections used in the truss designs, so an accurate check calculation is not possible. However it is known that with a stationary u.d.l. test load of 230 tons, one pair of girders deflected 1.50 in., or 1/1200 of the span. The loading represented 1.53 tons/ft. (Berridge, 1969).

Calculating backwards from the deflection formula $\frac{5WL^3}{384EI}$ and assuming a Young's modulus of 11,000 tons/sq. in. for the iron gives a value for I (moment of inertia) for the two girders of 10.6×10^5 in.^{4} i.e. the I-value for one girder was 5.30×10^5 in.^{4}.

It is known that the girders were 14 ft. 6 in. deep overall, and if the somewhat sweeping assumption is made that the neutral axis was near the midheight of the girder, then this would give a depth to the neutral axis of 7 ft. 3 in. or 87 in.

This value results in an elastic modulus for each flange of 6092 in.^{3}. The bending moment at midspan of the girder would be 2812 tons/ft. under the test load alone (230 tons) giving stresses of 5.54 tons/sq. in at the flange extreme fibres. But this test load equalled 1.53 tons/ft. Under the standard rail track live load of 1.0 tons/ft. the stresses would reduce to 3.62 tons/sq. in. The stresses due to the dead load of the girder would have to be added to this figure. Admittedly the calculation is very approximate and takes no account of the contribution of the web members, nor of the actual position of the neutral axis, but it does indicate that the design is not wildly inadequate.

The Crumlin viaduct spans are often criticised as having been too light and barely adequate, with nothing in reserve. But the deflection figure of 1/1800 of the span under 1 ton/ft. is not high. Also, an engineer seeking bids for a structure under a tight specification cannot expect to get something for nothing. Neither is he under any obligation to accept the lowest (or indeed, any) tender. Charles Liddell knew that in general you get what you pay for, and had he desired a more substantial bridge, he should have specified it.

There was one other innovative aspect of the Crumlin viaduct, and that was that it led the way in bridge manufacture as well as in bridge construction. There were 40 wrought iron trusses in the ten spans, and each of them was pre-fabricated complete at ground level before being hoisted into position, ready to receive the deck girders and flooring. This limited the work to be done aloft, and provided a safe non-temporary working platform on which to do it.

Thus the Crumlin viaduct, after Newark Dyke, showed what could be done with a Warren design, and the development of the girder bridge was taken one stage further. Further developments of the Warren type were to follow, particularly in the form of the double triangular truss, which was widely used to great advantage. This type will form the next aspect of study in the thesis.

Other Forms of the Warren Girder

The Double-Triangular Truss

The main shortcoming of the Warren truss was the wide spacing between panel points resulting from the equilateral triangle form of the web members. The cross-girders forming the deck had to be located at these points, and it was not advisable to land the cross-girders between them, otherwise local bending of the lower chord would have required a more substantial section.

This difficulty was first met at Newark Dyke in 1852 by the introduction of verticals from the apex of the web triangle to the midpoint of the lower chord. This solution was effective and has been employed many times down to the present day, including the 300 ft. span Warren truss used in the replacement of Brunel's Chepstow bridge in 1962. But these verticals contributed nothing to the overall strength of the bridge, and added significant dead load to be carried by the structure.

Another solution to the problem was the use of the double-triangular truss. Here a second set of bracing was superimposed on the original Warren pattern such that the panel width was halved, and the second set of web members were parallel to the first, and intersected them. This solved the problem of providing additional support points for the cross-girders and the members contributed to the overall strength of the girder. A bonus was that the deflection of the girder was also reduced, because strain energy was absorbed in the stressing of both sets of diagonals. Aesthetically too the double triangular from was attractive, as can be seen on the bridges of the West Highland Railway between Glasgow and Fort William. (Fig 37). The diagonals did not carry large forces, and as a result the gusset plates forming the connections to the chords were small. This type of bridge was best suited to a span of 60 - 70 ft., but it also was adaptable to the larger suspended spans of 350 ft. of the Forth Bridge (1883-90), where it harmonizes well with a similar system of bracing in the cantilever arms, and is a beautifully proportioned and elegant design.

Fig 37 The Double-Triangular Warren Girder.

Thomas Bouch and the Double-Triangular Truss

Perhaps the greatest exponent of this truss form was the prominent Victorian engineer, Thomas (later Sir Thomas) Bouch (1822-80). Bouch was an innovative engineer and emerged in the 1850s as a consulting engineer mainly in the development of railways in the North of England and in Scotland. In total he was the engineer for the construction of about 270 miles of railways, chief of which was the South Durham and Lancashire Union, 50 miles long. His largest railway in Scotland was the Edinburgh and Peebles line, 21 miles in length and for a long time a pattern for cheap construction. Bouch had a conviction that most civil engineering work of the day was over-designed and expensive, and his practice was to reduce structural sections and produce structures which were cheap. This made him popular in company boardrooms.

His railways demanded many bridges, including the Beelah viaduct with sixteen spans of 60 ft. each and 196 ft. high, and Deepdale, eleven spans of 60 ft. with a maximum height of 160 ft., both 1859. These bridges were double-triangular Warren girders, very economically designed on iron piers composed of groups of braced columns, and carried railway traffic for over a hundred years before being dismantled. Bouch also constructed a remarkable road bridge in 1871 at Newcastle, the Redheugh viaduct, with two spans of 240 ft. and two of 160 ft. These spans again were of the double Warren form supported by raking ties, and bore a striking resemblance to the cable-stayed bridges of today. (Fig 38).

Bouch again used the double Warren on his ill-fated Tay Bridge, blown down in the disaster of 1879. It was two miles long and had 85 spans, including the thirteen navigation spans which fell in the disaster. All were of the double Warren type, and were re-used in the rebuilt bridge (except the navigation spans) and carry rail traffic to this day. The cause of the disaster was the inadequacy of the bracing in the tall iron columns of the piers, and had nothing to do with the Warren spans.

Calculation of the Double-Triangular Truss

Since the double Warren is basically two Warren girders separated longitudinally by half a panel, the bridge is approximately calculated by separating the two systems and applying half the design load to each, then superimposing the results. This method is easy to apply and gives an accuracy to within 5% of the exact method, which involves solving the statically indeterminate truss.

The "superposition" method appears to have been first developed by D J Jourawski (1821-1891), who, after graduating from the Institute of Engineers in St Petersburg, became involved in railway work and with bridge calculation. He published his method in 1854 in Russia, and in France in 1856. It is still used to this day. (Timoshenko 1960).

In 1851 Robert Henry Bow of Edinburgh had published a treatise on braced structures, and Bouch had discussions with him on the calculation of Beelah and Deepdale viaducts. Bouch was no mathematician and it is likely he delegated the calculation of the girders completely to Bow. Bow's treatise is not easy to follow, and he does not give "worked examples" in his text, but he does give the impression that he knows his subject. (Bow, 1851). This is borne out by the fact that of all the double-triangular girders constructed, none seems ever to have been reported as having proved inadequate or failed. The method of superposition was also easy to apply, and easily understood, and hence the girder was popular and much used.

Bouch's Redheugh Bridge at Newcastle, 1871 (Fig 38)

This was a remarkable bridge for its time, embracing double Warren girders and structural ties connecting the third-points of the spans to towers on tops of the piers. It was a road bridge over the river Tyne having two main spans of 240 ft. and two side spans of 160 ft. Aesthetically it was an "unresolved duality", as was the Britannia bridge, but the six iron towers for the suspension chains helped to soften this aspect. (Cail, 1892).

The bridge had three river piers, each consisting of 4 iron columns 3 ft. diameter and braced. At the centre the roadway was 3 ft. higher than the earlier High Level Bridge of 1849. The bridge carried water pipes in the troughs of the lower chord girders, and the upper chords were of tubes 2 ft. 3 in. diameter which also served as a gas main between Newcastle and Gateshead.

The bridge carried a 20 ft. wide roadway between the trusses, and outside of them were 7 ft. wide cantilevered footpaths. The Board of Trade fixed the live load at 80 lbs./sq. ft.

Road bridge at Newcastle, 1871 by Thomas Bouch

Fig 38 Redheugh Bridge, Newcastle, 1871.
(Cail, 1892)

over the whole bridge, which gives a total of 1.21 tons/ft. run or 0.62 tons/ft run for each girder, higher than the standard railway load of 1.0 tons/ft. at the time.

The Warren girders had a depth of about 22 ft., giving a span/depth ratio of about 11:1. This is not a high value compared to other Warren designs (Newark Dyke was 16:1 and Crumlin 10:1) and in addition it is likely that Bouch made the spans continuous over the piers, as he did at Beelah, Deepdale and the Tay Bridge. Why then the necessity for suspension ties to the towers? The towers were substantial, being 12 ft. wide at the base and 64 ft. high, and six in number. The ties too, were substantial, being 15 in. wide, 3/8 in. thick and in groups of eight. The addition of the ties seems to stem from the erection method, which may have been by cantilevering outwards from the pier tops. A careful inspection of photographs suggests that the centre section of each main girder may have been a suspended span, as in the Forth Bridge. A contemporary account of the bridge construction states that Bouch worked to a tensile stress of 5 tons/sq. in. in the wrought iron, which had an ultimate stress of 22 tons/sq. in.

The erection method may have been tied to an Admiralty or other restriction on the obstruction of the Tyne waterway, as occurred at Chepstow. (Although there were twin spans over the waterway at Redheugh, the main shipping channel was under the north span). If this was so, and the bridge was indeed constructed by cantilevering out, then this was a "first" for the use of the method in Britain and perhaps the world.

As a footnote, it may be said that perhaps the bridge is a little uncharacteristic of Thomas Bouch. It almost appears over-designed, with its deep trusses, probable continuity at the piers and tie backs to the towers. But perhaps it shows Bouch at his best, innovative and inexpensive. But the bridge certainly displayed yet again the versatility of the braced girder in yet another form.

CHAPTER 6

AMERICAN DEVELOPMENTS AND BRITISH PRACTICE

The American scene

Review of British/American practice in the 1850s

Chapter 6

American Developments and British Practice

The American Scene

The thesis has so far reviewed the development of the girder bridge in Britain over 50 years, from 1820 to 1870, and it is now appropriate to turn to America and examine the development there over the same period.

In America generally timber was plentiful and cheap in the period 1800-50, and iron was expensive. There was a tendency therefore for bridges of all types of moderate spans to be built of timber. With the coming of railways this generated problems of heavy loading and the need to reduce deflection, and timber gradually gave way to iron. For a time a mixture of the two prevailed in bridge trusses of different types and the use of timber continued, but to a lesser extent. Eventually however, the advantages of iron - its greater strength, rigidity and durability, and the fact that it did not catch fire - held sway, and the used of timber for railway bridges disappeared. The distinctive early American covered bridges owed their appearance to the need to protect the timber bridge structure from the weather. Iron structures did not require such protection.

At first sight a review of American progress with girder bridges presents a bewildering array of engineers and the development of bridge types, firstly in timber then through the combined use of timber, cast-iron and wrought iron, and finally to iron alone. Many types began life as framed girders in timber, but had to be strengthened and stiffened by the addition of arched members to reduce deflection, and the final structural arrangement was a hybrid that was not a true girder. Such bridges have been rejected for inclusion in girder development, but are mentioned in passing.

Regarding calculation, it seems that early American timber bridges were designed empirically, often in a process of trial and error guided by failures. Sometimes a model was built and tested, and an attempt was made to scale up the model to the full-size prototype and obtain corresponding member sizes. But the combined arch-girder hybrids were complex and highly redundant, and the first hint of any analysis in practice comes from their use in Russia and investigations by D P Jourawski as late as 1847. The

American engineers seemed happy with this situation although there were many bridge failures. Steinman quotes a figure of 25 failures a year in the 1870s. (Steinman & Watson, 1957). Berridge puts the figure even higher at 27, (Berridge, 1969) though some of these could be attributed to climatic effects, when intense cold could be a cause of brittle fracture in cast or wrought iron.

Sometimes the use of iron for bridge girders was distrusted due to the long familiarity with the use of timber. Early in 1850 Lackawaxen bridge on the New York and Erie Railroad collapsed. This was a design by Nathaniel Rider, whose bridges were widely known and who was an experienced engineer. The collapse so shook the directors of the New York and Erie that they ordered the removal of all iron bridges on the railway, and re-designs were made in timber. This extreme measure led to the abandonment of two of Squire Whipple's iron bridges on the same railway. He too was an experienced engineer, who, he said, had built over one hundred bridges with one failure. Fortunately such panic measures were rare.

Early American Designs For Girder Bridges and their Designers

Theodore Burr (1771-1822)

The panel principle whereby a load is carried from the bridge deck to the upper chord and thence by the panel bracing to the abutment seems first to have been devised by Theodore Burr. This principle had in its roots the king-post timber truss, to which were added a series of panels on either side to make up the requisite span. It was defective in rigidity and had to be stiffened by combination with an arch. But the panel principle formed the basis of nearly all the early American bridge types, and later, overseas. In 1802 Burr built a bridge with four large arch reinforced trusses (154-180 ft. span) over the river Hudson at Waterford. Another of his bridges spanned the river Potomac at Washington, a long series of spans of 120 ft. each. This bridge had parallel chords and panels, but also incorporated internal raking struts to the abutments, which rendered it a hybrid. (Figs 39, 40, 41, 42).

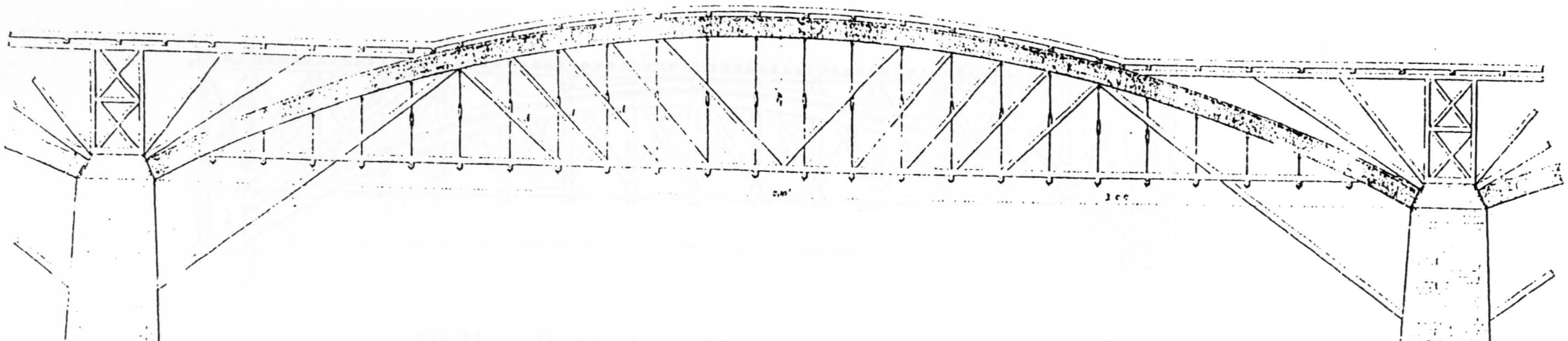


Fig 39 Early Timber bridge by Theodore Burr over Delaware River, Trenton, New Jersey (c.1810?) (Culmann, 1851)

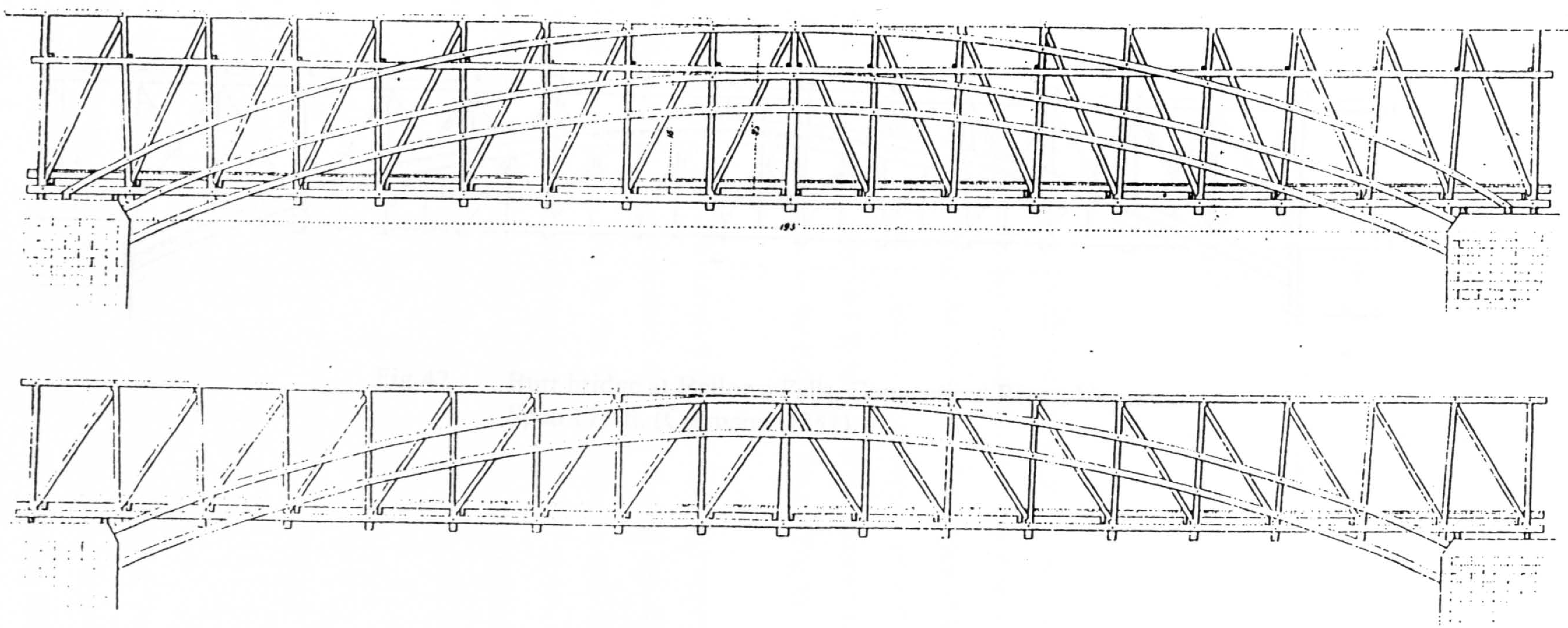


Fig 40 Burr timber bridge, inner and outer trusses, at Mill Creek, Cincinnati, Span 195 ft. (1815?) (Culmann, 1851)

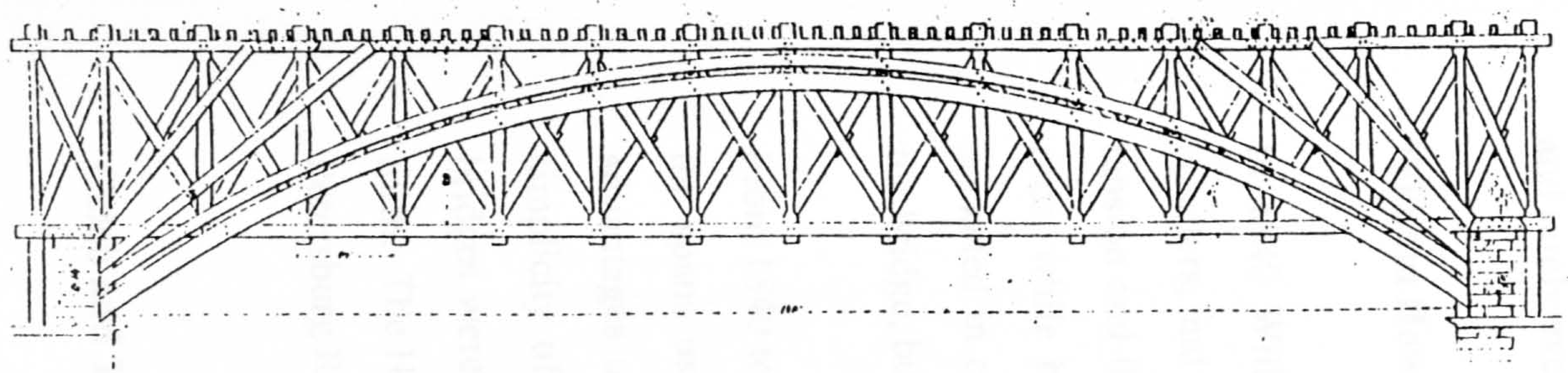


Fig 41 Burr bridge at Onion River, Burlington, Vermont.
Two 140 ft. spans. 1815. (Culmann, 1851)

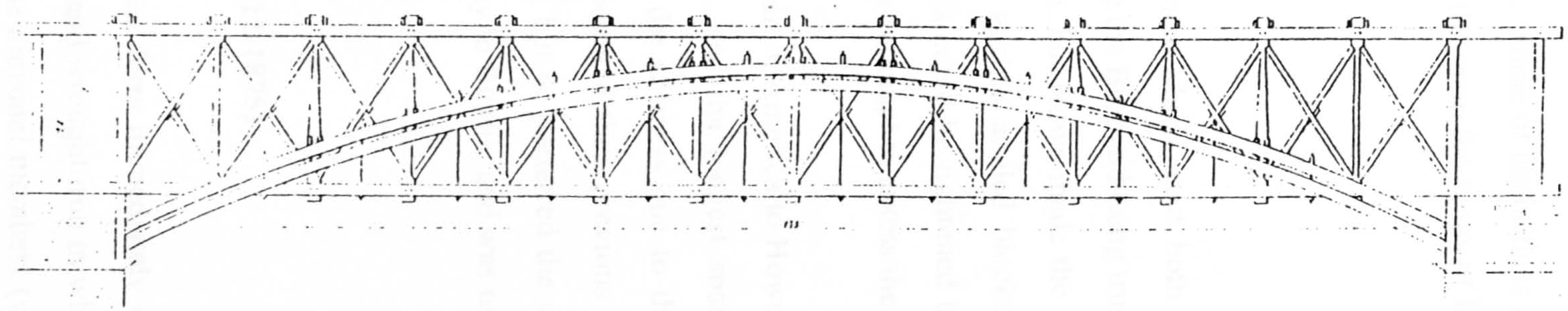


Fig 42 Burr bridge at Bellows Falls, Connecticut River, Vermont.
Span 175 ft. (Culmann, 1851)

Colonel Stephen H Long (1784-1864)

In 1837, S H Long introduced the idea of "counters" as they were known. This was cross bracing in the panels of trusses. He was originally in the U S Army's regiment of Topographical Engineers, but joined the Baltimore and Ohio railway in 1827. His panels had diagonal bracing, and although he understood the function of bracing, there is no record of him having calculated the forces. Some of his bridges contained internal raking struts, as Burr's did, and were hybrids. In the main the spans of Long's bridges were short and rarely exceeded 100 ft. (Figs 43, 45).

William Howe 1803-1852

In 1840 William Howe introduced a truss which used both the panel principle and counters, and was carefully detailed. Like the Burr and Long trusses, the verticals were in tension and the diagonals in compression, but Howe made the verticals of wrought iron rods, while his timber diagonals butted against angled blocks of hard wood, or were socketed in cast-iron shoes. The verticals could be tightened to improve the camber of the bridge, but this made a nonsense of any attempt to assess the forces in them. (Fig 46).

From 1840 to 1870 more bridges were built in America to Howe's designs than any other, the spans usually being 100 to 150 ft., and the longest span 300 ft. It had all the advantages of the panel system and of the stiffness due to the counter-braces, and its simplicity of design made it more economical then previous forms. Some of Howe's bridges were double intersection trusses, but he preferred the simpler single-intersection form. The Howe truss even found its way to Russia, and was used on the Moscow and St Petersburg Railway.

Caleb and Thomas Pratt (the latter 1812-1875)

In 1844 the Pratts patented a truss whose form immediately earned it the name of "N-truss". It was composed of both timber and wrought iron in which the new and important idea was introduced of using iron for the diagonal members (which were in tension) and timber for the vertical struts. The larger section of the timber member made it more suitable to resist compression and buckling, and it was also shorter in length than the

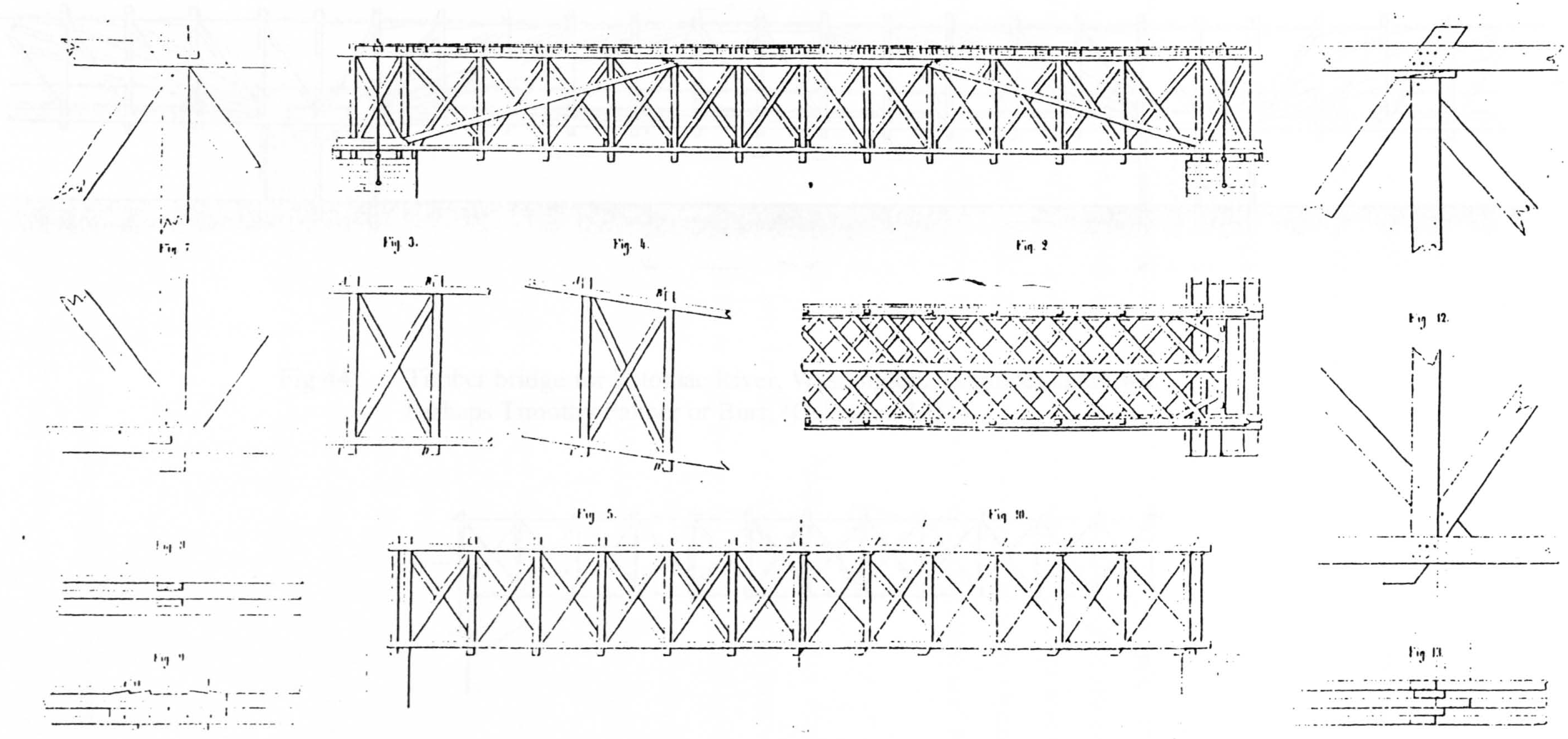


Fig 43 Colonel S H Long. Typical truss system. (Culmann, 1851)

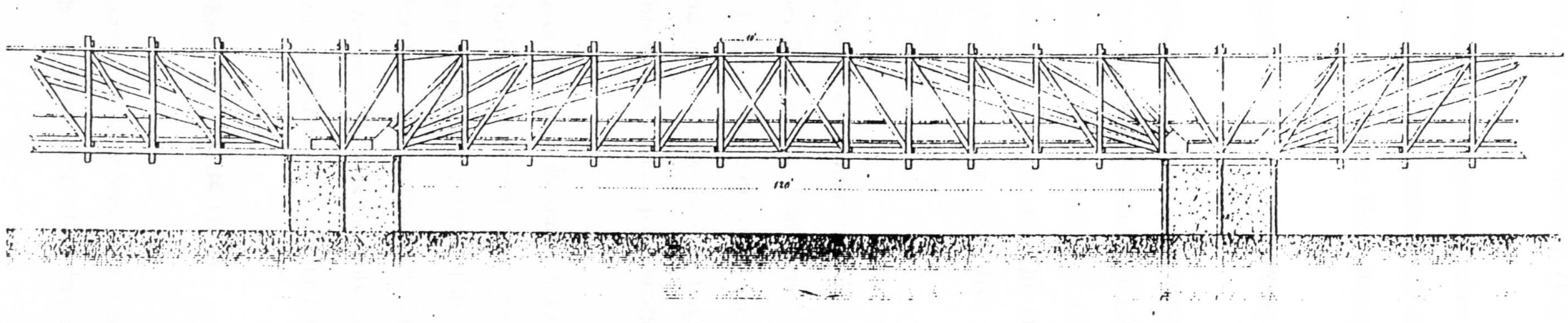


Fig 44 Timber bridge for Potomac River, Washington. Engineer unknown. Perhaps Timothy Palmer or Burr. (Culmann, 1851)

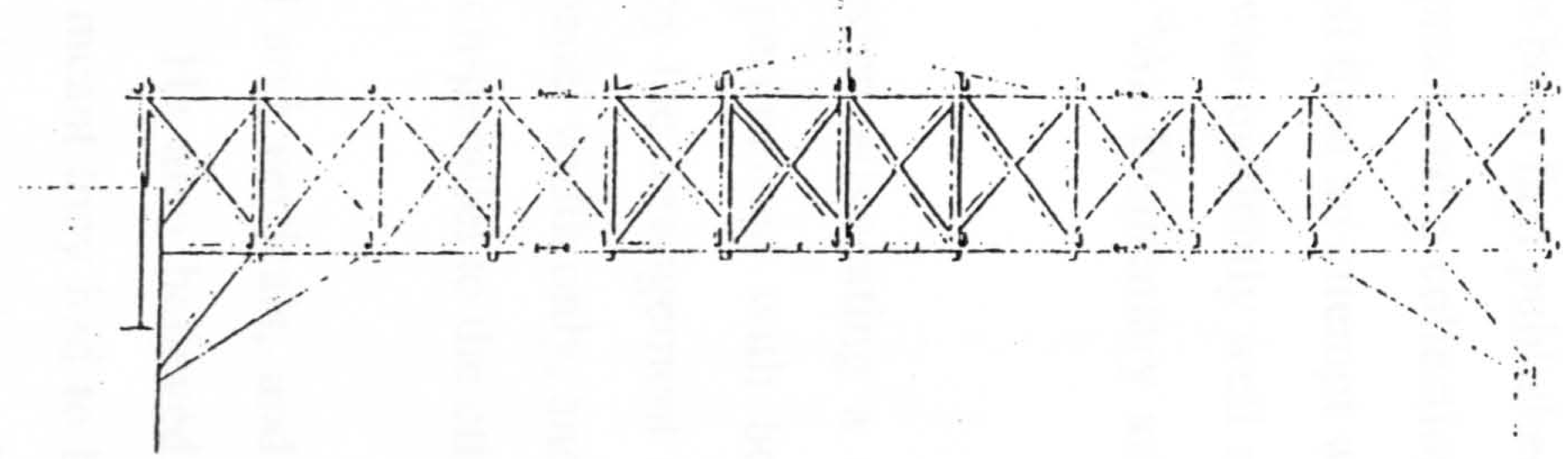


Fig 45 Timber bridge for Potomac River, Washington by S H Long. Series of 120 ft. spans. (Culmann, 1851)

diagonal. This was an economic arrangement but the slender diagonals required counter bracing in the centre panels to resist reversals of forces due to moving loads. From about 1850 the Pratt truss began to be built wholly in iron, using wrought iron for the diagonals and cast for the verticals. (Fig 48).

Initially the composite timber and iron Pratt truss showed more deflection than the Howe truss, but this disadvantage disappeared with the all-iron truss, and eventually the Pratt truss overtook the Howe in popularity.

Squire Whipple (1804-1888)

Information on truss calculation seems to have been first published in America by a New York engineer, Squire Whipple. In 1847 he produced a collection of Essays entitled "A Work on Bridge Building" wherein for the first time an attempt was made to analyse the forces in truss girders. This significant work was evidently well received, for in 1873 he published a more exhaustive textbook called "An Elementary and Practical Treatise on Bridge Building".

Whipple developed his ideas into bridge design advocating a double system of web members of the Pratt type, and he also experimented with bowstring girders. The Whipple truss however is easily recognised by the arrangement of the diagonals in the end panel, where the first diagonal spans one panel width only, and all the other span two. This means the end panel diagonal is always non-parallel to the others. (Fig 49).

Later Whipple favoured a tubular top chord and verticals, and parallel-chord trusses, which he reckoned were the most efficient. He also believed in substantial bracing between the top chords of his bridges, which meant they had to be deep enough to give clearance for locomotives, and this was not always possible. It was claimed that the Whipple truss was the only one which solved the problem of keeping the panel lengths within reasonable limits for long spans. (Whipple, 1847).

Ithiel Town (1784-1844)

Town was an architect who patented his lattice design for the webs of bridges in 1820. The bridges were of timber and the web members were of light sections built up in

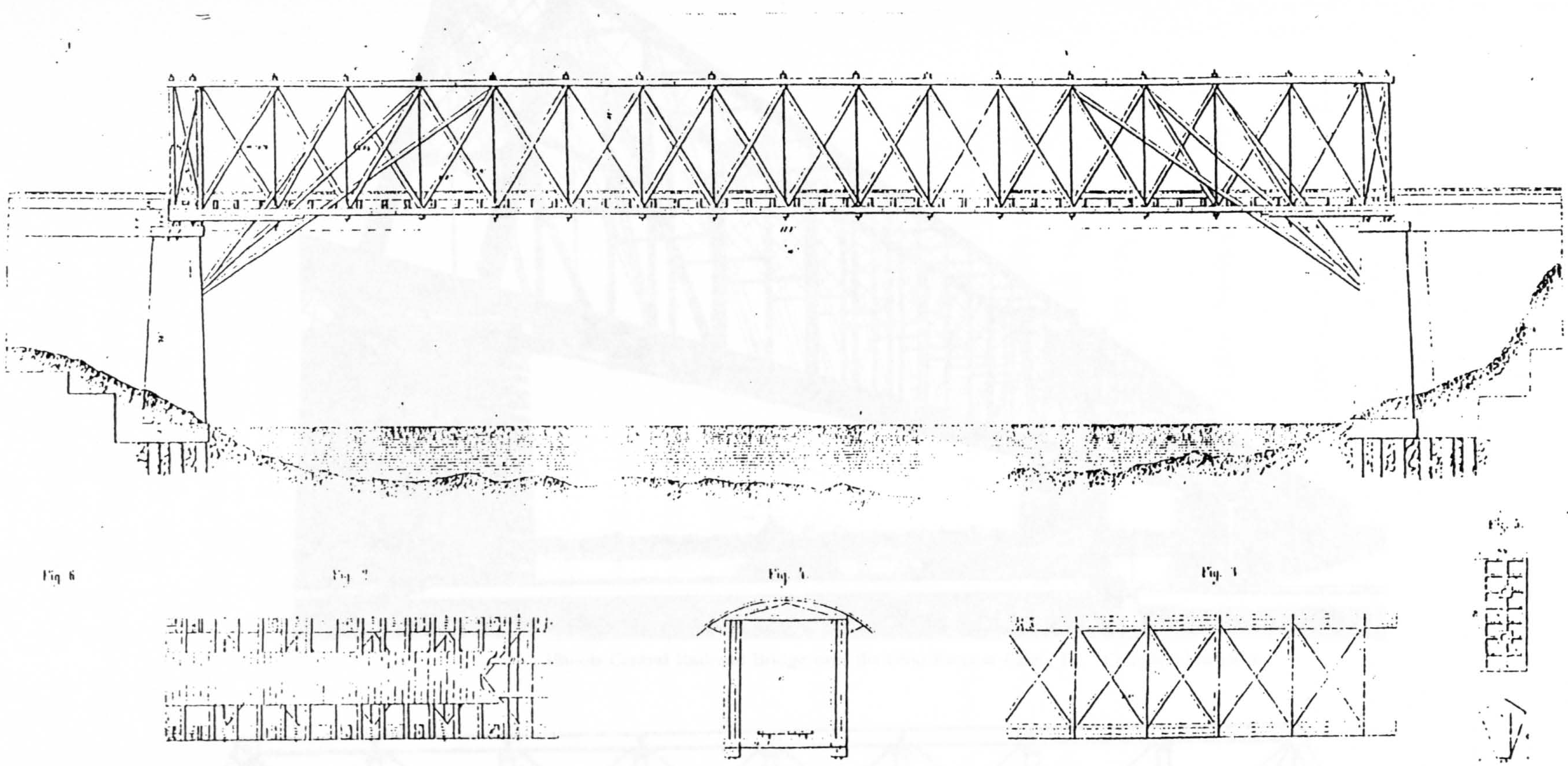


Fig 46 Howe timber bridge at Chilapac River, Connecticut. Span 112 ft.c. 1840. (Culmann, 1851)

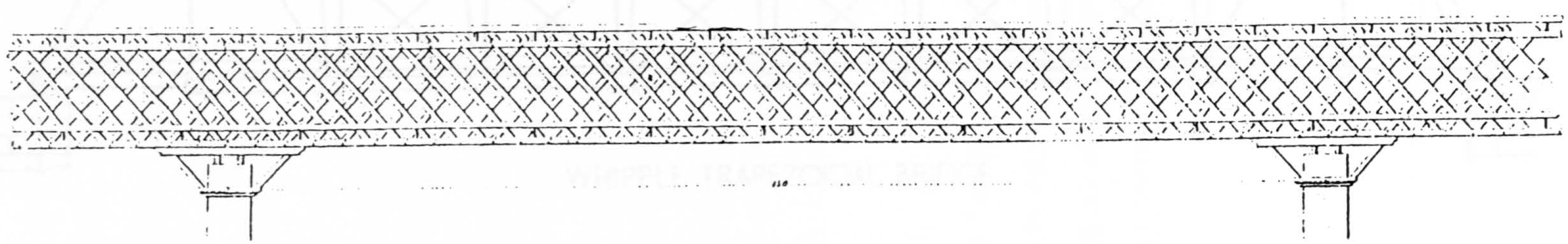
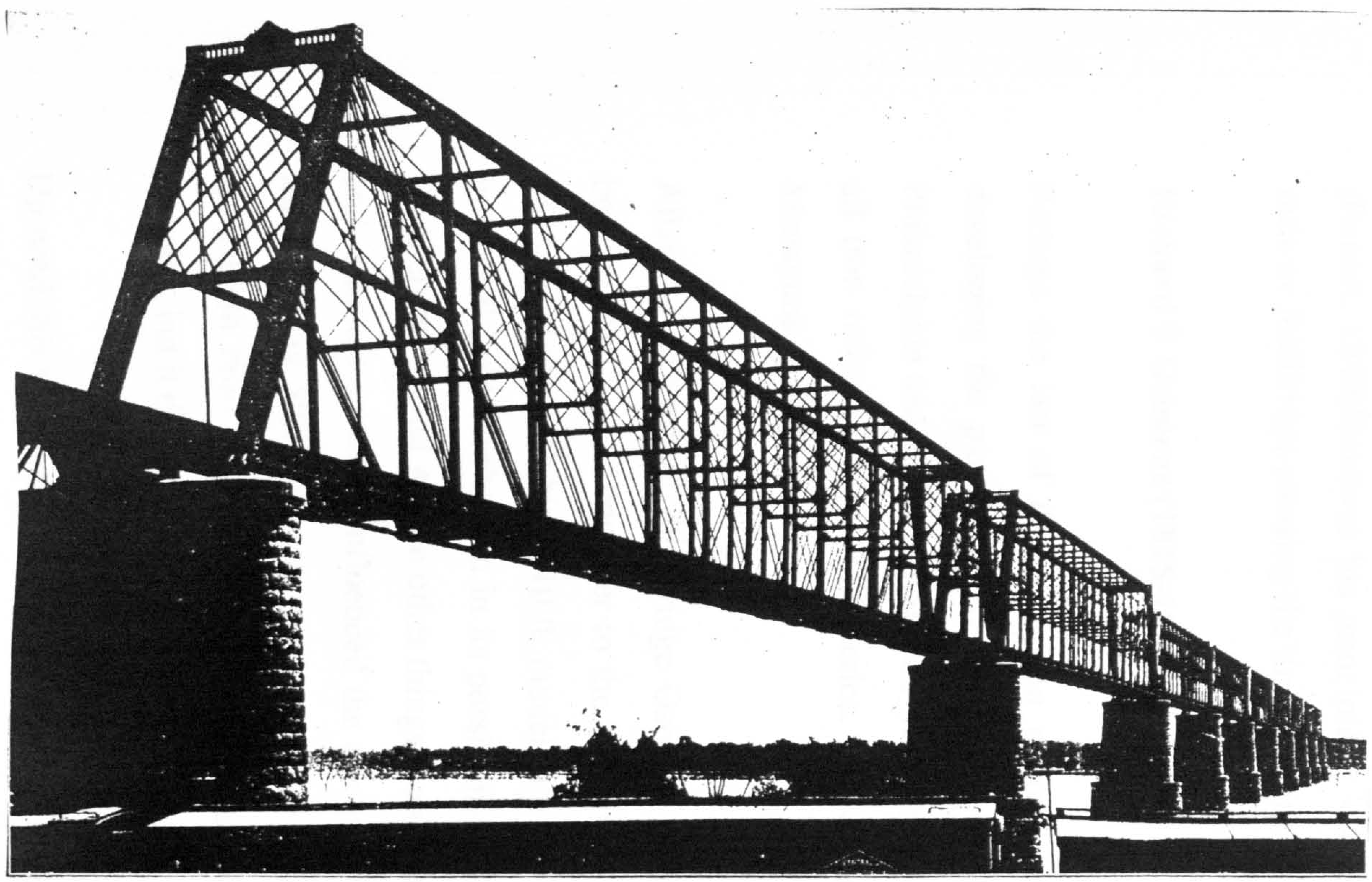


Fig 47 Ithiel Town. Twelve 150 ft. spans over James River, South Dakota. (Culmann, 1851)


Illinois Central Railroad Bridge over the Ohio River at Cairo, Ill. Completed in 1880.

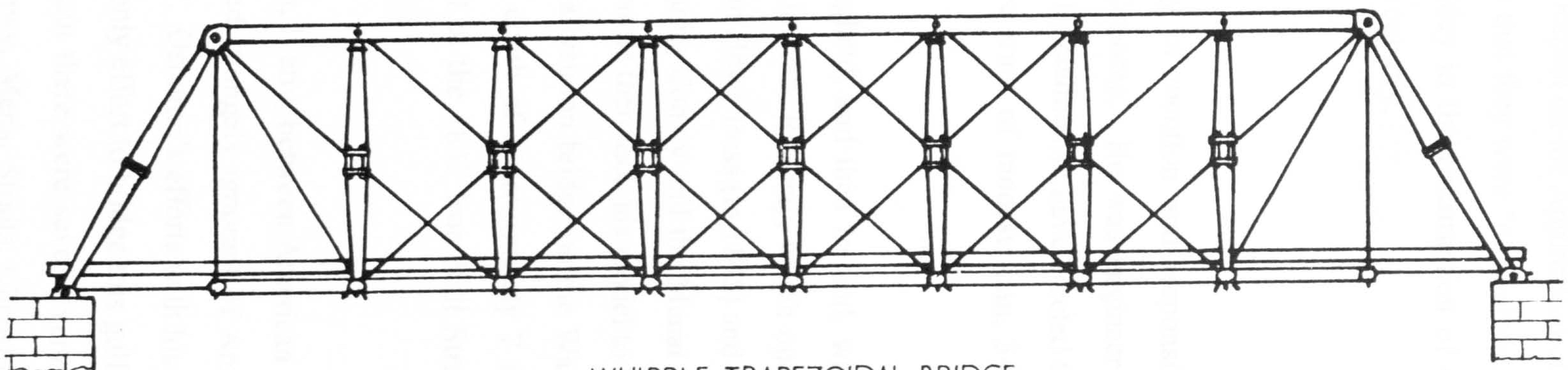

WHIPPLE TRAPEZOIDAL BRIDGE

Fig 49 Squire Whipple's early truss panel arrangement.

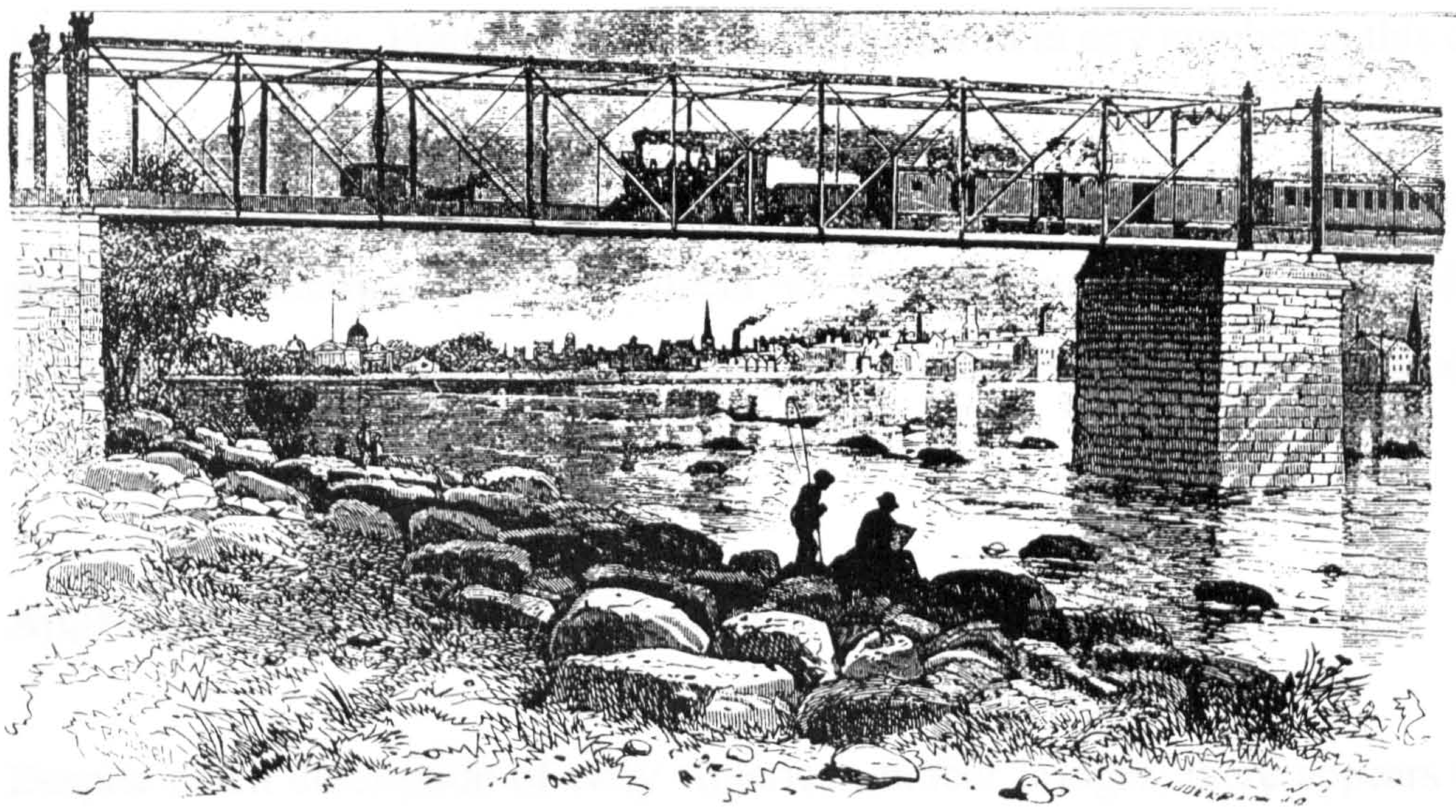
diagonal form, crossing one another usually at an angle of 90° (Fig 47). Because the members were light, they lent themselves to easy bridge construction at remote sites when transport was a problem. However the lightness of the sections meant they were vulnerable to buckling, and this disadvantage continued even when the bridges were constructed in iron. However several were built using Town's principle, including John MacNeill's bridge of 1844 over the Royal Canal in Dublin, having a span of 135 ft. Town's lattice design received scant attention in the "Report on the Application of Iron to Railway Structures " of 1849 when it was said curtly that they were "of doubtful merit". (Report, 1849)However his principle is still seen today in the construction of many a railway footbridge crossing the lines at stations in this country.

Richard B Osborne (1815-1899)

Perhaps the last of the American engineers worthy of mention and responsible for developing the parallel-chord truss girder is R B Osborne. He was engineer to the Philadelphia and Reading Railroad in 1842. In 1845 he claimed to have erected the first all iron railway bridge in America. It was a Howe truss of modest span, 34 ft., at Manayunk, Pennsylvania.

After the erection of this bridge Osborne visited England, and then Ireland, where he became involved as engineer to the Waterford and Limerick Railway, which opened in 1848. He took with him a 10 ft. model of a 200 ft. span Howe truss (in 1845) and tried to interest Robert Stephenson in its possibilities for bridging Conwy and the Menai Straits. Stephenson's mind was on other things, however. Osborne then took his model to Ireland and it almost certainly influenced the design of the Ballysimon bridge on the Waterford railway. (Fig 50). This was an 86 ft. span having a depth of approximately 7 ft. or a span/depth ratio of about 12·3. Osborne exhibited it at the 1847 Iron Rail Structures Enquiry, but it did not win favourable comment.

Up until this time it seems there was very little contact, if any, between American bridge engineers and those in Britain, and the British seemed strangely ignorant of American developments, almost as if they "didn't want to know". Osborne's efforts with his model seems to have been virtually the first and perhaps the only effort to bridge this gulf. This situation may have been exacerbated because in Britain there were several very large-scale bridges under design and construction (e.g. Conwy, Menai Straits, Chepstow and



Trenton Viaduct *on the Delaware River affords an example of the many Pratt trusses built in iron for railways in the U.S.A.*

Fig 48 Early Pratt truss in iron at Trenton, New Jersey.
Note unusual subdivision of lower chord panel length.

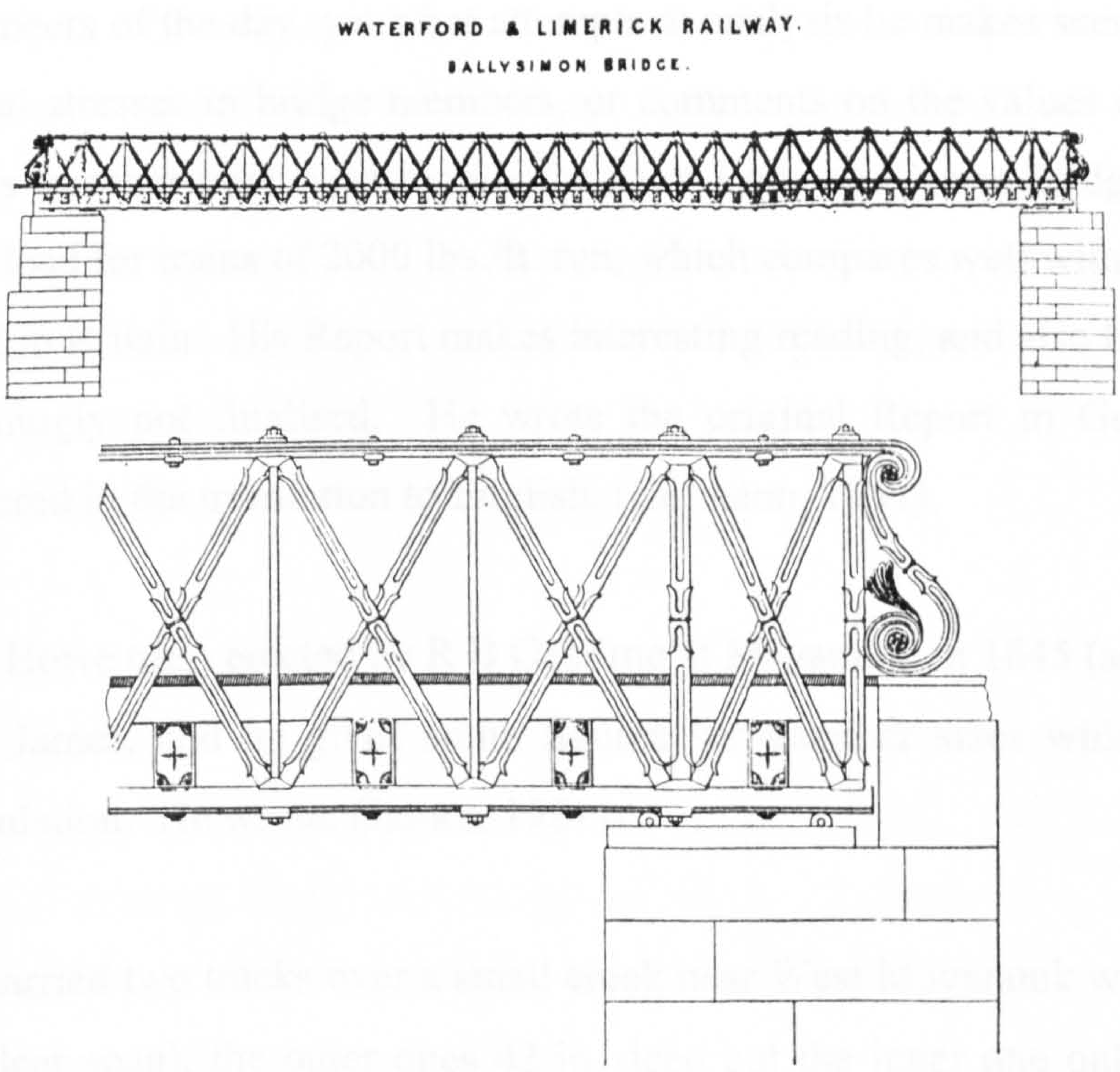


Fig 50 R B Osborne's influence on the Waterford and Limerick Railway, Ireland.
A Howe truss of 86 ft. span. (Berridge, 1969)

then Saltash), which dwarfed anything under construction at the time in America. It was years before Howe, Pratt or Whipple trusses appeared in any number in this country.

Another feature was that the open web girder was unpopular and the panel truss practically unknown until the advent of the Warren girder in the 1850s. So the development of the girder bridge took a different form in each country, and did not merge until much later in the 1880s and 1890s.

Approximate Calculation of a Howe Truss

Despite much description of early American timber bridges, there appears to be little in the way of member sizes given on which a calculation might be based to obtain some idea of strength.

The Bavarian engineer Karl Culman, who made a trip to America in 1849-50 commissioned by the Bavarian Government with the aim of compiling a state-of-the-art Report on the construction of timber bridges, is also disappointing regarding strength calculations. Culman was reckoned to have had a good technical education at the Karlsruhe Polytechnicum, an education superior to that of most American bridge engineers of the day, yet what attempts at analysis he makes seem always to stop short of actual stresses in bridge members, or comments on the values of forces calculated. He gives member sizes occasionally for Burr, Long and Howe bridges, and uses a maximum live load for trains of 2000 lbs./ft. run, which compares well with the value of 1 ton/ft. run used in Britain. His Report makes interesting reading, and also his attempts at calculation seemingly not finalised. He wrote the original Report in German, and it may have suffered in the translation to English. (Culmann, 1851).

The Howe truss erected by R B Osborne at Mayanunk in 1845 (see above) is described by J G James, and he gives some figures for member sizes which allows a rudimentary calculation. He wrote: (James, 1981)

"It carried two tracks over a small creek near West Mayanunk with trusses 42 ft. long (34 ft. clear span), the outer ones 42 in. deep but the inner one only 31.5 in. (apparently to clear the running boards of the rolling stock). The top and bottom chords were specially forged from pairs of iron bars 2 in. and 2 ¼ in. square respectively. The diagonals were 3 ¼ in. x 2 ½ in. cast iron, and the verticals were the usual round Howe rods".

Since the inner single girder must have shared the load of two rail tracks and was shallower in depth than the outer ones (31.5 in.) it was obviously the most highly loaded. Its span/depth ratio was about 14.5 if we take the distance between bearings as 38 ft. (34 ft. clear span).

The live loading, say, 1 ton/ft. run (50% of two tracks). There are no figures given for the self-weight of the bridge structure, but it may be assumed as approximately 50% of the live load, i.e:

Dead load = 0.50 tons/ft. run

Total load on middle girder = 1.50 tons/ft.

Bending moment at midspan = WL/8 = 271 tons-ft.

Area of chord = 2/(2.0 x 2.0) + (2.5 x 2.5) = 20.5 sq. in.

Lever arm between chords, say = 29.5 in.

Therefore stress in chord = M/A x d = 5.4 tons/sq. in.

This stress would apply equally to the upper chord (compression) and the lower chord (tension). Provided the upper chord was restrained in some way the stresses appear reasonable, and they must have been much less in the outer girders. It is agreed that the calculation is extremely rudimentary, with no account taken of the contribution of the bracing, but it does give some indication of the adequacy of the bridge. The live loading of 1.0 ton/ft. may also be overestimated for 1845 in America.

Other Applications of the American Trusses

The advantages of the braced girder began to be seen as a stiffener for suspension bridge decks and John Roebling employed such a girder in his design for his Cincinnati bridge begun in 1856 and his Brooklyn bridge begun in 1869. He used trusses also in his four-span Pittsburgh suspension bridge of 1858. Trusses were also used as part of the stiffened-chain Point suspension bridge in Pittsburgh, an unusual design constructed rather later in 1877. (Fowler, 1929).

These stiffening girders were of various forms, but the Howe type seemed to be preferred. Roebling also stiffened his bridges with additional radiating cables from the towers. Generally these methods for stiffening suspension bridges were effective. Often the stiffening girder was constructed in two halves with a pin-joint in the centre, as in the Brooklyn bridge, but in modern bridges the girder runs right through with joints only at the towers.

The Bollman and Fink Trusses

No review of the American scene in the development of the girder bridge would be complete without comment on two of the most remarkable truss types which came to fruition in the 1850s. These were trusses with no lower chords. The structure consisted of a deck which carried the railway loading and acted as a compression member spanning between the abutments, and anchoring various tension members of the bracing.

Wendell Bollman

Bollman was an engineer on the Baltimore and Ohio Railroad, the earliest in America. His first bridge of significance was built at Harper's Ferry, Virginia, in 1852 and carried railway traffic until 1893. The truss span was 124 ft., built up of 8 panels of 18 ft. depth, the upper chord and posts being cast-iron and the diagonal ties wrought iron. Sometimes light counter bracing was used on Bollman trusses to stiffen the structure, and sometimes a lower chord was added to provide a through bridge if headroom was restricted.

The Bollman truss however suffered from the disadvantage that there was a considerable difference in length between the ties joining to the same king post, and this produced unwelcome temperature effects. It was thus somewhat inferior to the Fink truss, the only other of it type. The form of Bollman's truss suggests it may also have been vulnerable to deflection. Only short spans were built. (Fig 51).

Albert Fink (b.1827)

Fink was born in Germany but came to America at an early age and worked alongside Bollman as an engineer on the Baltimore and Ohio. Like the Bollman truss, Fink's was based on the king post, and each panel had a single post in compression, which was

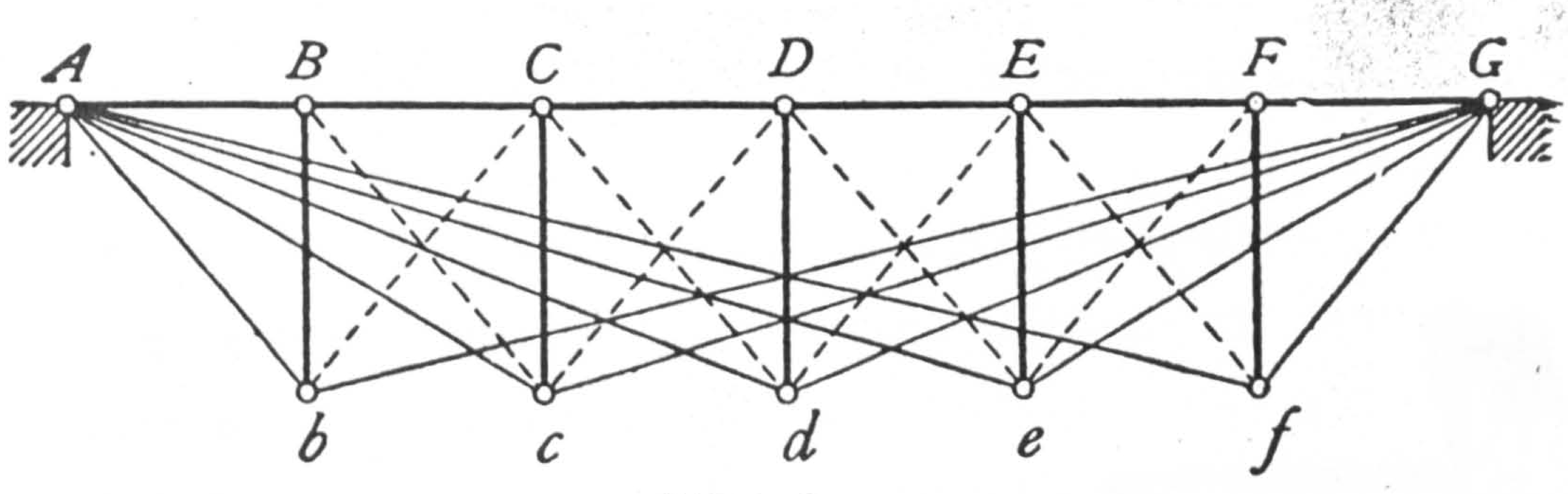
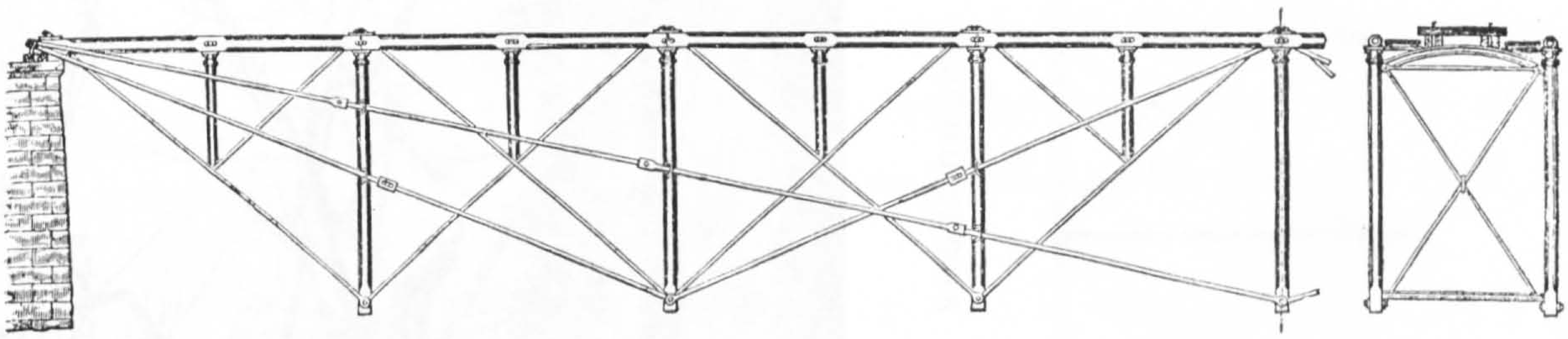


Fig 51 Wendell Bollman's truss arrangement. (Merriman & Jacoby, 1913)
Note disparity in ties' lengths to abutments.
The dotted lines were known as "counter ties", added to stiffen the structure.



Fink Truss.

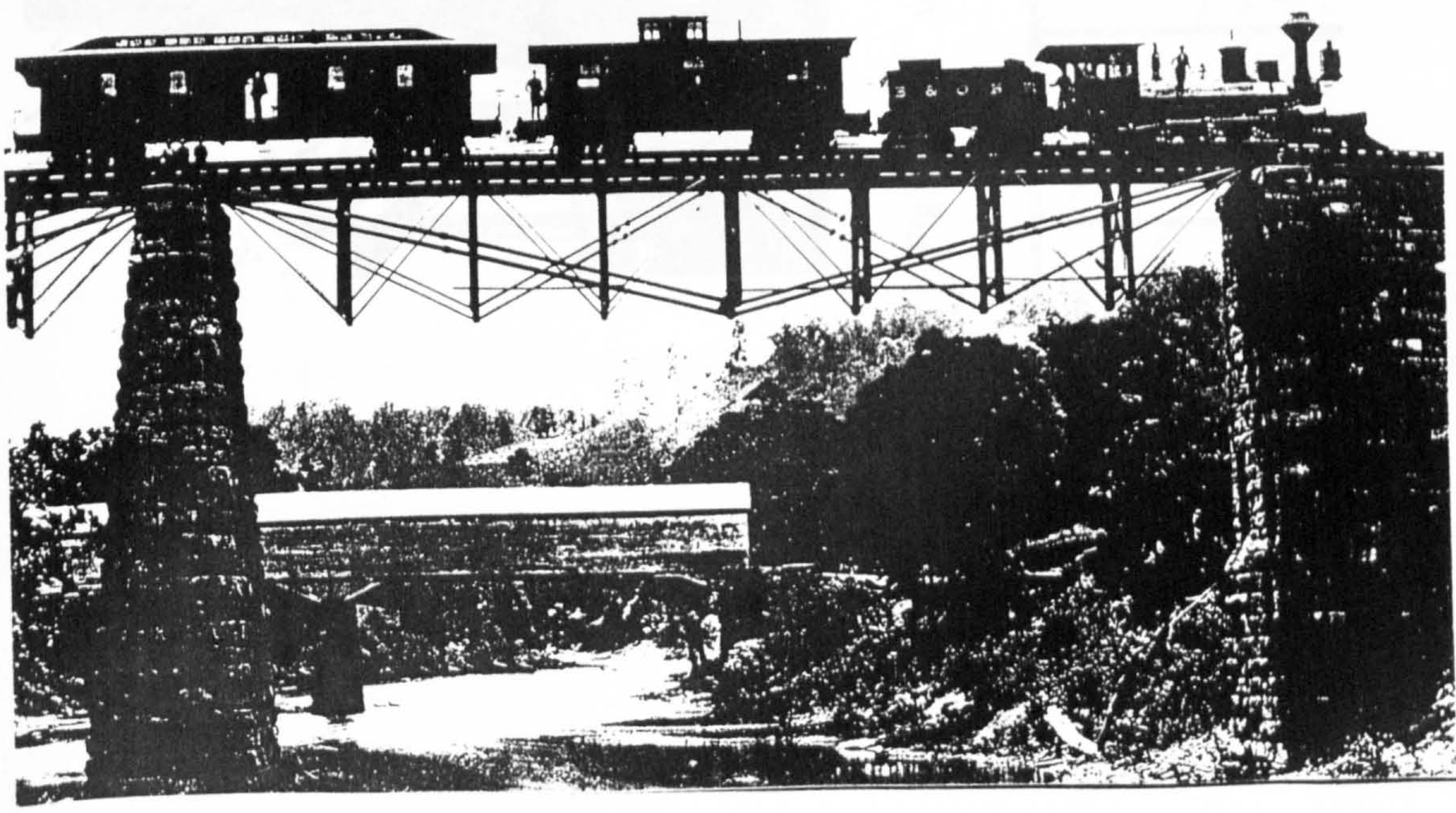
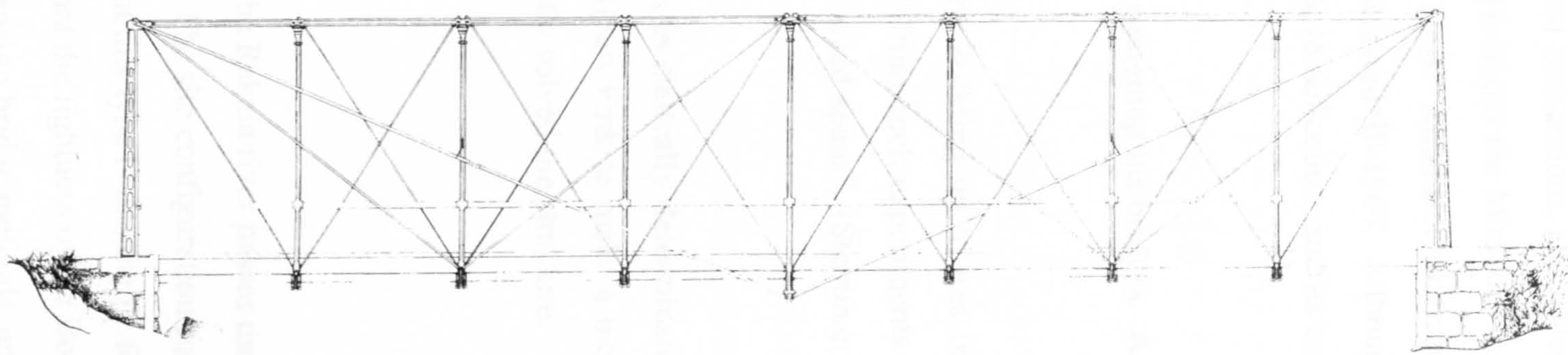


Fig 52 Albert Fink's truss, over the Monongahela River west of Clarksburg, West Virginia. No lower chord to truss. (c. 1875). (Brown, 1993)

Fig 52 Albert Fink's 1858 wrought iron truss



(1858, destroyed 1978)
South Branch Raritan River,
Hamden, New Jersey.
Trenton Locomotive & Machine Works, Fabricator.

carried by diagonal ties to the next panel point. This arrangement required a symmetrical form of panels – they generally numbered 8, with 7 king posts forming each sub-division.

The first Fink truss was built in 1852 over the Monongahela river at Fairmount, West Virginia. It had 3 spans of 215 ft. each and had pinned connections, and lasted till 1884. The longest Fink trusses ever built were of 306 ft. span across the Missouri at St Charles, 1871, but they had a short life and were replaced in 1884. Another Fink truss of 245 ft. span was built over the Ohio at Louisville in 1870 and lasted till 1904. Although the Fink truss was preferred to Bollman's it too was vulnerable to deflection, and its use generally came to an end about 1880. (Fig 52).

It is said that Fink employed an ingenious method in designing his bridges. According to a contemporary:

"Fink would go to work with pieces of tin and wire, building up trusses in miniature, testing strains and stresses carefully upon these, and from such experiments making his deductions and formulae for the construction of full sized spans". (Steinman & Watson, 1957).

Yet when the Fink truss is examined it will be seen as statically determinate and easily analysed panel by panel. There was no need for Albert Fink to make a model, though perhaps he did it to satisfy himself that he had correctly solved the structure.

A Bollman-Type Truss in New Zealand

Near Christchurch on New Zealand's South Island, the Rakaia river passes through a gorge which in the 1870s required a railway bridge. The site configuration suggested a suspension bridge design for ease of construction, but this type is unsuitable for railway loading. However, someone, somewhere, realised that the lightness of the Bollman bridge design might be adapted for erection by suspension bridge methods, and a design was proposed by a Scots Engineer, John Carruthers.

Carruthers had been engaged on the survey and construction of railways in America in the 1860s before arriving in New Zealand in 1871. He must have been familiar with current

American railway bridge practice, and with the work of Bollman and Fink. A great railway and public works programme took place in New Zealand in the 1870s, and plans for the Rakaia Gorge bridge were completed by 1877 under Carruthers, who by this time was Engineer-in-chief of the Public Works Department.

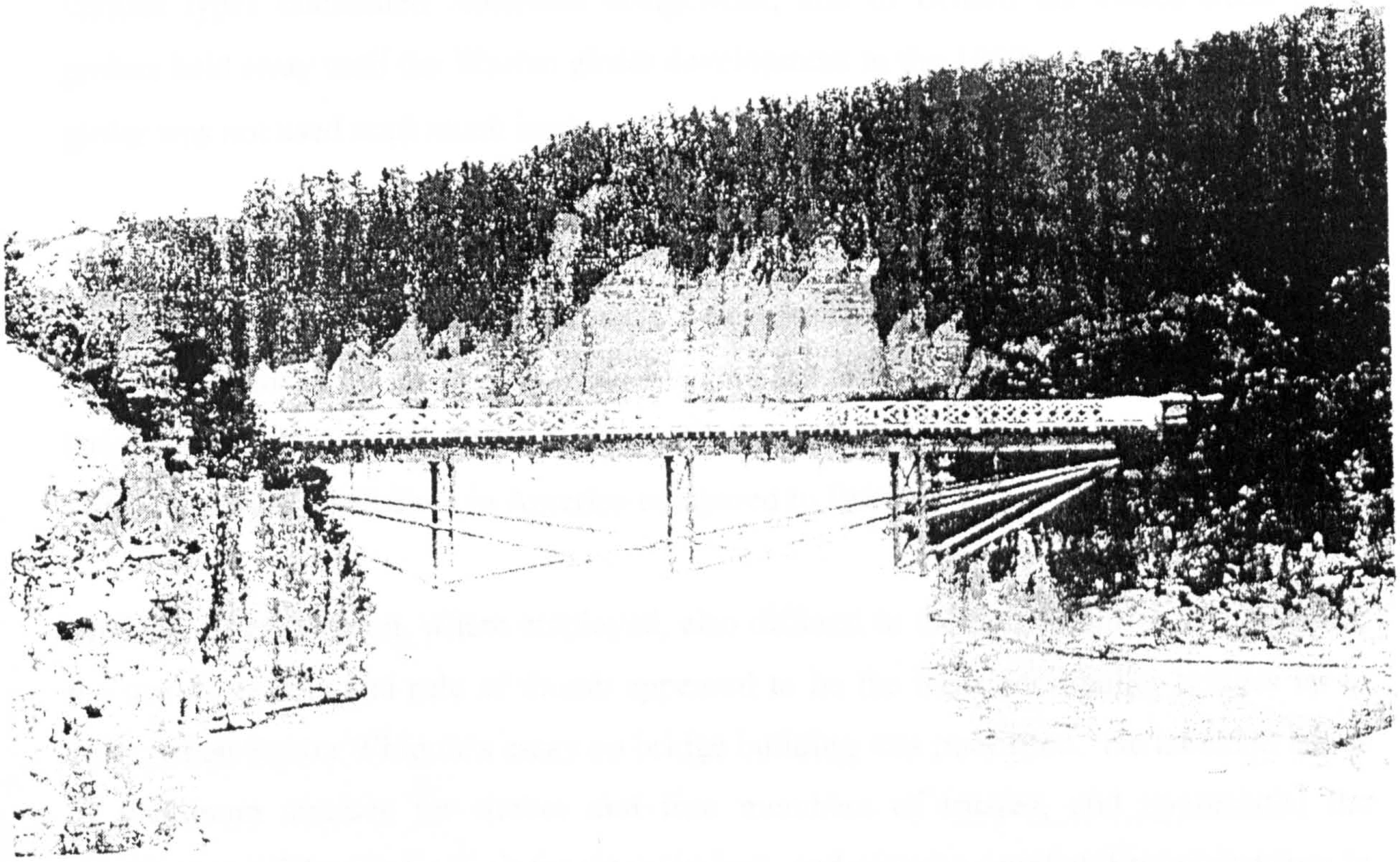
Normally in a Bollman truss the anchorage of the diagonal web members is to the ends of the top chords forming the deck of the bridge (Fig 51).; It is therefore self-anchoring, and the reactions at the supports are vertical. At Rakaia, however, the anchorage of the diagonals (there is no lower tension chord) is taken to points in the depths of the rock at the sides of the gorge. This means that the top chords are not under compression from forces is the diagonals, and only take local bending between the panel points of the truss, and as a result can be made lighter in cross-section.

The Rakaia span is 180 ft., divided into 4 panels of 45 ft. each. The depth of the verticals in approximately 24 ft., and slender hangers from the deck support the long diagrams between the verticals and the points of support (Fig 51A).

In erecting the bridge, the rock anchorages would be constructed first, then it is likely that temporary wire cradles would be slung across the gorge, and the bridge members erected. The erection method therefore to some extent must have generated the choice of the Bollman truss.

Tenders were invited for the supply of the ironwork from Britain in 1878, and by 1879 had arrived at the railhead for the bridge. A contract for the foundations and bridge erection was let in 1880, and the work was completed, after some delays, in 1882.

The bridge has Bollman's geometry and design principle, but because of the external anchorages, is best described as "Bollman-esque" today. The bridge is in a good state of repair and is preserved as an important part of New Zealand's engineering heritage. (Jones, 1994).



The Rakaia Gorge Bridge

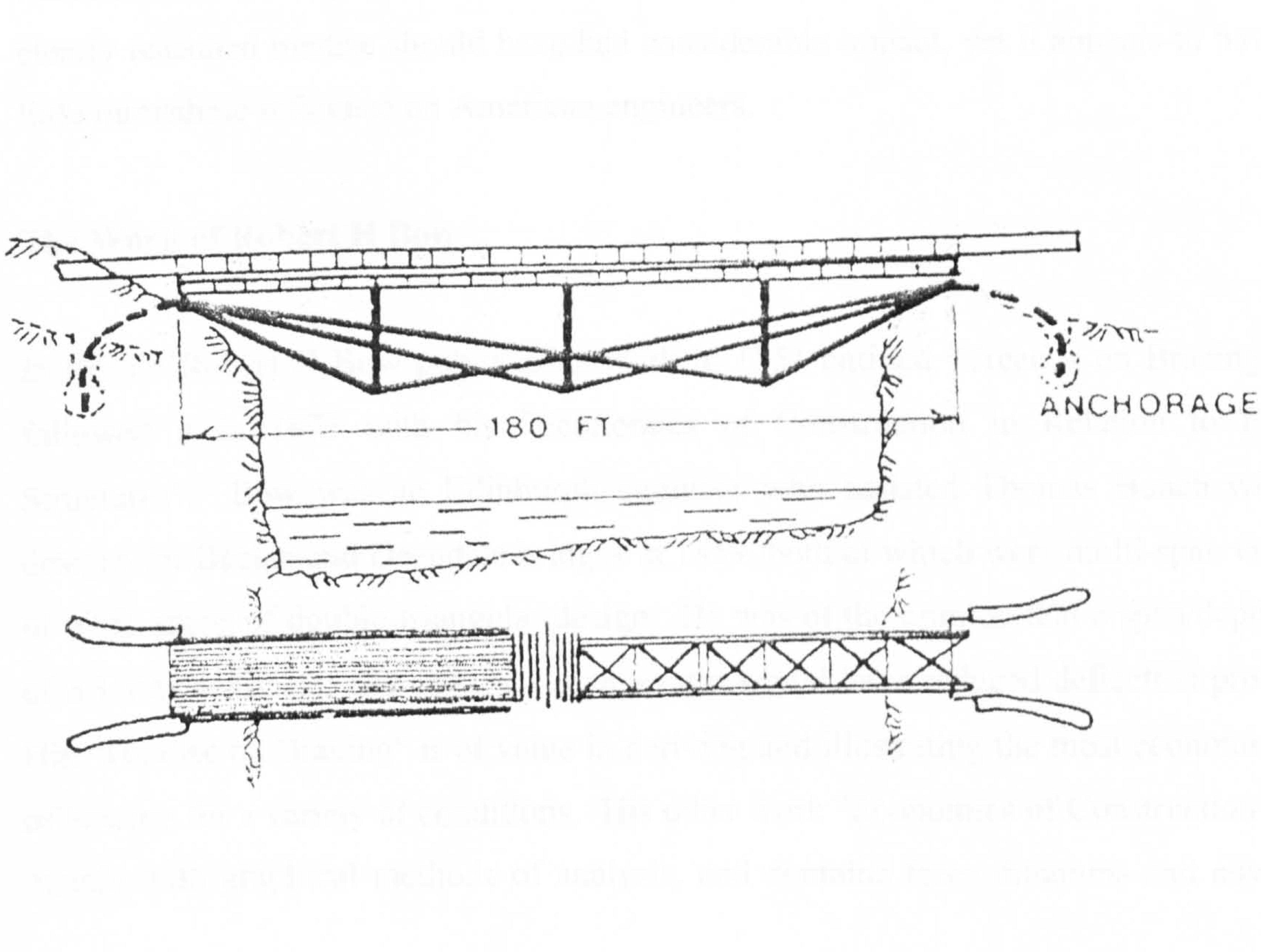


Fig 51A Rakaia Gorge Bridge, NZ (Jones, 1994)

Review of British/American Practice in the 1850s

From the foregoing it has been seen that the development of the girder bridge developed along somewhat different lines in Britain and America. The open-web panel girders of various types dominated American bridgework, and in Britain the closed-sided plate girders held sway until the Warren girder development in the 1850s. In America the plate girder was not used until much later.

Failures of American bridges were also numerous, and often seemed to be taken as a matter of course, the price of progress, whereas in Britain failures were taken seriously and investigated in detail, as with the Chester Dee bridge failure of 1847 and the Tay Bridge disaster of 1879. The differing attitudes to failure may have stemmed from the volume of work carried out in America compared to Britain.

Methods of calculation, where employed, also differed in the two countries. In America, the use of models and rule of thumb appeared to be the basis for smaller bridges up to 1847, when Squire Whipple's essay on bridge building was published. He included tables of maximum stresses for timber and iron members of trusses, and appreciated the significance of the elastic limit for wrought iron, and also the unreliability of cast-iron in tension. Since the majority of American bridge builders did not have the benefit of the mathematical training that could be had in the European engineering schools, Whipple's clearly reasoned treatise should have had considerable impact, yet it appears to have had little immediate influence on American engineers.

The Work of Robert H Bow

In Britain Robert H Bow published a work in 1851 entitled "Treatise on Bracing", and followed it in 1873 with his "Economics of Construction in Relation to Framed Structures". Bow was an Edinburgh engineer who assisted Thomas Bouch with his designs for Beelah and Deepdale bridges in 1859, both of which were multi-span viaducts of 60 ft. spans of double-triangular design. He was of the opinion that a span/depth ratio of 8:1 was desirable, and certainly such a ratio would have reduced deflection problems. His "Treatise on Bracing" is of value in deriving and illustrating the most economic form of bracing for a variety of conditions. His other work "Economics of Construction" deals mainly with graphical methods of analysis, and contains force diagrams and advice on

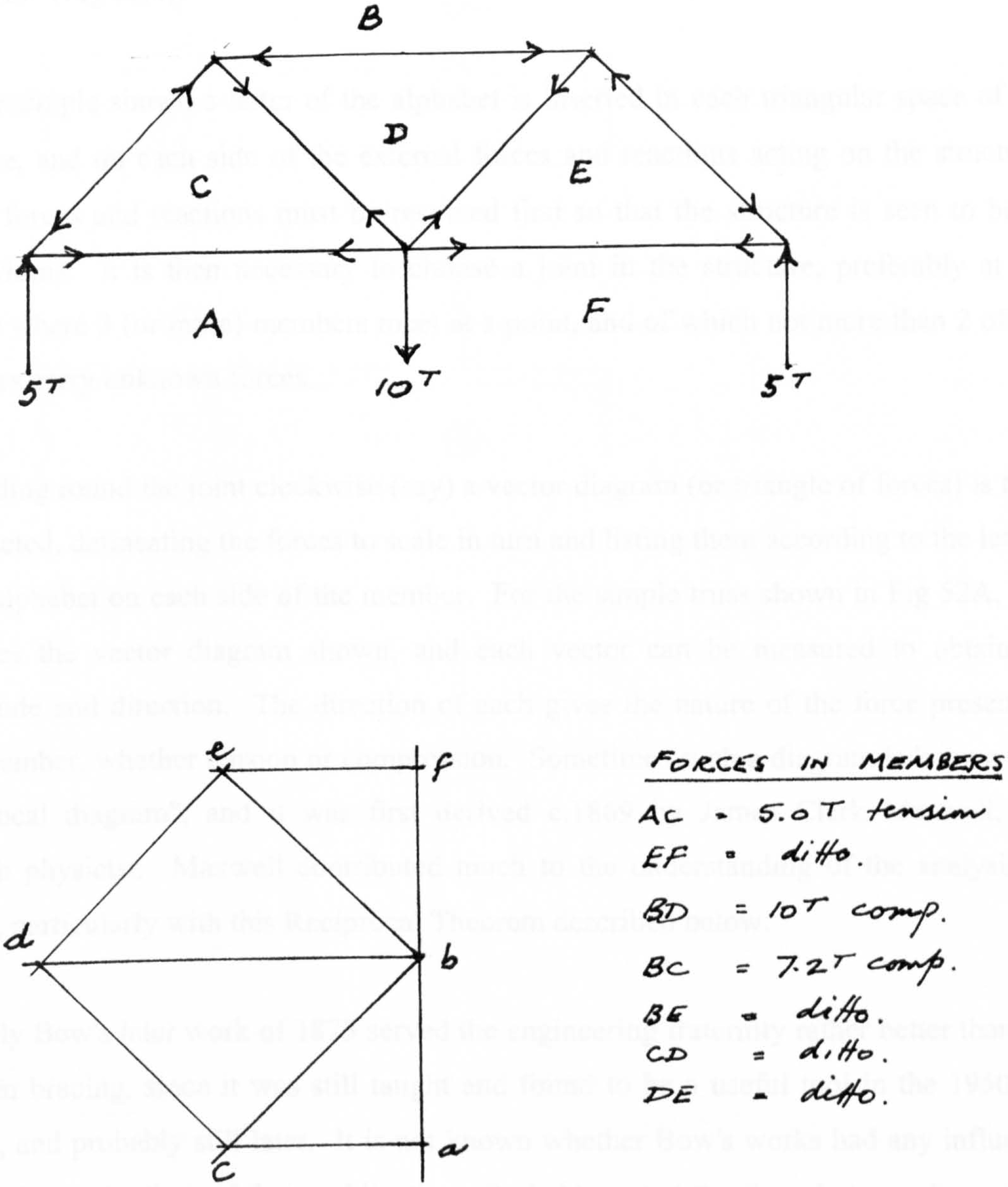


Fig 52A Bow's Notation,
Illustrating its use in analysing a simple truss.

drawing them for an innumerable array of structures. It also sets out his method of delineating the forces in members known as "Bow's Notation" which is of great value in constructing the force diagrams required in the analysis of triangular and other girder structures. (Fig 52A).

In the example shown a letter of the alphabet is inserted in each triangular space of the structure, and on each side of the external forces and reactions acting on the structure. (These forces and reactions must be resolved first so that the structure is seen to be in equilibrium). It is then necessary to choose a joint in the structure, preferably at the support where 3 (or more) members meet at a point, and of which not more than 2 of the members carry unknown forces.

Proceeding round the joint clockwise (say) a vector diagram (or triangle of forces) is then constructed, delineating the forces to scale in turn and listing them according to the letters of the alphabet on each side of the member. For the simple truss shown in Fig 52A, this produces the vector diagram shown, and each vector can be measured to obtain its magnitude and direction. The direction of each gives the nature of the force present in each member, whether tension or compression. Sometimes such a diagram is known as a "reciprocal diagram", and it was first derived c.1869 by James Clerk Maxwell, the Scottish physicist. Maxwell contributed much to the understanding of the analysis of frames, particularly with this Reciprocal Theorem described below.

Probably Bow's later work of 1873 served the engineering fraternity rather better than his work on bracing, since it was still taught and found to be a useful tool in the 1950s in Britain, and probably still later. It is not known whether Bow's works had any influence in America at the time of their publication. Probably not at the time, but may have done later.

The Work of James Clerk Maxwell in Structural Analysis

The other scientist in the field of structural analysis was James Clerk Maxwell (1831 - 1879), a polymath who made important contributions in the field of electromagnetism, colour vision, photo-elasticity, thermodynamics and many other aspects of science, including reciprocal diagrams and his Reciprocal Theorem, sometimes known as the Law of Reciprocal Deflections.

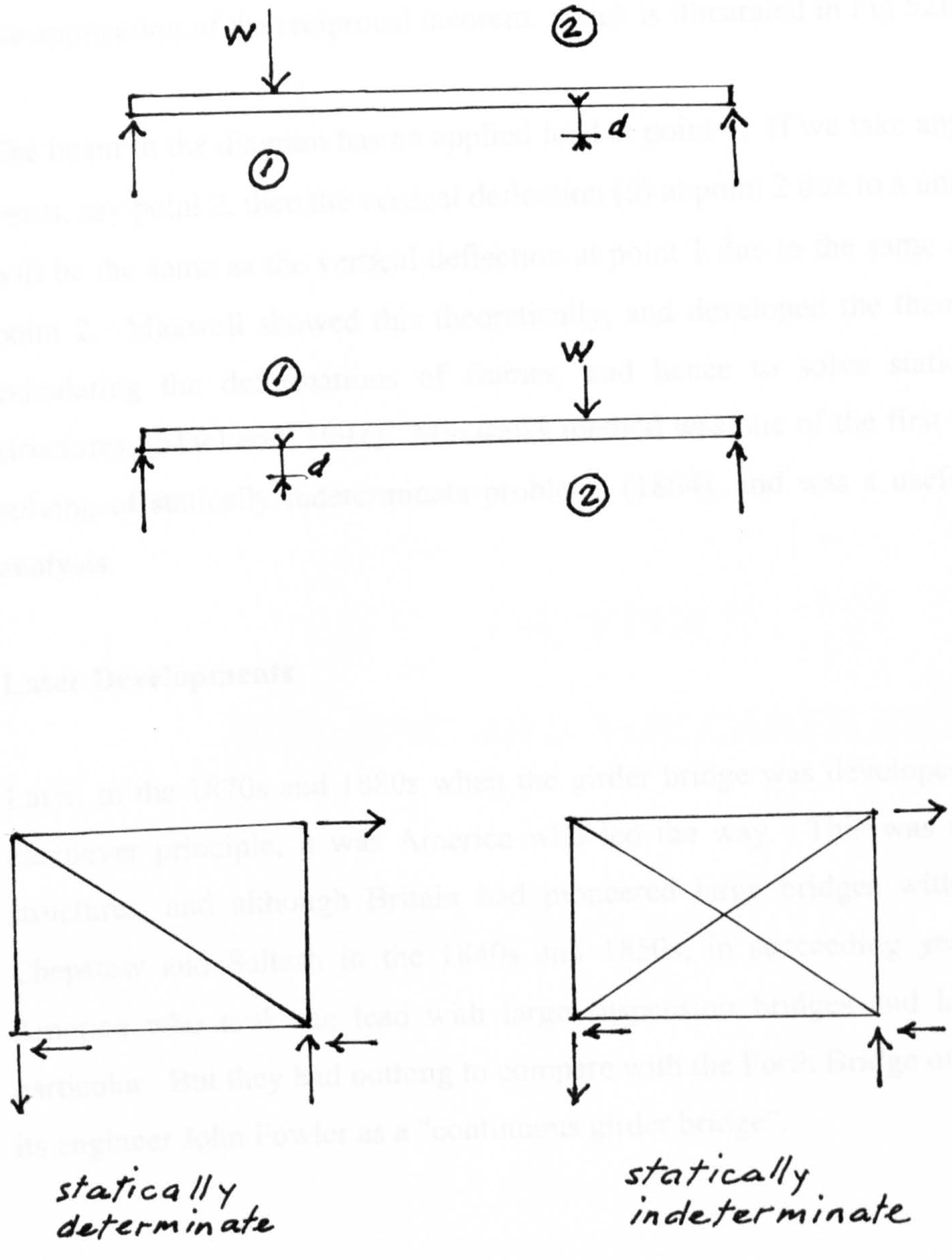


Fig 52B Clark-Maxwell's Reciprocal Theorem.

Much of the work in the solution of statically indeterminate structures involves the calculation of deflections or rotations. This work can often be reduced or simplified by the application of the reciprocal theorem, which is illustrated in Fig 52B.

The beam in the diagram has an applied load at point 1. If we take any other point in the beam, say point 2, then the vertical deflection (d) at point 2 due to a unit load W at point 1 will be the same as the vertical deflection at point 1 due to the same unit load applied at point 2. Maxwell showed this theoretically, and developed the theorem as a means of calculating the deformations of frames, and hence to solve statically indeterminate structures. (MacLeod, 1997). Maxwell's method was one of the first to be applied in the solving of statically indeterminate problems (1864), and was a useful tool in structural analysis.

Later Developments

Later, in the 1870s and 1880s when the girder bridge was developed and applied in the cantilever principle, it was America who led the way. This was in the field of large structures, and although Britain had pioneered large bridges with Conwy, Britannia, Chepstow and Saltash in the 1840s and 1850s, in succeeding years it was generally America who took the lead with large suspension bridges and large arch bridges in particular. But they had nothing to compare with the Forth Bridge of 1890, first known to its engineer John Fowler as a "continuous girder bridge".

CHAPTER 7

BUILDING BIG - THE CANTILEVER

The era of the cantilever bridge, 1867 - 1890

Chapter 7
Building Big – The Cantilever

The Era of the Cantilever Bridge

In the development of the girder bridge into the cantilever form the application of the braced girder reaches its peak, and it will necessarily require an extended study to appreciate the function of the girder and its suitability for the task.

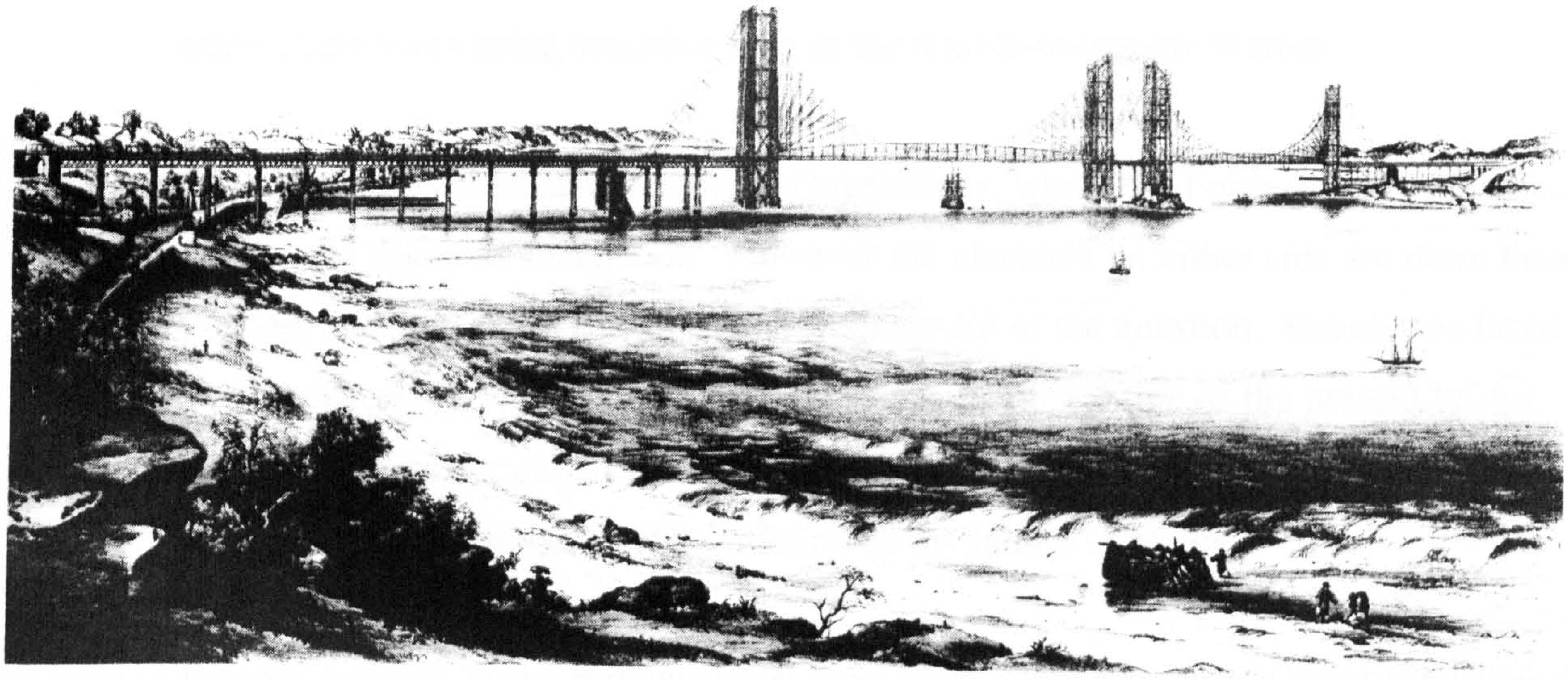
The greatest of all cantilever bridges is the Forth Bridge of 1883-90, on which the study will be based. But before work began there was a background to its design in which several proposals were made, and different forms of bridge and girders explored. It is of interest to review these, together with contemporary designs for cantilever bridges from America and from the continent.

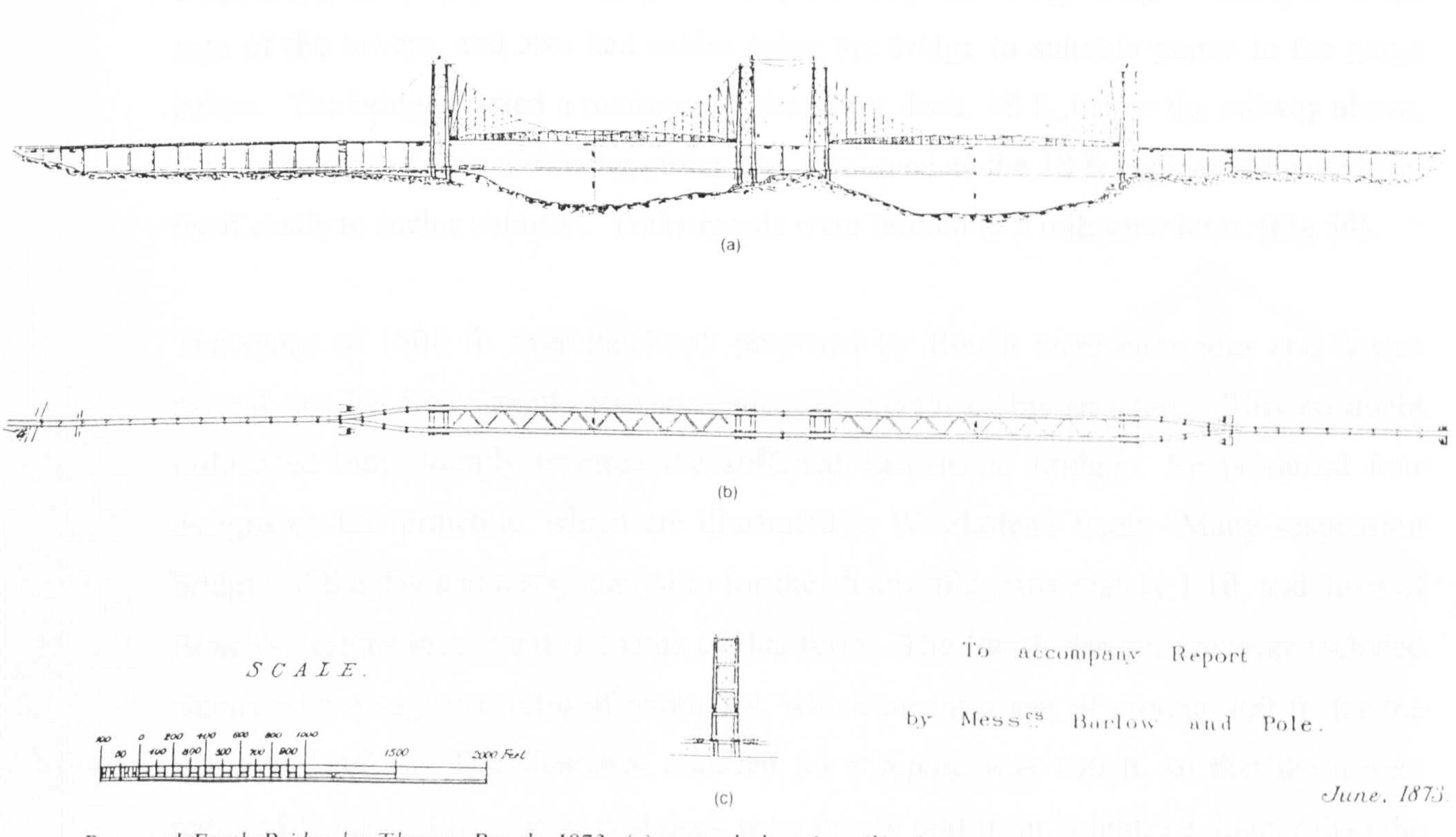
Preliminary Proposals

In the 1870s the career of Thomas Bouch also reached its peak. He was engineer to the North British Railway and had been commissioned to design and build a two-miles long bridge over the river Tay as part of the new main line from Edinburgh to Dundee. This bridge was completed in 1878, and Bouch was knighted by Queen Victoria later that year.

While engaged on the Tay bridge he had also been appointed to design and build a bridge over the Forth as the last link in the chain between Edinburgh and Dundee, and had produced preliminary designs. His earliest proposal, of 1865, was for a bridge at Charlestown, up-river from Queensferry. This comprised a 2 ¼ mile long viaduct having 62 wrought-iron close-lattice girder spans, rising to 125 ft. clearance for four 500 ft. navigation spans. These 500 ft span girders weighed 1170 tons and were 64 ft. deep. (Compare Britannia bridge's tubes of 472 ft. span and weighing 1587 tons, 30 ft. deep). Bouch's girders were to be floated into position and jacked up, as at Menai.

Numerous borings were made and revealed soft silt and clay to depths of 120 ft. and more. The solution to such foundation conditions was deemed impracticable, and the design was abandoned. But in 1865 at Charlestown we see a closed-lattice girder being





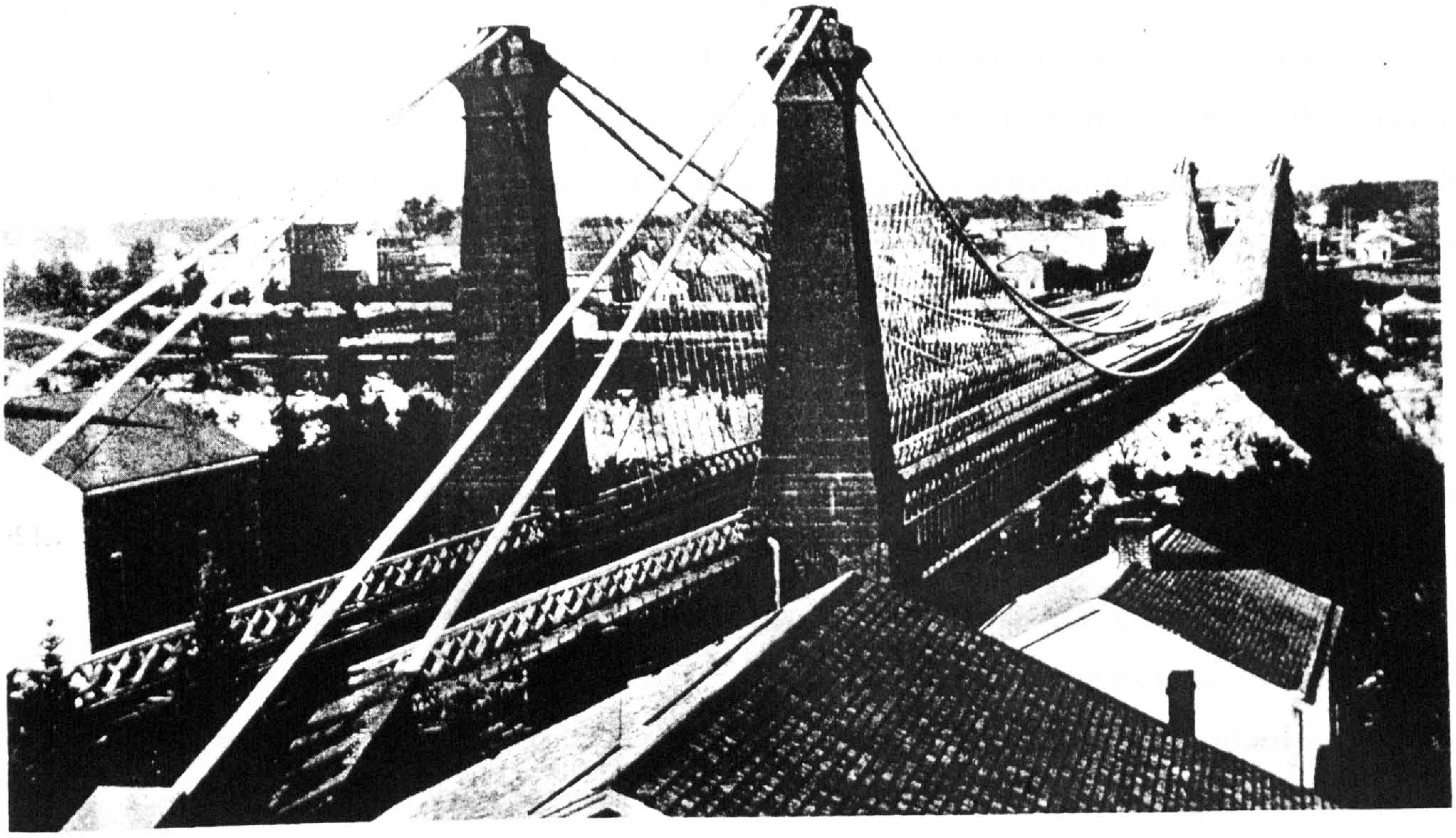
Proposed Forth Bridge by Thomas Bouch, 1873: (a) general elevation; (b) plan; (c) transverse elevation

Fig 53 Proposed bridge by Thomas Bouch, 1873 for crossing the Forth at Queensferry. (Shipway, 1990) chosen for an important bridge (a similar bridge at Runcorn was built 1863-68) despite other alternatives being available such as the double-triangular Warren.

In 1871 Bouch assessed a crossing at Queensferry, where the Forth estuary is divided into two by the island of Inchgarvie. However the channels on either side are more than 200 ft. deep, and supporting piers would have been out of the question. Bouch was faced with a twin-span bridge of 1600 ft. spans, more or less on the line of the present bridge. This was a colossal prospect, as the largest girder span so far constructed at the time in Britain was the Britannia bridge of 460 ft., followed by Saltash at 455 ft.

Bouch's solution to the problem must have been simmering in his mind for a long time. He came up with the idea of a stiffened suspension bridge to carry rail traffic, just as Stephenson 30 years before had had the same idea for the Menai Straits, but Stephenson had come to the conclusion that his tubes were adequate on their own without assistance from suspension chains. In the 19^{th} Century only one successful stiffened suspension bridge for rail traffic had been built - that over Niagara in America by John Roebling in 1855. The span of the bridge was 821 ft. and it lasted for 42 years. This bridge had no stiffening girder, but had radiating auxiliary cables (a Roebling design feature) from the tops of the towers, and also had cables tying the bridge to suitable points in the gorge below. The bridge carried a roadway on the lower deck, 18 ft. below the railway above. It is a mystery why no stiffening girder was employed as the 18 ft. gap would have leant itself easily to such a solution. Train speeds were limited to 3 miles per hour. (Fig 54).

The spans of 1600 ft. over the Forth proposed by Bouch were enormous and it was natural that his first thought was to reduce the weight of his structure. This no doubt influenced him strongly towards the stiffened suspension bridge. He produced four designs on this principle, which are illustrated in Westhofen's book. Many suspension bridges of the day had a sag/span ratio for the chains of approximately 1:10, and three of Bouch's designs incorporated chains of this form. The fourth design, however included chains with a sag/span ratio of about 1:4, which meant a sag of around 400 ft. for the spans of 1600 ft. The clearance required for shipping was 150 ft. so that the towers required to support the chains and deck were nearly 600 ft. in height, i.e. higher than the present road bridge towers (550 ft.). This was the design favoured by Bouch. (Fig 53).



Suspension bridge carrying rail traffic. Roebling, 1855

Fig 54 Stiffened suspension bridge by John Roebling, 1855, for rail traffic over Niagara gorge. Span 821 ft. (Shipway, 1990)

Road bridge at Newcastle, 1871 by Thomas Bouch

Fig 57 Hybrid cantilever design at Newcastle, 1871, by Thomas Bouch.

Bouch apparently chose this low sag/span ratio to reduce the stresses in the chains, which were of steel (not wrought iron) and stressed to 10.0 tons/sq. in., high for these days. His three other designs (Fig 55) had chains stressed to 8.0 tons/sq. in., and as they had less sag, they must have had a greater area and weight of material to reduce the stress to this figure. For comparison, a maximum stress of 7.50 tons/sq. in. was adopted for the steel of the present Forth rail bridge, but only after prolonged negotiation with the Board of Trade by the engineers.

It was not clear why Bouch favoured this design, with its very large cable sag implying increased length of chains, increased height of towers, increased load on foundations, increased resistance to wind and of course, increased cost. Also, the large sag meant that the chains themselves were more prone to oscillation than chains with a shallow profile. The usual elegant curve of the chains generally lends suspension bridges an attractive appearance, but the large sag of the chains on Bouch's bridge made it look cumbersome. There were no suspended side spans to match the 1600 ft. main spans, and this too was an awkward feature.

Distortion of the chains was to be avoided by tying them in position with radiating chain ties fixed to the end of the stiffening girder, which was 50 ft. deep at midspan, 18 ft. deep at the ends and of lattice construction. Also the chains were anchored to the stiffening girders at their midpoints; an unusual feature. Here again we see Bouch choosing a lattice girder, with its high obstruction to the wind and small members prone to buckling (and awkward to maintain).

Bouch must have wished for confirmation that his design was feasible, for he invited an independent report as early as 1873 from two eminent engineers, W H Barlow and William Pole. Wind pressure for design purposes was assumed to be 10 lbs./sq. ft., on the advice of the Astronomer Royal. This figure was approved by Barlow and Pole in their report on Bouch's design, and seems to have been the norm for design in Britain at that time. Bouch separated the rail tracks in his bridge into two distinct structures 100 ft. apart, bracing them to provide a rigid system against the horizontal force of the wind. The chains for each of the two 1600 ft. spans were anchored separately, so that the loading on one span did not affect the other. Temperature effects were also carefully assessed and suitable provision was made to accommodate them.

ALTERNATIVE PRELIMINARY DESIGNS FOR THE FORTH BRIDGE.

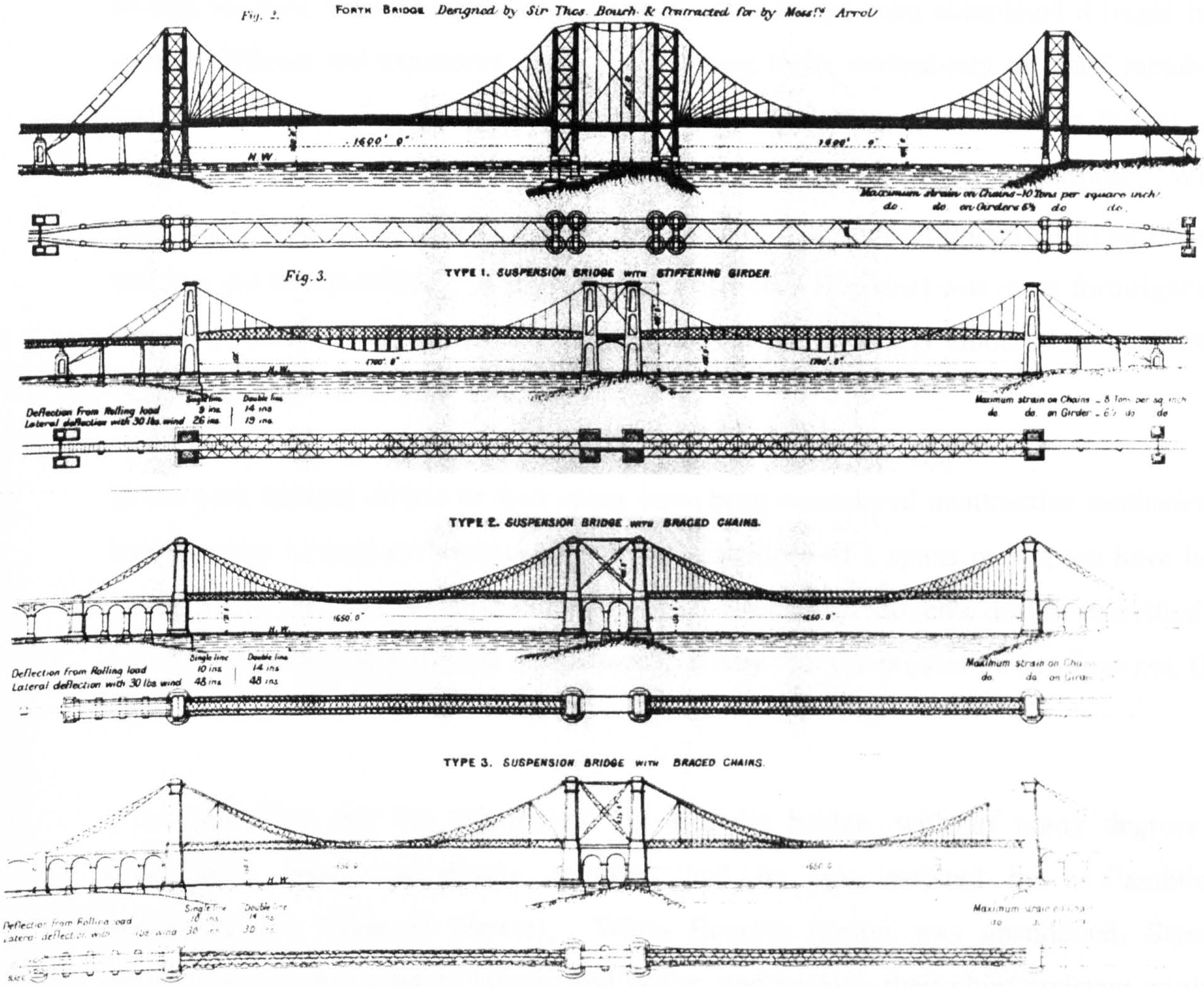


Fig 55 Alternative designs for Forth bridge by Thomas Bouch. (Shipway, 1990)

The design seemingly gained Parliamentary and Board of Trade approval without any detailed investigation other than the private report by Barlow and Pole, and construction began in 1878, with William Arrol of Glasgow as constructor. After the fall of the Tay Bridge in 1879 work was discontinued, and only a single foundation pier on the Inchgarvie rock remains to this day. Had Bouch's design been completed it might have proved difficult and expensive to construct owing to its multiplicity of small members. Barlow and Pole thought that it would require "great accuracy in manufacture and erection" and seemed to indicate a lingering doubt on its fitness for its purpose: "While we raise no objection to Mr Bouch's system, we do not commit ourselves to an opinion that it is the best possible". A modern-day writer (H J Hopkins) was more forthright and less kind when he commented that the design was "painstakingly laboured to the point of being an oddity". (Hopkins, 1970).

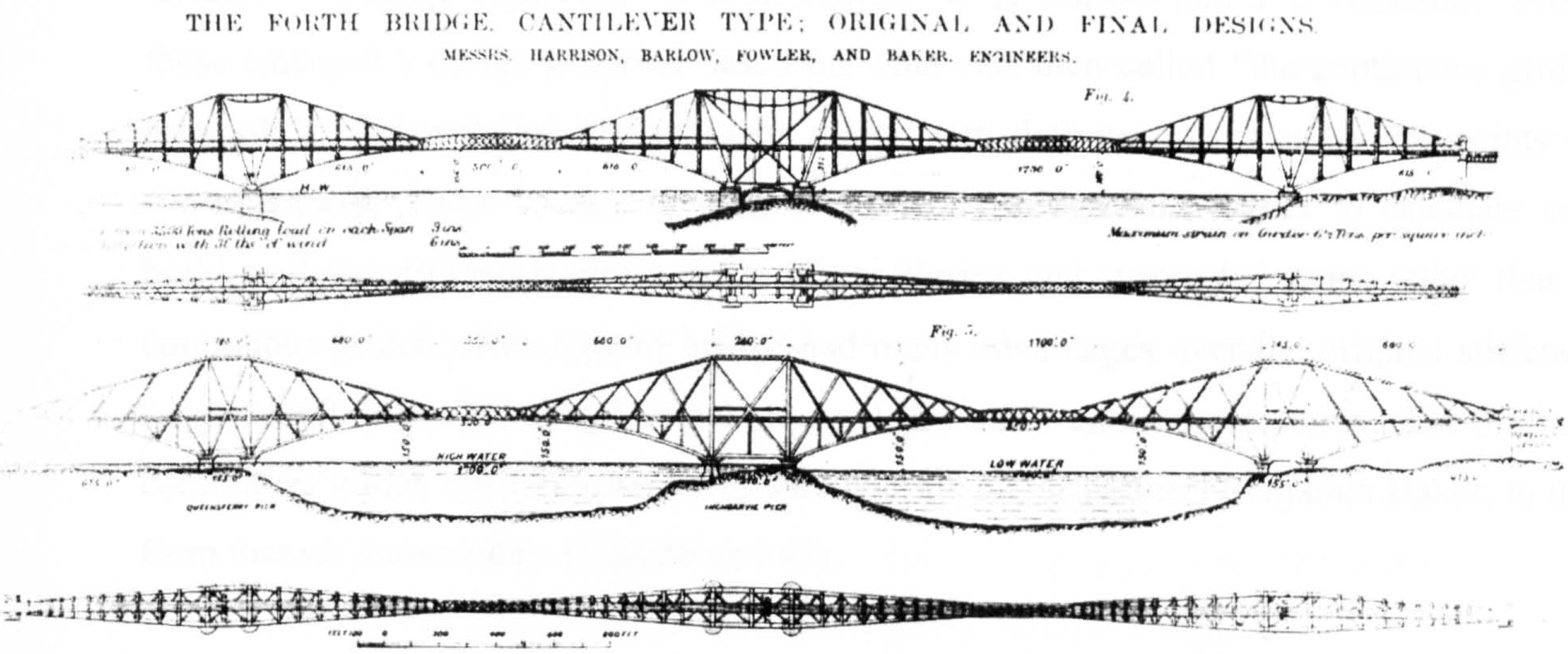
In the past, bridges of two or four spans have been considered unattractive aesthetically because they formed an "unresolved duality". Bridges of 3 spans or 5 spans have been preferred for the sake of appearance. It can be seen that Bouch's design embodied an unresolved duality of awkward appearance in a way that the present bridge does not. (Fig 53).

It is interesting that the calculations for Bouch's bridge, with its many degrees of redundancy, were exceedingly complex, and he was assisted by a Cambridge mathematician, Allan D Stewart. When Bouch's design was abandoned, Stewart transferred his allegiance to Fowler and Baker, and became their chief assistant on their cantilever design. His name appears on the commemorative plaque on the present bridge.

The Continuous Girder Proposals by Fowler and Baker, 1881

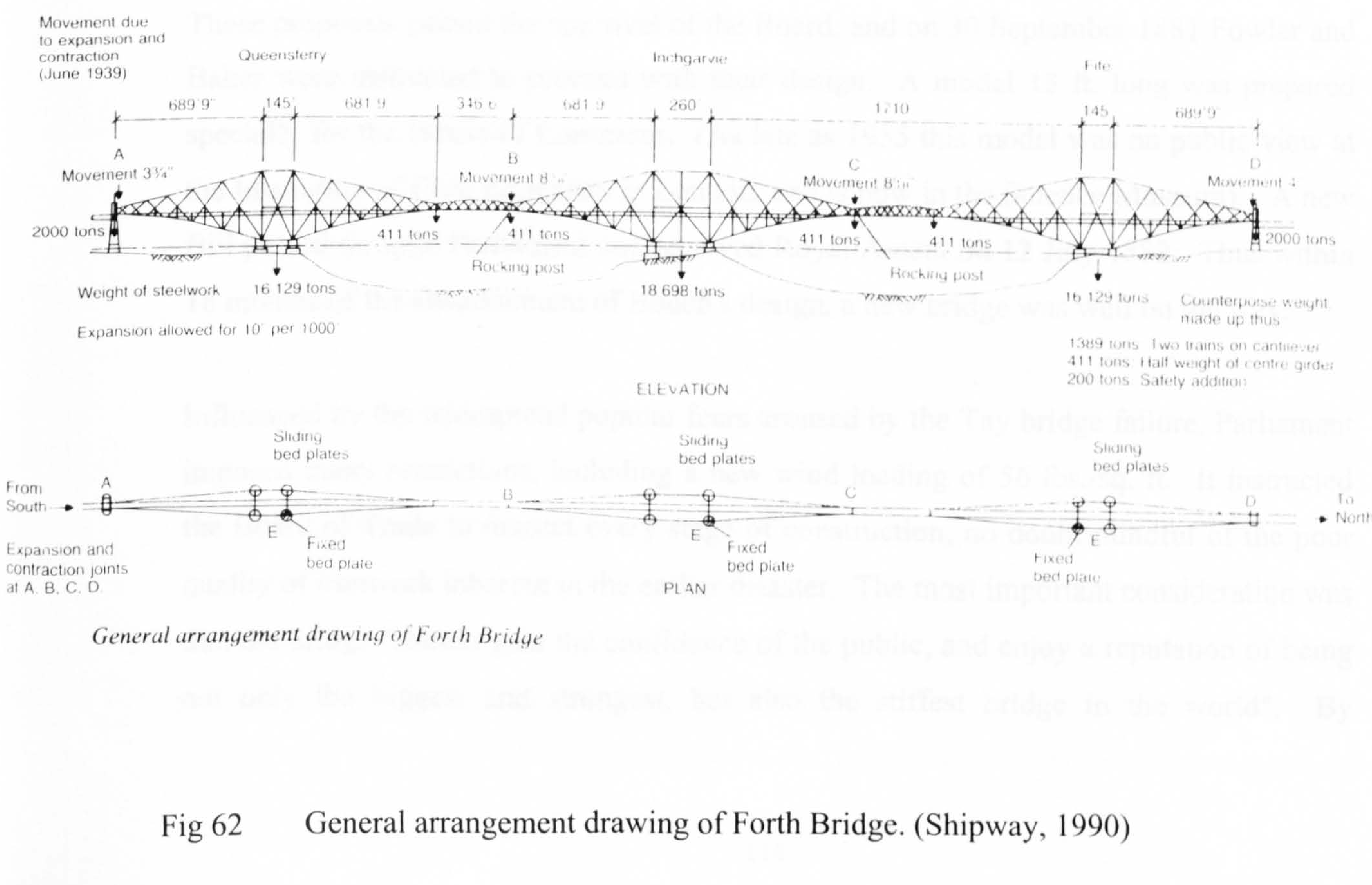
After the Tay Bridge disaster in 1879 the public and the press were loud in their criticism of Bouch's Forth Bridge design, and there was a massive loss of faith in the project. It soon became known that new and stringent regulations were to be introduced by the Board of Trade to control standards, and it was obvious that Bouch's design would have to be revised, particularly as it was designed for a wind pressure of only 10 lbs./sq. ft.

The Railway Board formally abandoned Bouch's design on 13 January 1881 (i.e. a full year after the disaster) which indicates they gave very serious consideration to the matter.



The Forth Bridge: original and final design by Fowler and Baker, and others

Fig 56 Original and final design for Forth cantilever bridge by Fowler and Baker, and others. (Shipway, 1990)



General arrangement drawing of Forth Bridge

Fig 62 General arrangement drawing of Forth Bridge. (Shipway, 1990)

(Meanwhile, Bouch had died in October 1880). They cancelled the existing contracts and paid compensation where necessary. New proposals for a bridge were invited from the Board's consulting engineers, Sir John Fowler, W H Barlow and T E Harrison. From these emerged a design proposal based on what was then called "the continuous girder principle". This entailed a continuous girder with definite breaks at chosen points of contraflexure. These breaks, or hinges, made the bridge form easier to calculate and build, and transformed it into a series of cantilevers and suspended spans rather than a continuous girder. This type of bridge had many advantages over the original stiffened suspension bridge, including being more rigid and stable under construction. The original continuous girder was modified by Fowler and his junior partner, Benjamin Baker, to the form that we know today. (Figs 56 and 62).

The original design had cantilever arms based on immense N-girder or Pratt truss panels with large 500 ft. suspended spans in between. The Fife and Queensferry cantilevers, based on two support points, were not completely stable under construction until the landward arms had been completed. The revised design had a cantilever structure based on the double-triangular form, and shorter suspended spans of 350 ft. The Fife and Queensferry cantilevers with four support points were also designed to be self-supporting at all stages of construction. This is the bridge we know today - the revised arrangement gave the impression of stability and strength.

These proposals gained the approval of the Board, and on 30 September 1881 Fowler and Baker were instructed to proceed with their design. A model 13 ft. long was prepared specially for the House of Commons. (As late as 1955 this model was on public view at the Institution of Civil Engineers in London, and is now in the Science Museum). A new Bill passed through Parliament and received Royal Assent on 12 July 1882. Thus within 18 months of the abandonment of Bouch's design, a new bridge was well on the way.

Influenced by the widespread popular fears aroused by the Tay bridge failure, Parliament imposed many restrictions, including a new wind loading of 56 lbs./sq. ft. It instructed the Board of Trade to inspect every stage of construction, no doubt mindful of the poor quality of ironwork inherent in the earlier disaster. The most important consideration was that the bridge "should gain the confidence of the public, and enjoy a reputation of being not only the biggest and strongest, but also the stiffest bridge in the world". By "stiffness" they undoubtedly meant lack of noticeable movement or deflection under the passage of the trains or the violence of the winds.

In the 18 months of discussion between the abandonment of Bouch's design and the acceptance of Fowler and Baker's, the following criteria emerged as the basis for the design of the bridge:

(a) The maximum attainable rigidity, both vertically under the rolling load and laterally under wind pressure.

(b) Facility and security of erection so that at any stage the incomplete structure would be as secure against a hurricane as the finished bridge.

(c) That no untried material be used in the construction, and that no steel be used which did not comply with the requirements of the Admiralty and Lloyds.

(d) That the maximum economy be attained consistent with the fulfilment of the preceding conditions.

The proposed continuous girder bridge complied more fully with these criteria than the previous stiffened suspension bridge design.

Baker himself, (on whom the main burden of design and supervision fell) had strong views on the requirements of a design, and as early as 1873 had stated in a lecture:

"Of all the numerous practical considerations and contingencies to be duly weighed and carefully estimated, before the fitness of a design for a long span railway bridge can be satisfactorily determined, <u>none are more important</u> than those affecting facility of erection".

The Present Form of the Bridge

The present form of the Forth Bridge is familiar to everyone, and we think of cantilever bridges of all types as being commonplace. But it was not always so. Before 1881, when Fowler and Baker put forward their proposals, the cantilever bridge was unknown in

Britain, and the designers of the new bridge were continually asked to justify it. This was important, as there would ultimately be no point in a bridge that did not inspire confidence in those who would be using it.

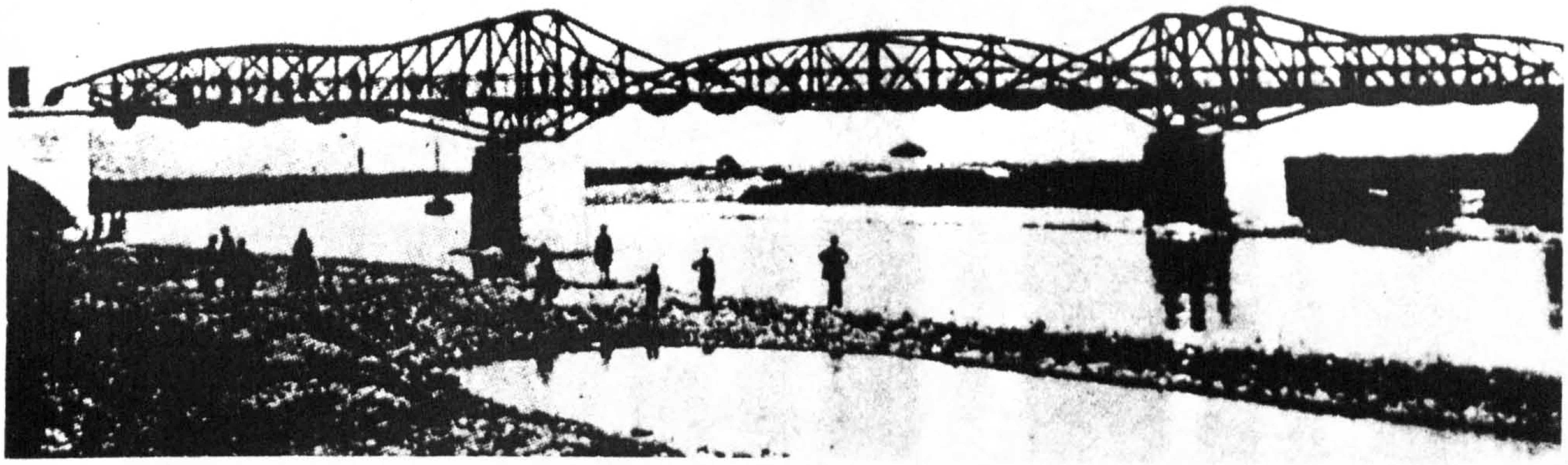
In particular Baker was asked about his novel use of "cantilevers". He explained that the term means no more than "bracket" and that an ordinary balcony or shelf is a form of cantilever. There was nothing novel even in the design of cantilever bridges; they had been built in China more than 200 years before, and probably earlier than that. There were also more recent examples of road and railway bridges in Europe and the USA built on the same principle, including the strange hybrid structure which Bouch had built across the Tyne at Newcastle in 1871. (Fig 57).

As early as 1867, Benjamin Baker (then aged 27) had published a series of articles in "The Engineer" advocating the use of cantilevers supporting a girder system as the most effective means of providing bridges of long span. He and Fowler had some experience of designing such bridges, for in 1864 they had proposed a bridge of 1000 ft. span on the cantilever system for a railway crossing of the Severn. The span was subsequently reduced to 600 ft. with 300 ft. side-spans, and a contract was let for the works, which did not proceed due to financial problems. Again in 1871 they had proposed designs and estimates for a second proposal to cross the Severn in two spans of 800 ft. each, but in this case also the design did not reach the construction stage. Fowler had taken part in the discussion on a Paper to the ICE as far back as 1850 on the continuous bridge at Torksey over the river Trent, which showed that even at that early stage 30 years before the Forth Bridge design he had an accurate grasp of cantilever principles. (Torksey, 1850).

Other Earlier Cantilever Designs

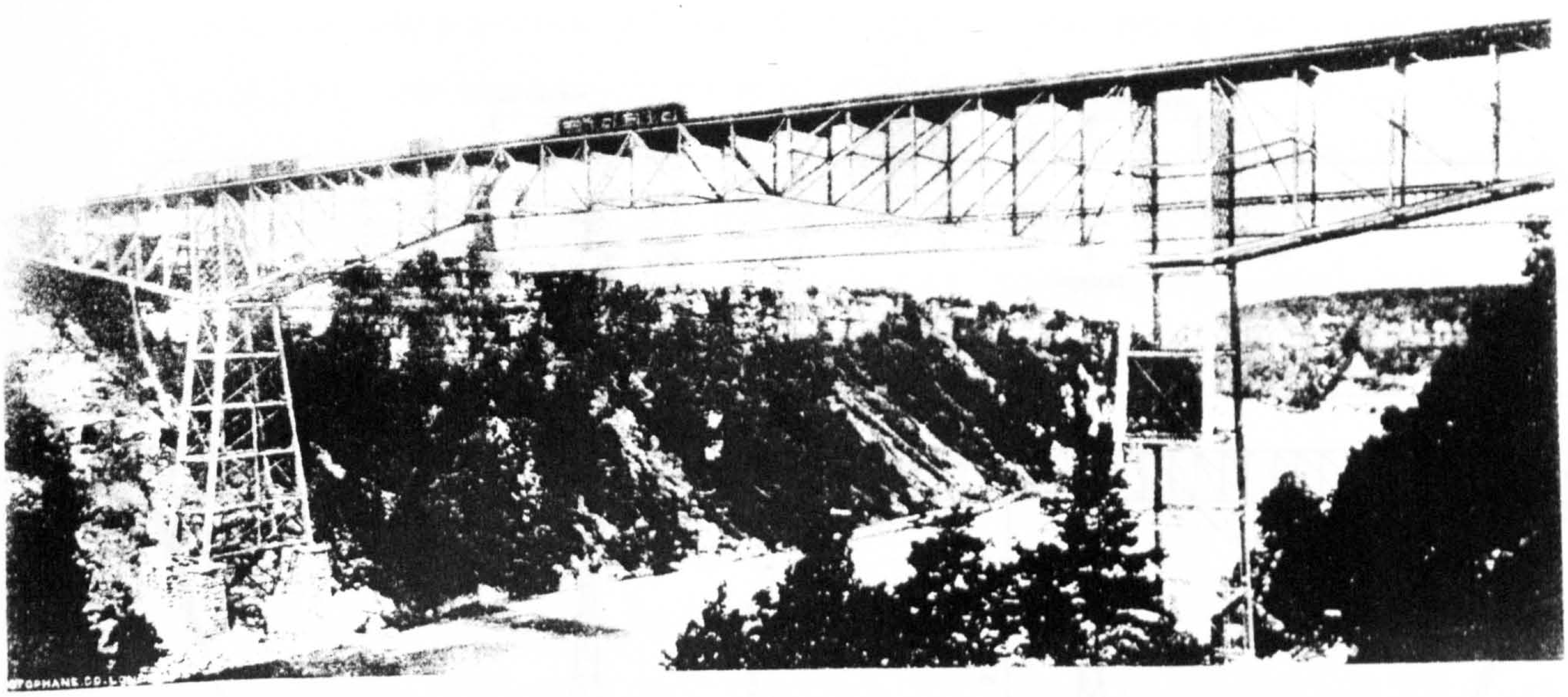
In addition to Fowler and Baker's designs, other engineers in Europe and America had produced at least three cantilever bridges before 1883.

(i) In 1866 the Bavarian engineer Heinrich Gerber was granted a patent for a design known as the "Gerber Girder", or in English speaking countries a cantilever girder. This was a continuous girder over several spans in which hinged joints were inserted so that the harmful influence of minor settlement of supports was eliminated. The first bridge built by Gerber employing this system was in



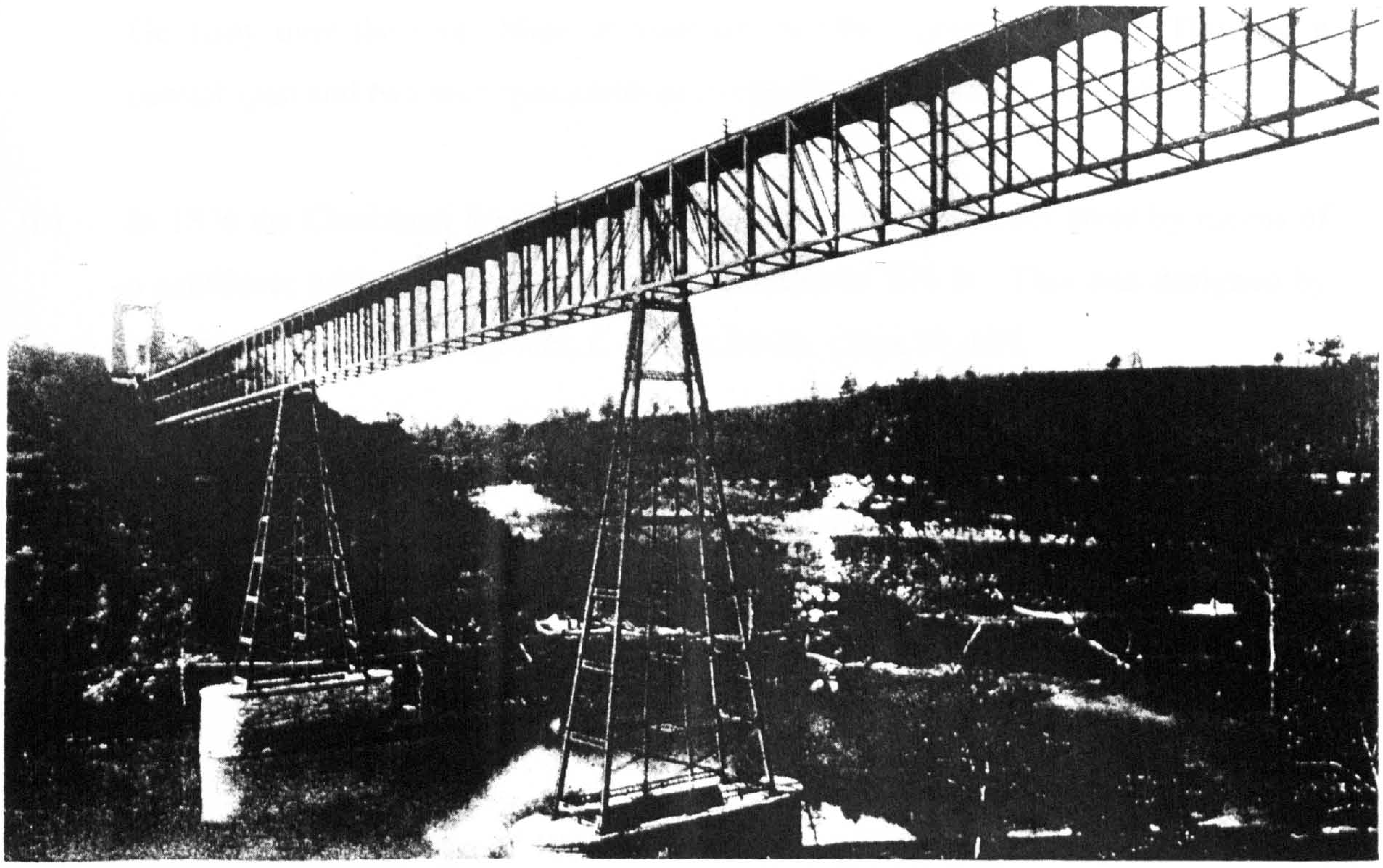
Cantilever 'Gerber' bridge at Hassfurt, Germany, 1869

Fig 58 Heinrich Gerber cantilever design at Hassfurt, Germany, 1869. Length 426 ft. (Shipway, 1990)



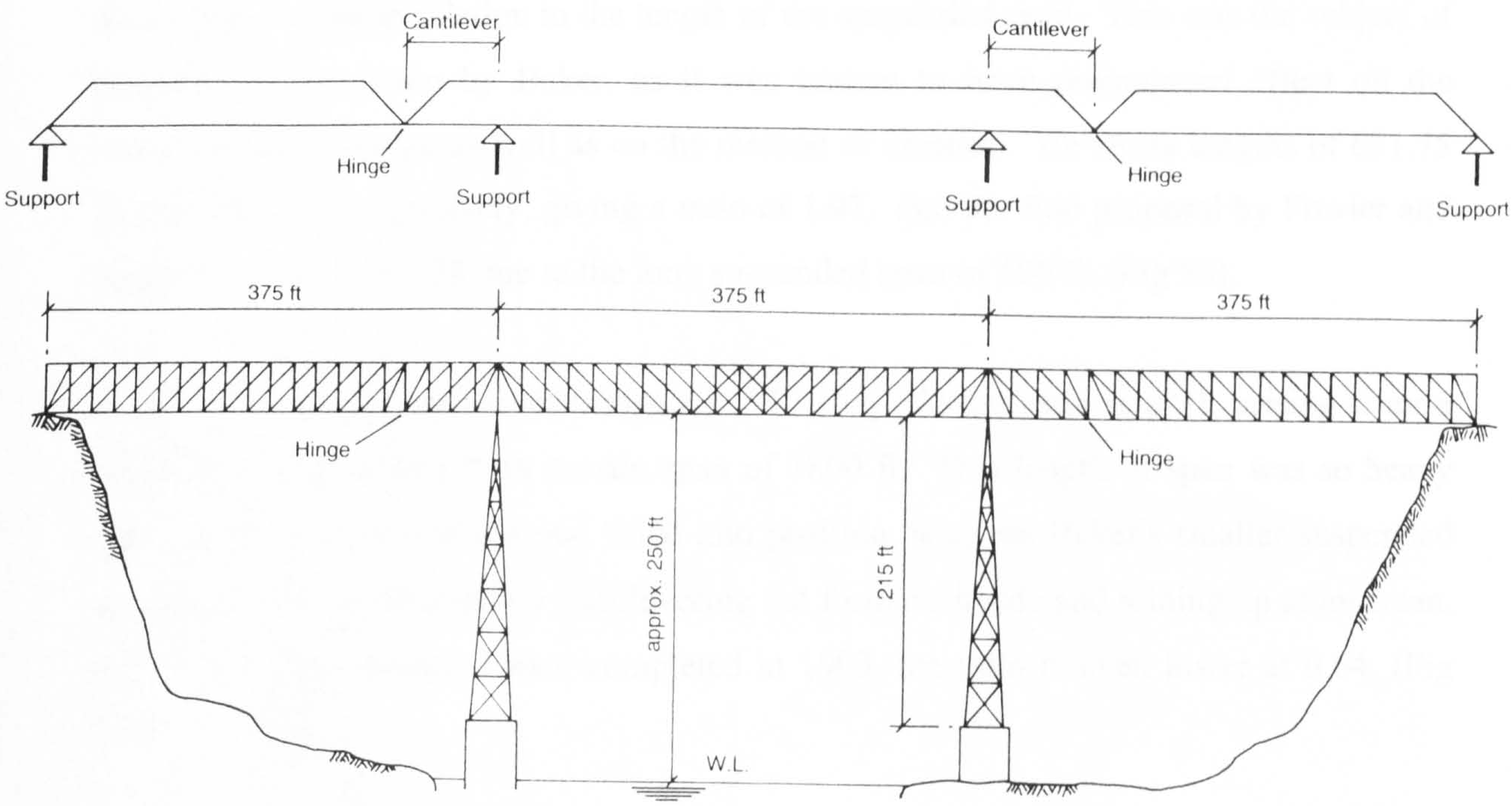
Niagara Cantilever Bridge—1883

Fig 61 Niagara Cantilever bridge, 1883, by C C Schneider. Main span 495 ft. Roebling's bridge in background. (Shipway, 1990)



Cantilever bridge over the Kentucky River, 1876

Fig 59 Kentucky River cantilever bridge, 1876, by C Shaler Smith. Three spans of 375 ft. (Shipway, 1990)



Articulation diagram for Kentucky River bridge

Fig 60 Articulation Diagram for Kentucky River bridge. (Shipway, 1990)

Germany over the river Main at Hassfurt in 1867. (Fowler, 1929). This had a central span and two side spans and an overall length of 426 ft. (Fig 58).

(ii) In 1876 the Cincinnati Southern Railway crossed the Kentucky river by means of a cantilever bridge which had three equal spans of 375 ft. This was designed by an eminent American engineer, C Shaler Smith. (Figs 59, 60).

(iii) In 1883, the year work began at the Forth, a notable cantilever bridge was completed over the Niagara river by another leading American engineer, C C Schneider. This had a total length of 910 ft., was 239 ft. above the water, and had a main span of 495 ft. It had pin-connected members, and was constructed in the amazingly short time of 10 months.

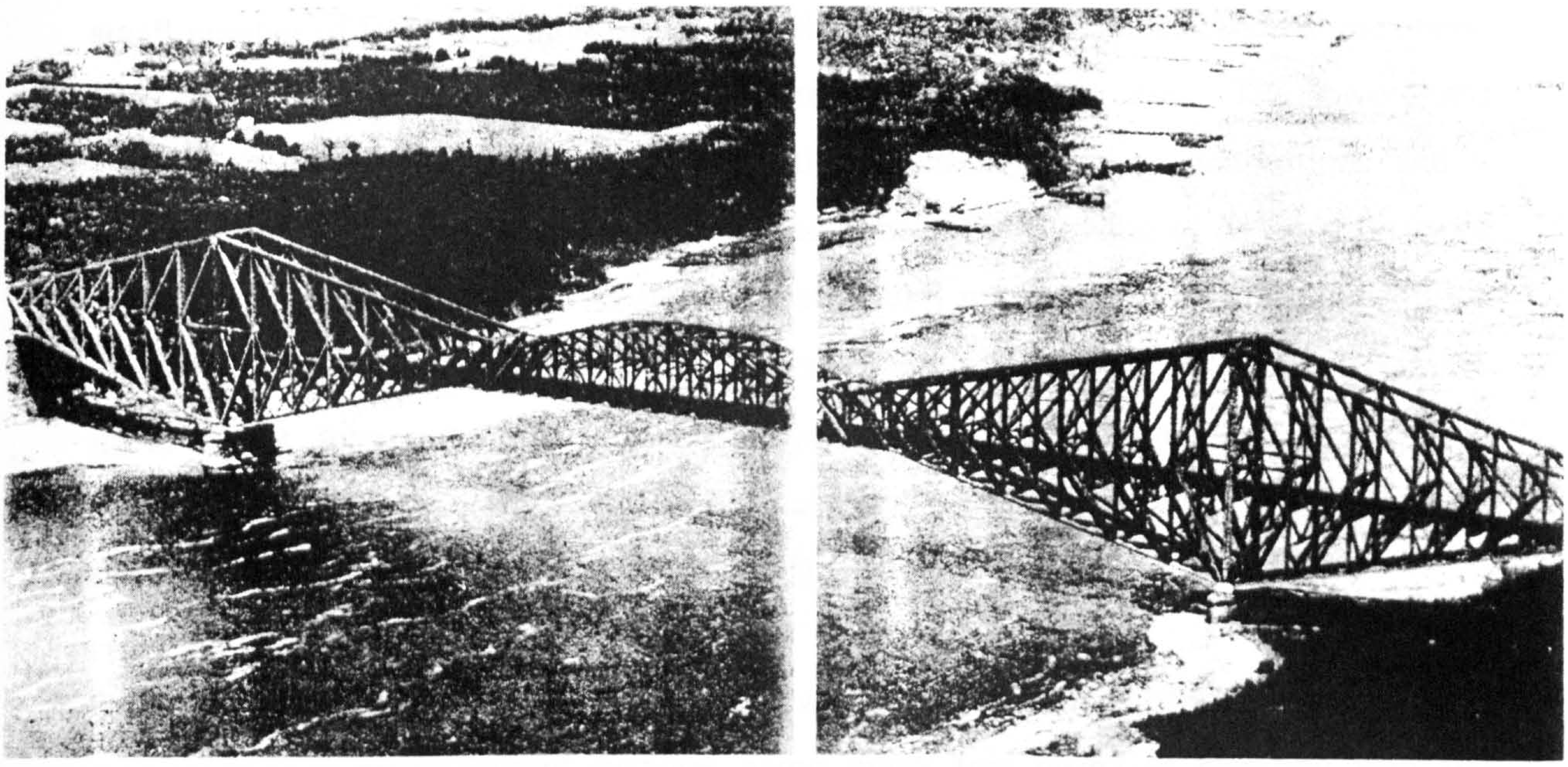
There was also Bouch's bridge of 1871 at Newcastle, which had main spans of 240 ft., and which has already been described.

Forth Bridge Basic Geometry

One of the earliest questions affecting the form of the continuous girder was the length of the cantilever arm in relation to the length of the suspended span. This was the subject of elaborate investigation by Baker, as it was known to have pronounced effect on the economy of the design as well as on the method of erection. He chose lengths of 681.75 ft. and 346.5 ft. respectively, giving a ratio of 1.97. But the first proposal by Fowler and Baker had a ratio of 1.23 due to the long suspended span of 500 ft. (Fig 56).

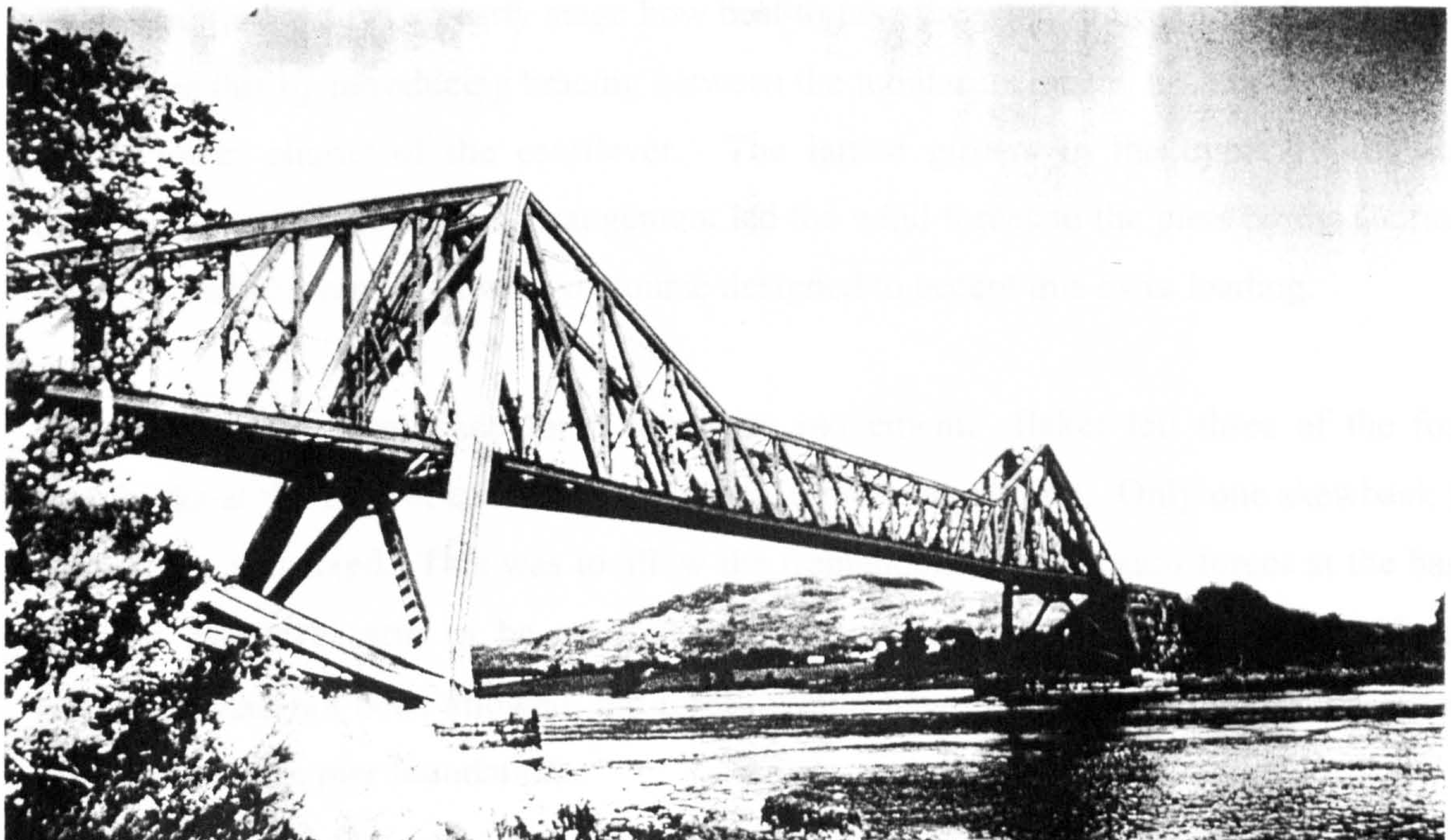
In the later Quebec cantilever bridge of 1917 (Fig 63) the ratio is 0.91, arising from a suspended span of 640 ft. in a main span of 1800 ft. This length of span was so heavy that it had to be floated out and lifted into position, whereas Baker's smaller suspended spans could be constructed by cantilevering out from each side and joining up at midspan. At Connel Ferry bridge, Oban, completed in 1903, the ratio is even lower at 0.64. (Fig 64).

Thus the Forth Bridge has rather short suspended spans compared to other designs of the period, but the proportions appear attractive, and they look right.



Quebec Bridge of 1917

Fig 63 Quebec Bridge, 1817. Span 1800 ft. (Shipway, 1990)



Connel Ferry Bridge, Oban, 1903

Fig 64 Connel Ferry bridge, Oban, 1903. Span 525 ft. (Shipway, 1990)

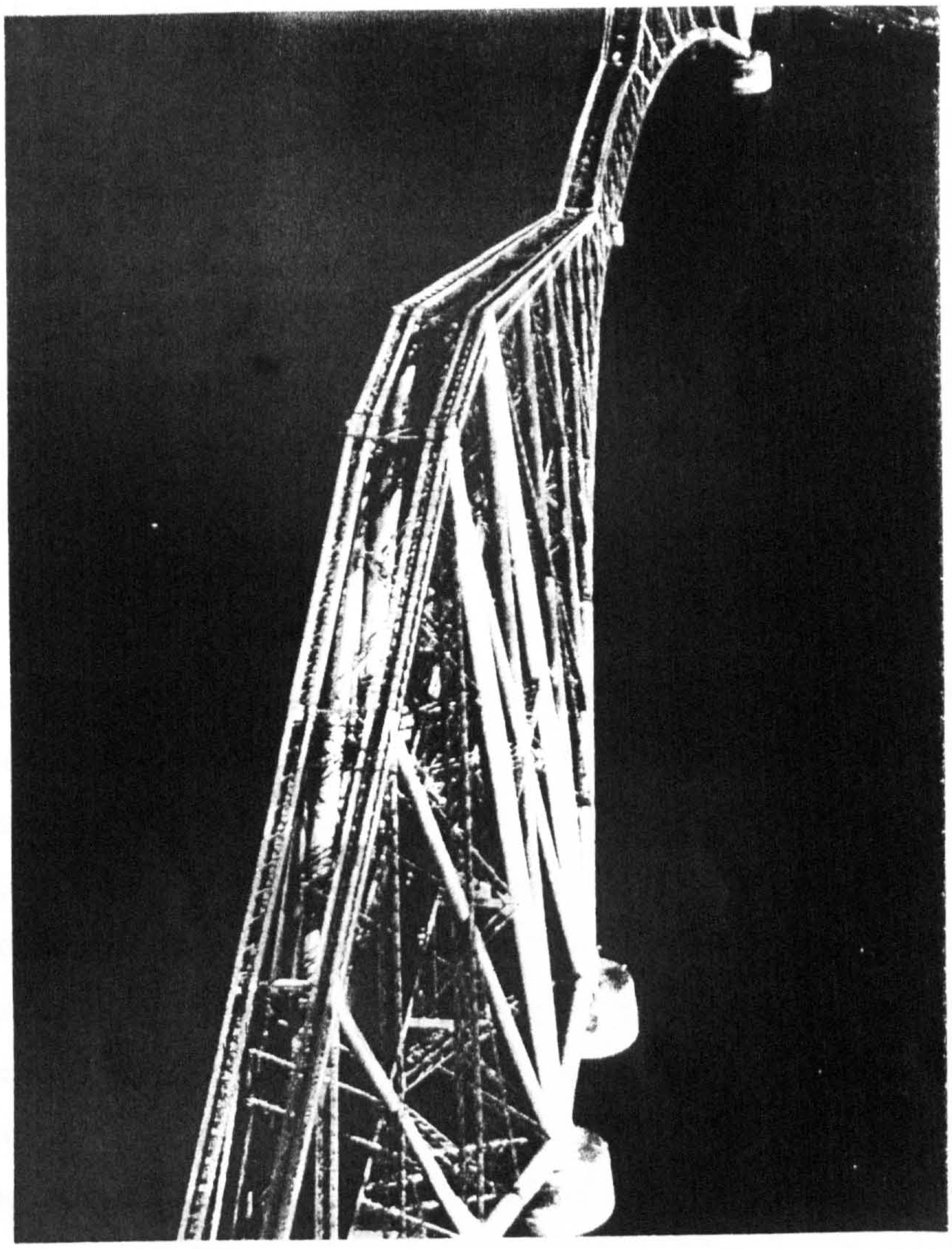
Another feature of the continuous girder affecting its form was Baker's decision to have a small number of large structural members rather than a large number of small members. Bouch's 1873 suspension bridge design illustrated the latter case. The towers and stiffening girders were of lattice construction, generating many connections and a probable substantial maintenance problem had it been built, owing to its multiplicity of members. Baker's choice of few but large structural members was emphasised by his selection of tubes for the compression members of the bridge. (Fig 65). These struts were up to 343 ft. long and 12 ft. in diameter, yet appear of slender proportions and even matchstick-like from a distance. (The vertical tubes over the piers have a length/breadth ratio of 28.6 as compared to 21.5 for a match!)

Tubes were chosen because they are known to be the most efficient shape for a compression member, curved surfaces being much less prone to buckling than flat surfaces. Baker accepted that they would cost more to construct, but that this would be outweighed by the structural advantages. Nevertheless, the intersection of the curved tubes at the skewbacks and elsewhere were extremely complex and difficult to design and detail as well as to construct.

Baker had to decide at an early stage how best to take the wind forces to the foundations, and he did this by introducing bracing between the tubular members, both in the webs and in the lower chords of the cantilever. The lattice girders in the upper chords and diagonals are unbraced. This arrangement led the wind forces to the piers by the shortest route; the tubular members were of course designed to accept this extra loading.

Careful provision was made for temperature movement. Baker left three of the four skewbacks at the base of each cantilever free to move by sliding. Only one skewback in each group was fixed. This was to allow the tremendous compression forces at the base of each cantilever arm to be resisted by the horizontal strut joining them. If this movement had not been allowed, the forces would have produced an overturning effect on the respective pier foundations.

Perhaps the most noticeable feature of the bridge, however, is the "Holbein" straddle, i.e. the wide base of the cantilevers tapering to narrow widths both vertically and horizontally. (Fig 66). The German artist, Hans Holbein, was wont to straddle the feet of male subjects in his pictures. Sir John Fowler knew this, and when he met James



Inclined structure of Forth Bridge - the Holbein straddle

Fig 66 The inclined structure of the Forth bridge - the Holbein straddle. (Shipway, 1990)

Nasmyth of steam-hammer fame by chance at a Holbein exhibition in London after the Tay bridge failure, he remarked that the bridge would not have fallen had it had a straddle to its piers resembling Holbein's characteristic pose. Hence the Holbein straddle, which resulted in a batter of about 1:7.5 throughout all the vertical members. This arrangement gave an impression of great stability, but also gave rise to much complexity in calculation, drawing and detailing, and in construction.

Materials and Allowable Stresses

In the late 1870s when the Forth Bridge was being designed, steel remained a comparatively untried material for bridges, though it had been employed since the 1850s in several ships, including the "Columba", the most famous of all the Clyde steamers, which was built in 1878 and queened it on the Clyde for more than 50 summers. Steel was considered a new material, and some engineers had reservations about its brittleness as compared to wrought iron. It was also impossible to define its character completely by chemistry and ingredients, and testing of specimens was necessary to obtain sufficient proof of its quality and strength.

In the rebuilding of the Tay bridge conservatism held sway, and wrought iron was used. Corresponding maximum stresses of 5.0 tons/sq. in. in tension and 4.80 tons/sq. in. in compression were adopted. However steel could offer a 50% increase on these working stresses, and was obviously a great attraction when long spans were concerned, as its weight differed little from that of wrought iron. Fowler and Baker therefore approached the Board of Trade to find out the maximum stress which could be adopted in design. At that time the Board was allowing 6.5 tons/sq. in., but Baker showed that by improving the quality of the steel, 7.5 tons/sq. in., i.e. a quarter of the ultimate strength of the steel would be reasonable, and the Board accepted this figure. Its value of 6.50 tons/sq. in. had been based on steel with an ultimate strength of 26 tons/sq. in., whereas Baker proposed a minimum of 30 tons/sq. in. for the bridge. Nowadays a stress of perhaps 10.6 tons/sq. in. tension would be allowed on a steel of this ultimate stress.

Baker was aware of the dangers of fatigue, and limited the stress in the wind bracing, which was subject to alternate tension and compression, to 5.0 tons/sq. in. Elsewhere on the bridge the working stress was to be 3.33 tons/sq. in. if the stresses alternated in this fashion. He recognised that in the case of hurricane winds the repetitions of stress would be few and far between, and so allowed a higher stress.

The Board of Trade regulations for the use of steel in the bridge were minimal, simply stating that the working stress should not exceed 25% of the ultimate. No distinction was made between tensile and compressive stress, or between stresses produced by dead and live load, either alone or in combination. No Codes of Practice existed to give guidance. Fowler and Baker therefore derived their design rules for stresses by careful thought and experiment, and their caution and insight has been amply repaid in the 110 years' life of the bridge so far. Fatigue has not been a problem in the main structure, but has necessitated some repairs to the internal viaduct. In the latter, of course, live load plays a much greater part than dead load, although in the main structure the reverse applies.

In the structure of the internal viaduct, N-trusses were used in spanning between support points through the main cantilever frame. This was one of the earliest uses of the Pratt girder in Britain. It is not known why Baker departed from the harmonious use of the double-triangular truss in favour of the Pratt. He had of course considered the Pratt configuration for the first design of the cantilever bridge, but this had been discarded in favour of the double-triangular arrangement which was adopted.

In the manufacture and construction of the tubes and other members, Baker insisted that as far as possible all plates and bars were to be bent when cold. Where heating was essential, no work was to be done on the material after it had cooled to a "blue" heat. He also insisted that the steady pressure of hydraulic riveting was to be used in place of hammering wherever possible, and that annealing would be required if the steel had been overworked in any way. He allowed no punching of holes or shearing of edges and specified that all plates were to be planed at the edges, and all holes drilled. In particular holes were to be drilled through multiple thickness of plates and angles after assembly.

These recommendations show the care with which Baker considered the material and its connections. Nothing was left to chance, painstaking attention to detail prevailed throughout. Having chosen the quality of steel and the working stresses, Baker than designed his structure boldly and confidently to the set limits. Incidentally the writer has not been able to locate any mention of Baker employing extra material or thickness to allow for the effects of corrosion in such an exposed location. In the event his confidence in the effective maintenance of the bridge was not misplaced, and it has been exceedingly well painted and maintained over the last 110 years.

Analysis - Forces in the Struts and Ties

None of Baker's published Papers give any hint of the methods used in calculating the structure. His book "Long Span Railway Bridges" published in 1873 does not go into structural analysis or stress calculation; it outlines general principles. Recourse to the Mitchell Library in Glasgow, where some of the original Arrol drawings are stored, indicated that no calculations have been filed. It has therefore not been possible to locate any relevant calculations, but the method of calculation is known.

As already mentioned, Baker was assisted in his calculations by a Cambridge mathematician, Allan D Stewart, who had earlier assisted Bouch in his calculations for the stiffened suspension bridge. For this rather more complex bridge Stewart is known to have used what he called "diagrams of forces" evolved by James Clerk Maxwell, who had evolved the load-displacement method and reciprocal theorems for analysis of structures.

Allan Stewart presented a Paper to the ICE in 1892 on "Stresses and deflections in braced girders" (Stewart, 1892). In those days the words "stress" and "strain" were commonly used where "force" is the equivalent term today, and the Paper is actually about forces and deflections. In it, Stewart shows how a method of calculating which he calls "the principle of elastic forces" can be applied to double-triangular girders and other redundant structures with great accuracy and states that this method was used in the calculation of the Forth Bridge structure. He also gives an example of its use for the calculation of deflection at the counter-weighted shore arms of the Fife and Queensferry cantilevers. Stewart's name appears on the plaque on the bridge as Fowler and Baker's chief assistant, and there seems little doubt that he was responsible for the basic calculations of the Forth Bridge.

It will be seen that the bridge is almost entirely composed of girders of the double-triangular arrangement of an even number of panels in each span, i.e. the approach girders, the central supported spans, and the cantilever arms. As mentioned earlier, this form of truss, if the upper and lower chords are straight, is calculated approximately by separating the double-triangular system into two single systems, each loaded with 50% of

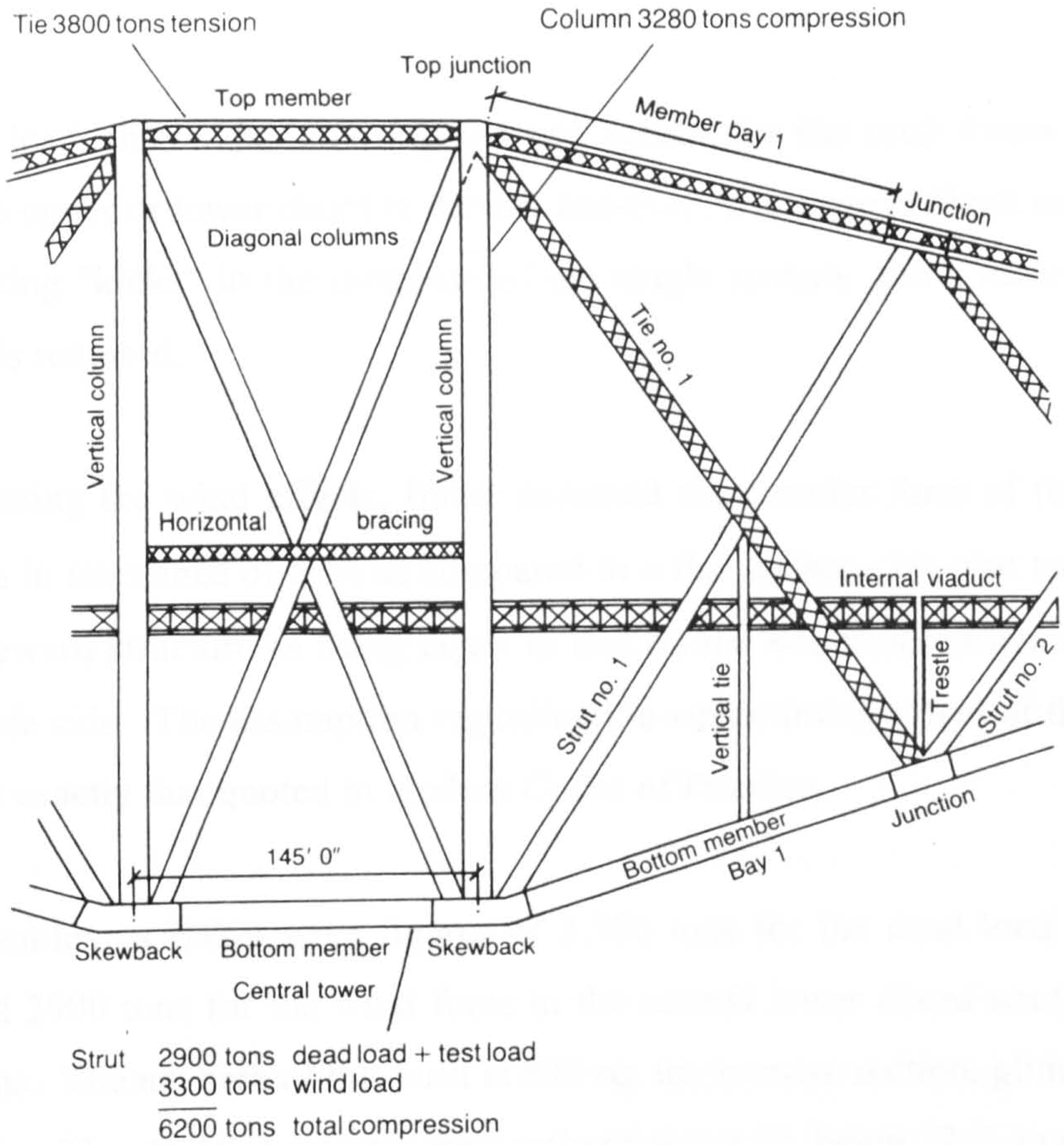


Fig 67 Baker's figures for forces in members, Forth bridge. (Shipway, 1990)

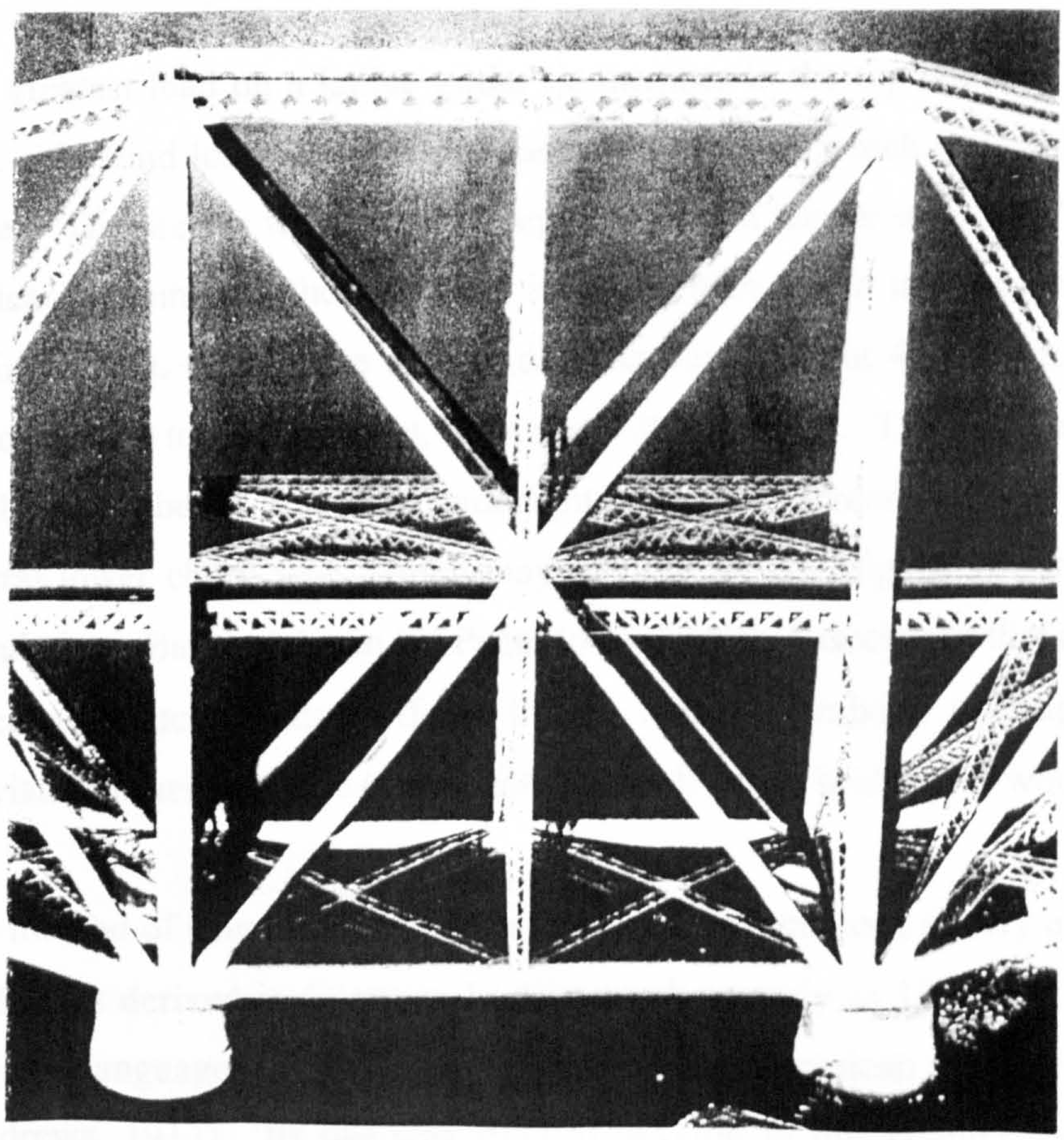


Fig 65 Inchgarvie Tower, Forth bridge. Notesuperfluous upper central member, not present in Baker's later final design, Fig. 56. (Shipway, 1990)

the total load, then superimposing the two results for the total forces in the members. When the upper or lower chord is curved, however, it is more difficult to apply because of the resulting "kinks" in the members of the single system, and a more exact method of analysis is required.

In calculating the wind effects, Baker assumed the circular form of the tubes to give a reduction in resistance of 50% as compared to a flat surface. He also took the wind force in the leeward structure as being equal to that on the windward structure; an assumption on the safe side. The assumption regarding the streamlining effect of the tubular surface is almost exactly that quoted in modern Codes of Practice.

In the cantilevers Baker gave figures of 3,300 tons for the dead load and the train test load, and 2900 tons for the wind force in the central lower chord strut, giving a total of 6,200 tons. The net area of this strut is 830 sq. in. in cross-section, giving a stress of 7.47 tons/sq. in. The strut has a slenderness ratio of about 29, being 12 ft. in diameter with 1 ¼ in. plating and 145 ft. long, but restrained by some fixity at the ends. The allowable total stress was 7.50 tons/sq. in., lower than would be allowed today, and no additional 25% increase was allowed for wind loading.

The greatest load on a lattice girder tie member in the top chord is approximately 3800 tons. No wind load was assumed for these members which makes the calculation rather easier. The area in tension is 506 sq. in. giving a tensile stress of 7.50 tons/sq. in. The vertical column over the skewback is 343 ft. high, 12 ft. in diameter, and the shell is of plating 5/8 in. thick. The area in compression is about 468 sq. in., and Baker gives a force of 3280 tons in the strut, i.e about 7.0 tons/sq. in. The strut has a slenderness ratio of about 34, being braced at midheight,. The slenderness ratio is thus higher than the central lower chord strut, which may account for the slightly lower stress shown in the calculation. Baker states in his Paper to the British Association that no attempt was made to calculate temperature effects in the bridge members, although of course ample provision for temperature movement was made in the bridge as a whole. (Fig 67).

The method of using influence lines for forces in members, widely used in railway bridge work, was derived in Germany by Weyrauch as early as 1873, and first appeared in the English language in 1887 in a Paper to the American Society of Civil Engineers (Andrews, 1913). Its use was slow to develop in this country, and it is unlikely that influence lines were used in the calculation of forces in the cantilevers, but might have had some application in the girders of the approach spans or those of the internal viaduct.

The magnitude of the structure and the forces deriving from it takes it beyond the range of simple modes of calculation, which are customary in structures of a more ordinary kind. Influences which are usually small and can be handled easily have in this case to be carefully studied, particularly the dead load of the tubes and other members spanning between the panels, stiffness of the joints, torsion effects in the cantilevers due to trains on one line only, and the effects of the wind bracing on the tubes where the bracing does not reach a base reaction point. The locked-in stresses due to erection procedures would add to these problems, as would temperature effects of many kinds, including sunshine on one side only of the bridge.

Baker forecast that the heaviest train crossing the bridge would not deflect the 1710 ft. span more than 4 inches. In this he was remarkably accurate as a deflection of 3 ½ in. has actually been measured on the bridge. He also calculated that a wind pressure equivalent to 30 lbs./sq. ft. over the centre 1700 ft. span would bend the bridge laterally less than 9 in. He did not give a figure for a pressure of 56 lbs./sq. ft., perhaps because it was unlikely that such a pressure would ever act over the whole structure.

It is interesting that Baker in his public lectures up to the year 1885 gave the estimate for the weight of steel in the cantilever structure of the bridge as totalling 42,000 tons, excluding the approach viaducts. In a lecture in 1887 he gave the figure as 45,000 tons, after erection of the steelwork had begun. However, the final figure given by Westhofen is 51,000 tons, 21% higher than the original estimate. (Westhofen, 1890).

Westhofen wrote of alterations to the design and the increase of cross-section in various parts. The extra steel may have been necessary because of the weight of the cranes and other plant on individual members during erection, where cantilevering out over long lengths took place.

The internal stiffening of the great tubes may not have been refined in detail, and probably the internal viaduct was not fully designed at the estimating stage. Other complex interpenetrations of tubes such as the twelve skewbacks are unlikely to have been fully detailed at that stage either, and may have required more steel than was allowed for. There are many reasons why the original estimate may have been low, but if Baker used a figure of 42,000 tons in his calculations, he must have designed with a good margin in hand to allow for an increase in dead load of 21%.

Most views of the bridge are from the shore close by, where foreshortening gives it a heavy appearance. Seen from the road bridge, however, the Forth Bridge is slender and graceful, leaping like a greyhound over the vast span of water between the cantilevers. In spite of the constraints of high wind loads and low steel stresses, Baker achieved a graceful and elegant design.

It is undoubtedly the greatest continuous girder of all time.

CHAPTER 8

SUMMING UP THE DEVELOPMENT OF THE GIRDER BRIDGE

Discussion

Conclusions

Chapter 8

Summing Up the Development of the Girder Bridge

Discussion

The study has sought to trace the development of the girder bridge from 1820 to 1890 and of necessity it has been somewhat brief with many omissions. The American scene has yielded information of a different sort from Britain. Britain often made striking advances with bridges that were solutions to particular problems, but which were not, or rarely, repeated. This situation seems rarely to have occurred in America.

In the early days the availability of timber in America had a strong influence on girder design, leading to the use of panel girders often accompanied by arch stiffening. In Britain cast-iron was widely used in the absence of timber, and this generated the Hodgkinson beam and its accompanying formula for strength calculation. Thus the strands of development in the two countries stemmed from the materials available.

Efforts to lengthen the span of cast-iron girders led to abortive developments of the bolted girder and the trussed girder, and after the Chester Dee bridge failure of 1847 the use of cast-iron declined.

Wrought iron came to the fore, and maintained its supremacy until replaced by steel in the 1880s. Steel offered a 50% increase in strength for much the same weight as wrought iron. But wrought iron allowed the construction of some spectacular bridges. These were the tubular bridges at Conwy and the Menai Straits, and Brunel's bridges at Chepstow and Saltash.

Conwy and Menai both had plated, closed webs. Chepstow was the forerunner of the open-web girder in Europe, and a landmark design. Saltash likewise had an open web, but its unique form was never repeated. Likewise the tubular bridges - they were a brilliant solution to particular problems of the time, but their form was soon to disappear. Menai bridge, and Torksey, pioneered continuity between multiple spans and this was a striking advance, and one of the greatest innovations of the 19th Century.

The advance in America of the open-web girder was slow to be followed in Europe, but eventually the development of the Warren truss swept all before it, and it remained popular for many years. The plate girder remained a popular solution too, but mainly for spans below 100 ft., and it allowed high span:rise ratios where headroom was a problem, yet was not vulnerable to deflection.

The open-web girder reached its climax in the continuous girders of the Forth Bridge, completed in 1890. There the use of the double-triangular Warren girder on a gigantic scale was a brilliant solution to the many problems of construction and erection of spans which were a breath-taking advance on all that had gone before. These spans were marginally increased in the Quebec cantilever bridge (Fig 63), finally completed in 1917, 27 years after the Forth, but never again was the girder bridge to be constructed on such a scale. It was a fitting climax to a century of development of man's answer to the problems of railway bridging.

The feature that perhaps offered some restriction to the early development of the girder bridge was the absence of convenient and accurate methods of calculation. Navier's Lecons of 1821 contained the basis for accurate calculation of the bending moments and bending stresses in beam design, and it seems surprising that they were not more widely known and used. Hodgkinson's Formula had the advantage of being simple and easily applied, but it was at its best limited and approximate. Whereas the design of the Conwy and Britannia bridges (striking advances though they were) suggest a heaviness and lack of refinement, there is no such impression with the contemporary Chepstow and Saltash bridges by Brunel.

This fact may indicate that Brunel had the benefit of greater skill in calculation. He was French-educated and could speak the language fluently, and this may have given him access to Navier's works and a greater understanding of the mathematical approach to the design of structures.

In contrast, the feature that perhaps most advanced the progress of the girder bridge was the development of the panel concept leading to the open-web girder. The basis of this is seen in the early designs of the American Theodore Burr, and the concept was quickly grasped by his compatriots Long, Howe, Pratt, Whipple and Warren. The panel arrangement led to a simplicity of form and construction once the necessity for an accompanying relieving arch was eliminated. It also led to easier calculation, yet the concept was slow to take root in Europe, and seemed to have to wait to be independently derived by Brunel at Chepstow.

The closed-web girder remained in service as the plate girder, once it had rid itself of the cumbersome box concept, and became a favourite maid-of-all-work for smaller spans under heavy loading. The problem of buckling of the top flange required much experiment and analysis over the years before Brunel's elaborate curved flanges and Fairbairn's boxes became a thing of the past. One advantage of the plate girder, apart from its ease of construction, was that it lent itself readily to the adoption of continuous spans.

The development of continuous spans was another feature which led to an advance in girder design. The bending moments obtaining in multiple simply-supported spans could be reduced and savings made when distributed over the supports, resulting in shallower, less highly stressed girders. Midspan deflection was also reduced and was a further advantage. Continuity was easily applied to plate girder construction and with care was also easily applied to the triangular girder, as in the double-triangular Warren spans of the Tay Bridge of 1878. Here again the lack of a convenient and easily applied method of calculation may have hindered the adoption of continuity before 1850.

But perhaps the greatest impetus or lack of impetus in the development of the girder bridge could be said to lie in the springs of invention. The simple beam sufficed for many purposes, but the little Gaunless trusses of 1823 were an innovation that in Britain opened the door to the development of framed structures. But why was Gaunless left on the shelf and never repeated with a framed structure until the early 1850s when Chepstow, Newark, Dyke and Crumlin appeared? The answer seems to lie in the difficulties of calculation - it may have seemed too risky to attempt a larger span. Yet the complexity of the trussed girder appeared to deter no-one until the Chester Dee disaster.

The enormous spans at Conwy and Britannia and the resolution of the many difficulties led to an appreciation of construction on a large scale, culminating in the application of continuity at Britannia. The design of Chepstow with its open webs and gigantic panels was a further step forward, but the unique arrangement of the spans at Saltash, though impressive, was not. Finally, the invention of the continuous girder or cantilever system led to the construction of still greater spans, culminating in the gigantic, yet elegant, Forth Bridge of 1890.

Development had to await the arrival of ideas, methods and materials. Design, always a slow, irregular and halting journey, often from one setback to another, had to await the awakening of men's minds to the possibilities of new forms. The springs of invention had to flow. Once these possibilities had been grasped, however, development was often rapid. Only 25 years separated the little Gaunless trusses from the gigantic spans of Conwy and Britannia, and down the years the efforts of the bridge builders were to lead on to even greater things, and in their own way play their part in transforming the world.

Conclusions

The previous section's discussion on the development of the girder bridge was discursive, commenting on different aspects of its history, and of the engineers who were responsible for its progress over the years, and of influences which either hindered or advanced that progress.

Conclusions require a different treatment, and can be rendered more tersely. The thesis has been confined to the development of the girder bridge between 1820 and 1890, which on examination can be traced to the near-simultaneous occurrence of a number of factors or demands. Sometimes these demands were in conflict and solutions had to be an optimum in meeting the objectives, e.g. girders often had to be of shallow construction depth, yet of limited deflection. Most of these demands recurred continually during the 19th Century in various forms, and if not in conflict were often dependent on the others in the course of girder development, e.g. large spans were often desirable, but foundation conditions were unsuitable or inadequate to accept the resulting heavy loading.

It is concluded that the factors or demands that led to the development of the girder bridge were as follows:

1. The advance of the railways both in Britain and America, and their need for direct routes and a near-level track. Heavy loads and the need to restrict deflection had to be accommodated.

2. The demands of sites offering difficult foundation conditions leading to multiple spans and often restricted construction depth.

3. The need for economy in the use of materials, resulting in the employment in turn, of timber, cast-iron, wrought iron and steel. Generally optimum use of material was basic to the form of the bridge.

4. The need to recognise and employ innovation in the service of engineering solutions, some more successful than others. Of such were the emergence of the compound trussed girder, continuity between spans and the use in turn of new materials and higher stresses.

5. The existence of engineers and contractors of the required towering vision, ability and courage to develop designs and organise their construction. Among such were Robert Stephenson and I K Brunel, who with their bridges at Conwy, Menai, Chepstow and Saltash led the way in the solution of seemingly impossible problems, culminating in the successful bridging of the Forth in 1890 by Sir John Fowler and Sir Benjamin Baker, assisted by Sir William Arrol.

The development of the girder bridge was not always smooth, and disasters and failures occurred, though not many in Britain. In America there seemed to be a greater degree of abandon in design, but the greater number of failures there may have been partly due to the increased number of bridges compared to Britain.

In conclusion, it has to be repeated that development had to await the arrival of ideas, methods and materials available. Design, always a slow, irregular and halting journey, often from one setback to another, had to await the awakening of men's minds to the possibilities offered by new forms. The springs of invention had to flow. Once the possibilities had been grasped, however, development was often rapid. Only 25 years separated the little Gaunless trusses from the gigantic spans of Conwy and Menai, and down the years the efforts of the bridge builders were to lead on to even greater things, and in their own way play their part in transforming the world. The girder bridge as it developed became indispensable to the construction of bridges of all kinds, and continues to do so to the present day.

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APPENDIX A

APPROXIMATE CALCULATIONS FOR
(a) THE CHESTER DEE TRUSSED GIRDER BRIDGE
(b) THE CHEPSTOW BRIDGE MAIN SPAN
(c) THE SALTASH BRIDGE MAIN SPAN

(These calculations were first prepared when the writer was a consultant to Robert H Cuthbertson & Partners, Edinburgh, hence the headings on the sheets)

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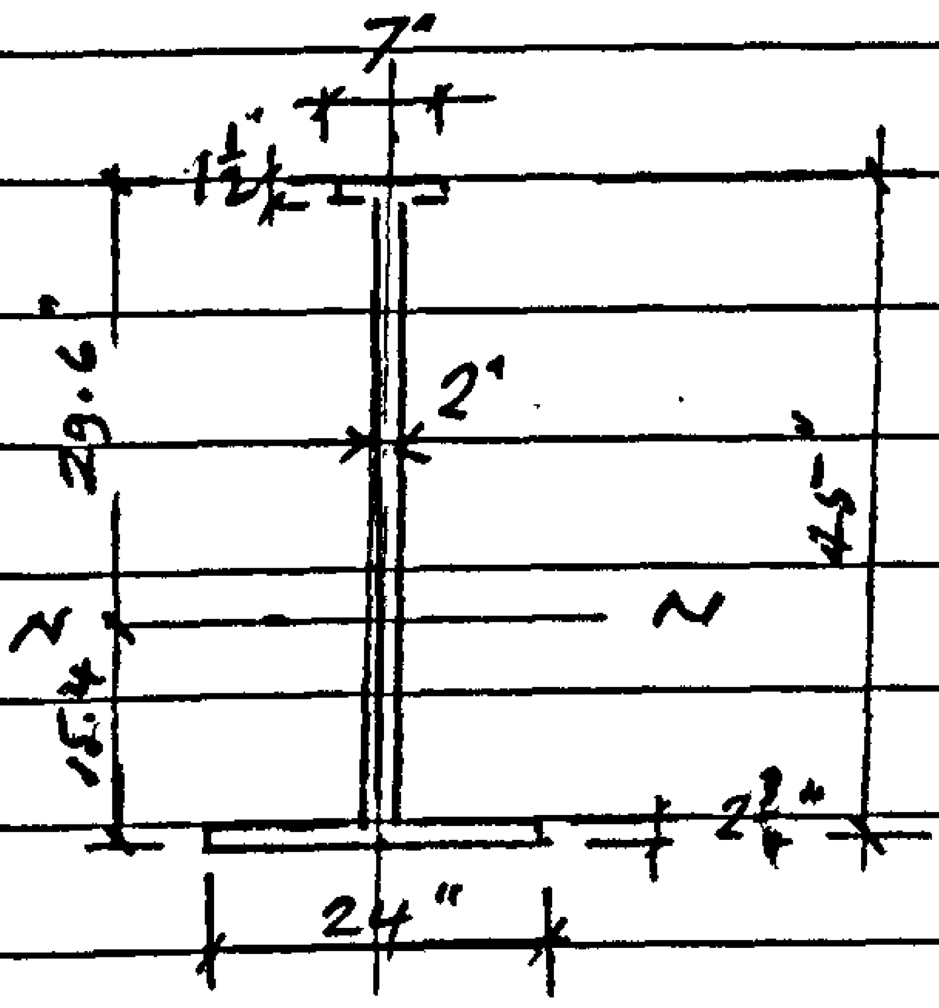
Chester Dee Bridge

Main Girder



Calculation of M. of I.:

Position of N-N:

Σ M abt base of section:

$$N\text{-}N = \frac{(7\times1.5\times44.25)+(2\times41\times23)+(24\times2.75\times1.38)}{(7\times1.5)+(2\times41)+(24\times2.75)}$$

$$= \frac{(464.6 + 1886 + 95)}{(10.5 + 82 + 66)}$$

$$\therefore N\text{-}N = \frac{2441.6}{158.5} = \underline{15.40"} \text{ above base}$$

$$I_{NA} = \tfrac{1}{12}\times7\times1.5^3 + (7\times1.5\times28.85^2) + \tfrac{1}{12}\times2\times40.8^3 + (2\times40.8\times7.70^2)$$
$$+ \tfrac{1}{12}\times24\times2.75^3 + (24\times2.75\times14.03^2)$$
$$= 2.0 + 8739 + 11319 + 4838 + 42 + 12991$$
$$= \underline{37{,}931 \text{ in}^4}$$

$$Z_{min} = \frac{I}{y} \quad \therefore Z_{top} = \frac{37{,}931}{29.6} = \underline{1281 \text{ in}^3} \text{ (MIN)}.$$

$$Z_{btt} = \frac{37{,}931}{15.4} = \underline{2463 \text{ in}^3}.$$

Area of girder = $\underline{158.5 \text{ in}^2}$. Wt/ft = $\frac{158.5}{144} \times 490 = 540$ lb/ft.

Say 0.25 ton/ft. ($\underline{8.20 \text{ kN/m}}$)

Span = 98 ft = ($\underline{29.90 \text{ m}}$) ∴ Wt = 245.2 kN or $\underline{24.5 \text{ tons}}$

+ top sections, misc. plates, etc = about 2.3 tons

Wt of chains = 30 in² = $\frac{30}{144} \times 105 \times \frac{490}{2240}$ = $\underline{4.8 \text{ tons}}$/gdr.

Wt of ballast = 25 tons = $\underline{12.5 \text{ tons}}$/girder

Wt of deck (timber, rails, etc) = $\underline{6.5 \text{ tons}}$ (say)

Wt of train (1.0 ton/ft) = 0.5 × 100 = $\underline{50 \text{ tons}}$/gdr.

∴ Self Wt = 26.8 + 4.8 = $\underline{31.6 \text{ tons}}$. (gdr + chains)

Eccentric load = 12.5 + 6.5 + 50 = $\underline{69.0 \text{ tons}}$/gdr.

Calculations by	Concept checked by	Arithmetic checked by
Name [signature]	Name JSS	Name [signature]
Date	Date	Date

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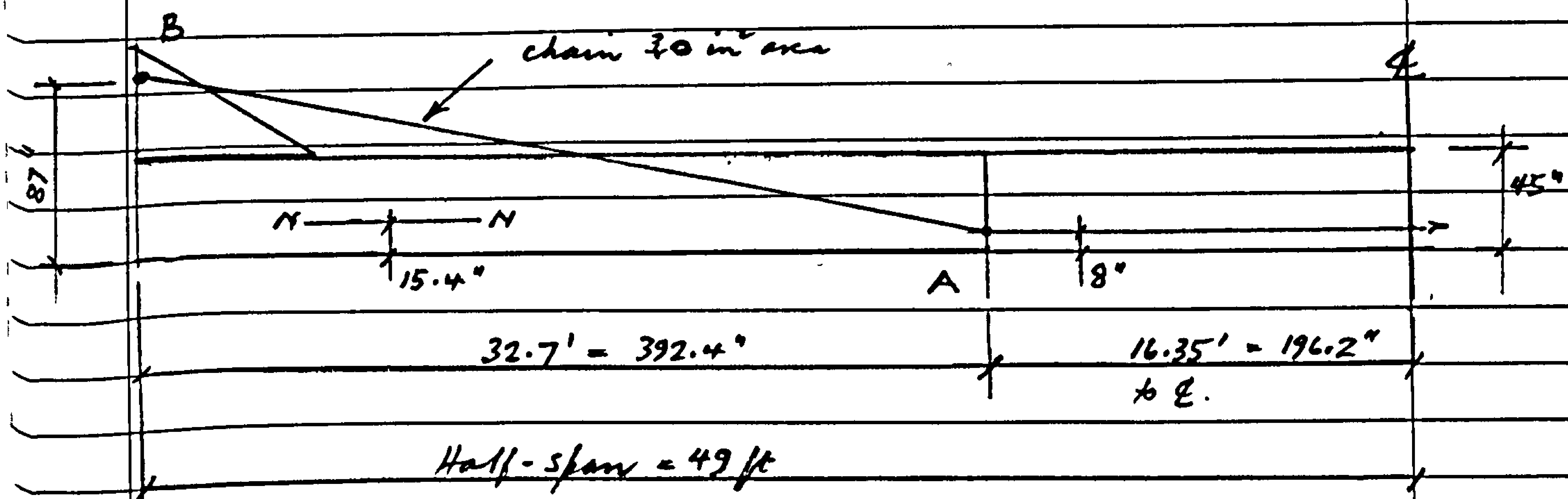
Chester Dee Bridge
Main Girder

$$I_{yy} = \left(\tfrac{1}{12}\times 1.5\times 7^3\right) + \left(\tfrac{1}{12}\times 10.8\times 2^3\right) + \tfrac{1}{12}\times 2.75\times 24^3$$
$$= \tfrac{1}{12}\left(514.5 + 326.4 + 38016\right)$$
$$= \tfrac{1}{12}\times\left(38856.9\right) = \underline{3238\ in^4}$$

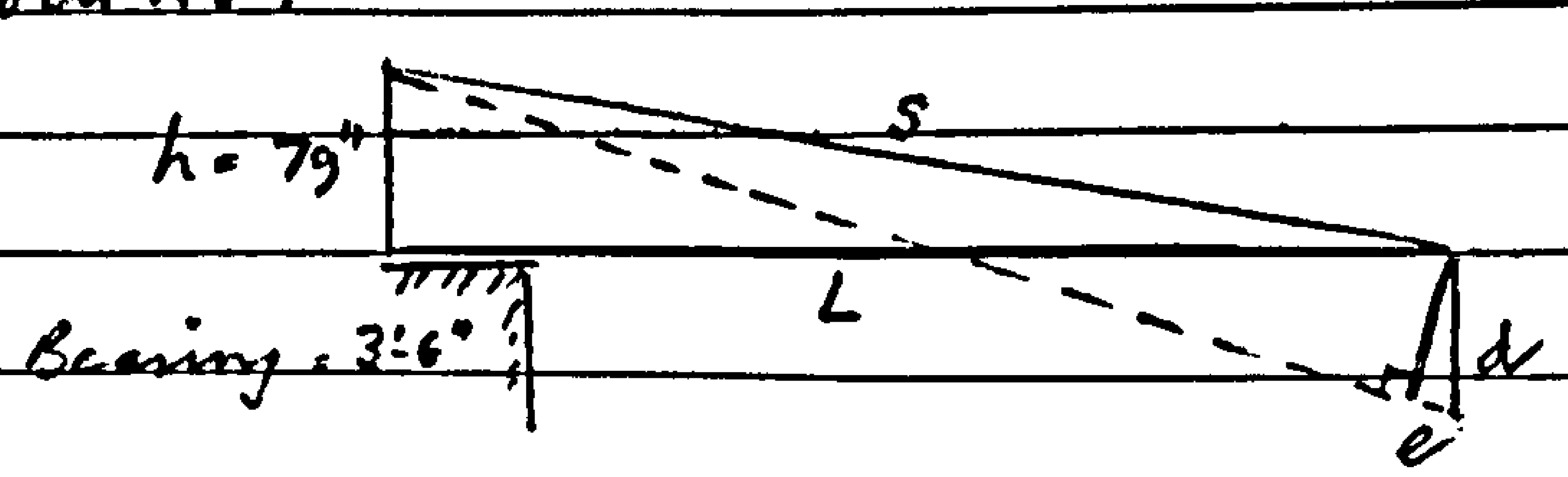
$$k_{yy} = \sqrt{I/A} = \sqrt{\tfrac{3238}{158.5}} = \sqrt{20.4} = \underline{4.52\ in}$$

General Arrangement:
Half-span of girder:



Loading: self wt = 31.6 T i.e. 0.32 T/ft.
Imposed (eccentric) = 69.0 T = 0.69 ton/ft.

Simmons/Walker report says chain was pre-stressed by forcing down the connection at A by 2.5". We get extension of chain in link AB as follows:



$h = 87 - 8 = 79"$
$L = 392" + 42" = 434"$
$d = 2.5"$

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Sheet No. 3.

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Chester Dee Bridge:

Extension of chains by pre-stressing: contd

$$S = \sqrt{L^2+h^2} = \sqrt{434^2+79^2} = 10\sqrt{18.83+0.62} = 441 \text{ in.}$$

and $\dfrac{h}{S} = \dfrac{e}{d}$ ∴ $\Delta = \dfrac{2.5\times 79}{441} = 0.448''$

The half-span of the chain is under the same tension. Therefore strain is

$$\text{strain} = \frac{\Delta}{L} = \frac{0.448}{441+192} = \frac{0.448}{633} = \frac{1}{1413}$$

and $E = 12{,}500$ T/in² for wrought iron

and $\dfrac{\text{stress}}{\text{strain}} = E$ ∴ stress $f_t = \dfrac{12{,}500}{1413} = \underline{8.85 \text{ T/in}^2}$

This is a high tensile stress for W.I.

Total force in chain, area 30 in²

$$F = 8.85\times 30 = \underline{265.5 \text{ tons.}}$$

This is an over-simplification because of:

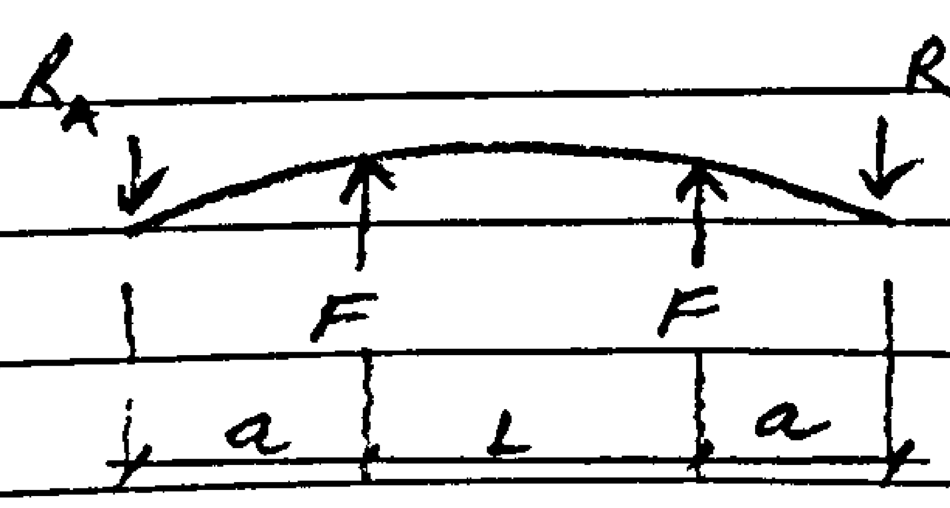
Upward force at $\frac{1}{3}$ point:

$$F_u = \frac{h}{S}\times 265.5 = \frac{79}{441}\times 265.5 = \underline{47.6 \text{ tons}}$$

This upward force must cause an upward deflection in the girder which would form part of the total induced deflection of 2.50".

∴ Revise calculation to allow for both upward deflection of the girder + downward deflection of the chains, to equal 2.50"

Formula for upward deflection of girder:



Deflection at load points

$$\Delta = \frac{Fa^2}{6EI}(2a + 3L)$$

(see Arrol Handbook p.27)

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Chester Dee Bridge:
Extension of chains by pre-stressing, etc contd
upward deflection of girder, contd

here $F = 47.6$ tons.
$a = 32.7$ ft $= 392''$
$l = 392''$
$E = 7{,}000$ T/in². for cast-iron.
$I = 37{,}931$ in⁴. (p. 1).

$$\therefore \Delta = \frac{Fa^2}{6EI}(2a + 3l) = \frac{47.6 \times 392^2\,(5 \times 392)}{6 \times 7{,}000 \times 37{,}931}$$

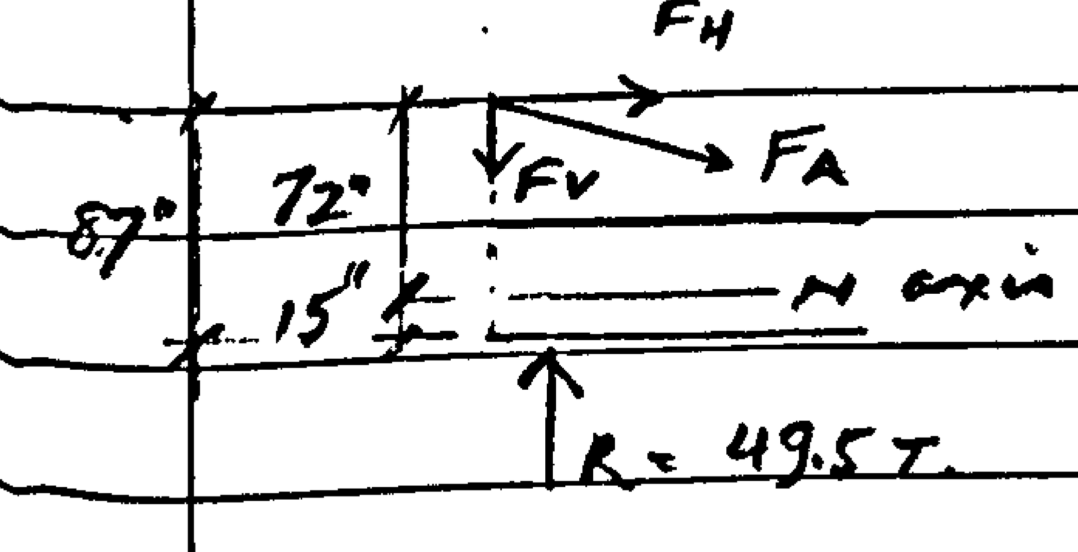
$$= \frac{14{,}336 \times 10^6}{1593.1 \times 10^6} = 9.00'' \quad (10.50'' \text{ for } E = 6{,}000 \text{ T/sq.in})$$

$\therefore$ Total deflection is 2.50" downwards for chains, and 9.00" upwards for girder.
$\therefore$ Total deflection is $\Delta = 11.50$ in. for a load of 47.6 tons at the jacking point. 13.84"

But actual deflection induced was 2.50 in only.
$\therefore$ Actual load was: $F_A = \frac{2.50}{11.50} \times 47.6$ tons
i.e. $F_a = 10.35$ tons.
say 10.4 tons vertically. (2.55 T to each bolt under full load)
$\therefore$ Load in chains $= 10.35 \times \frac{s}{h} = \frac{10.35 \times 441}{79} = 57.8$ tons.

$\therefore$ Stress in chain $= \frac{F_A}{A} = \frac{57.8}{30} = 1.93$ T/in².

Moment induced in girder by chain: $F_V = 10.35^T$



$F_H = \sqrt{57.8^2 - 10.35^2} = 56.9$ tons.
ΣM abt ℄ of girder: $R = 1.01 \times \frac{98}{2} = 49.5^T$.
$\therefore M_c = (49.5 \times 50.7) + (56.9 \times 6.0) - (10.35 \times 52.5) + (10.35 \times 16.4) - (49.5 \times 24.5)$
$= (2510 + 341 - 543 + 170 - 1213)$
$= 1268$ T-ft. (sagging M)

Compare: Free BM $= \frac{WL}{8} = \frac{1.01 \times 98^2}{8} = 1213$ Ton-ft. (< 1268)
i.e. chains do not assis

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Chester Dee Bridge

Stresses in girder:

Loads: (see p.1.)

Dead Load = 0.32 Ton/ft run.

Live load = 0.69 ton/ft. run.

∴ Total = 1.01 Ton/ft. run.

$\therefore$ Max BM = $\frac{WL^2}{8} = \frac{1.01 \times 98^2}{8} = 1213$ Ton-ft

Z of girder (top flange) = 1281 in³. (p.1)

Z of girder (bott. flange) = 2463 in³. "

$\therefore$ Bending Stress (compression) = $\frac{M}{Z} = \frac{1213 \times 12}{1281} = 11.36$ T/in²

Bending Stress (tension) = $\frac{M}{Z} = \frac{1213 \times 12}{2463} = 5.91$ T/in²

These stresses are high. (for cast iron).
These are not the stresses produced by the train, but what it must have been originally designed for. (1 Ton/ft run + D.L.)

Torsion effects have been neglected, also impact.

check deflection under this load:

Total load = 1.01 × 98 = 99 tons.

$\Delta = \frac{5WL^3}{384EI}$ here $W = 99$ tons

$L = 98$ ft $= 1176''$

$= \frac{5 \times 99 \times (1176)^3}{384 \times 7{,}000 \times 37{,}931}$ $E = 7{,}000$ T/in² (cast iron)

$I = 37{,}931$ in⁴

$= \frac{495 \times 1.63 \times 10^9}{0.384 \times 7.0 \times 37.93 \times 10^9}$

$= \frac{807}{102} = 7.91''$ at centre. (theoretical)

This would produce a rotation at the ends:

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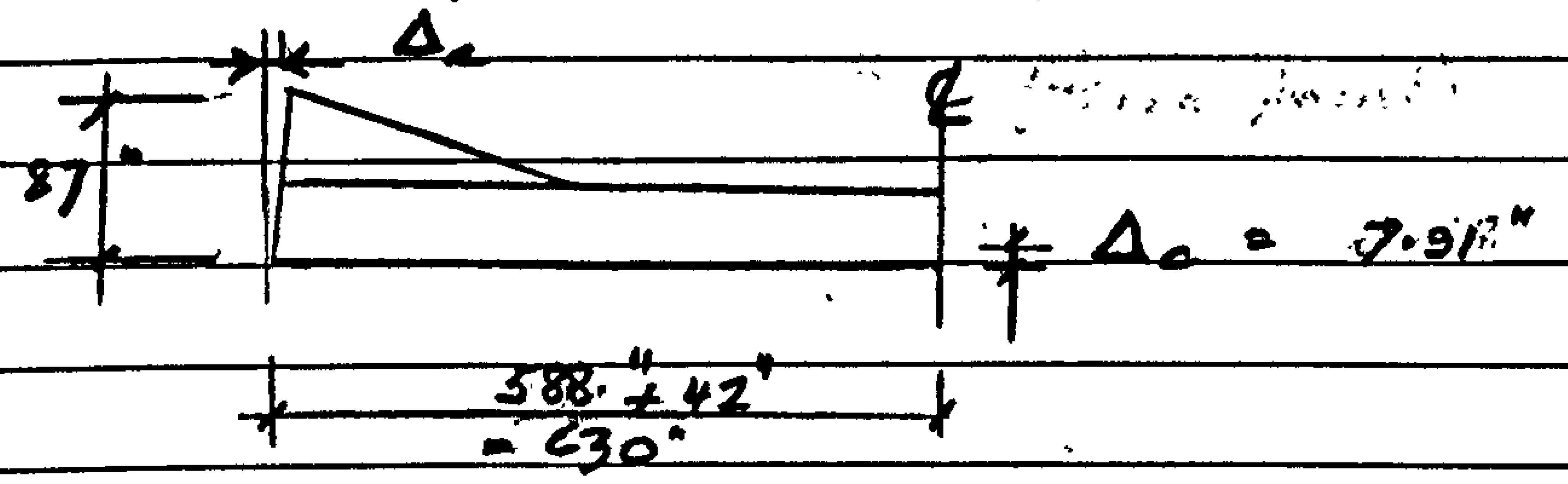
Chester Dee Bridge:

Rotation of ends of bridge girder:



$\therefore$ Approx value of $\Delta_e = \dfrac{7.51 \times 87}{630}$ (approx.)

$\therefore \Delta_e = \underline{1.03"}$

This would tend to slacken the chain, such that all load would be released.

Extension of chain = Strain × L.

here strain = $\dfrac{1}{1413}$ (p.3.) L = 441 + 192 = 633 (half span)

$\therefore \Delta = \dfrac{633}{1413} = \underline{0.44"}$ under load

This shows that all strain in the chain would be lost, and hence all tension.

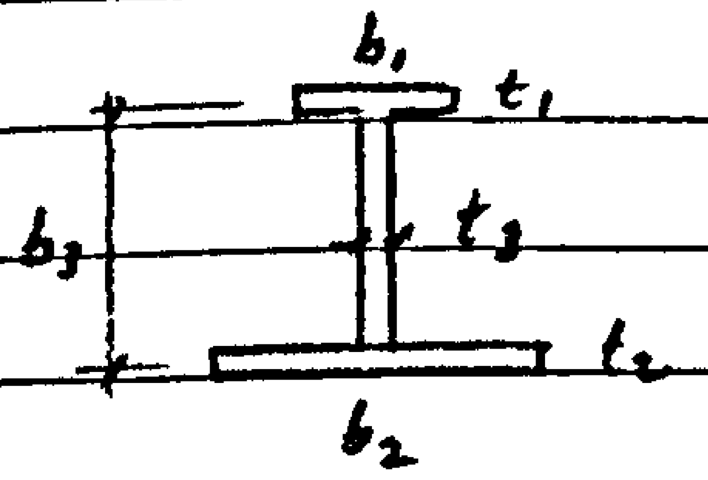
So from 2 causes the chains were useless:

1) The pre-stress was nil, and actually increased the bending in the girder.

2) The geometry of the rotation of the ends of the fixing point of the chains meant that the force in the chain was released.

Consider torsion effects:

Torsion constant J:



$$J = \frac{b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3}{3}$$

Substitute values:

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Chester Dee Bridge:

Torsion effects: see formula p.6.

here $b_1 = 7''$ $t_1 = 1.5''$

$b_2 = 24''$ $t_2 = 2.75''$

$b_3 = 42.9''$ $t_3 = 2''$

$$\therefore J = \frac{b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3}{3} \text{ in}^4$$

$$= \frac{7 \times 1.5^3 + 24 \times 2.75^3 + 42.9 \times 2^3}{3}$$

$$= \frac{(7 \times 3.38) + (24 \times 20.80) + (42.9 \times 8)}{3}$$

$$= \frac{23.7 + 499.2 + 343.2}{3} = \frac{866.1}{3} \text{ in}^4$$

$$J = \underline{288.7 \text{ in}^4}$$

Torsion Moment (see sketch, p.8)

The lever arm on the flange varied, because of the presence of the chain. Take 7" as lever arm, which is conservative.

Load = live load, etc only = 0.67 ton/ft.

$\therefore$ Torsion $M = 0.67 \times \frac{7}{12} \times \frac{98}{2} = \underline{19.15 \text{ Ton-ft}}$ at support.

Torsion $M = 0.67 \times \frac{7}{12} \times \frac{32.7}{2} = \underline{6.39 \text{ Ton-ft}}$ at third point

Torsion constant $C = \frac{J}{r}$ approx $J = 289$ in^4

i.e. $C = \frac{289}{29.6} = \underline{9.76 \text{ in}^3}$.

$\therefore$ Torsion shear stress $= \frac{T_0}{C} = \frac{6.39 \times 12}{9.76} = \underline{7.86 \text{ T/in}^2}$ in top flange.

This is partly a tension stress and is high

Conclusion: flange buckling due to high compressive stress and high torsion stress.

Plus: Top flange lacked straightness

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Name J. S. Shipway	Name JSS	Name [illegible]
Date [illegible]	Date	Date

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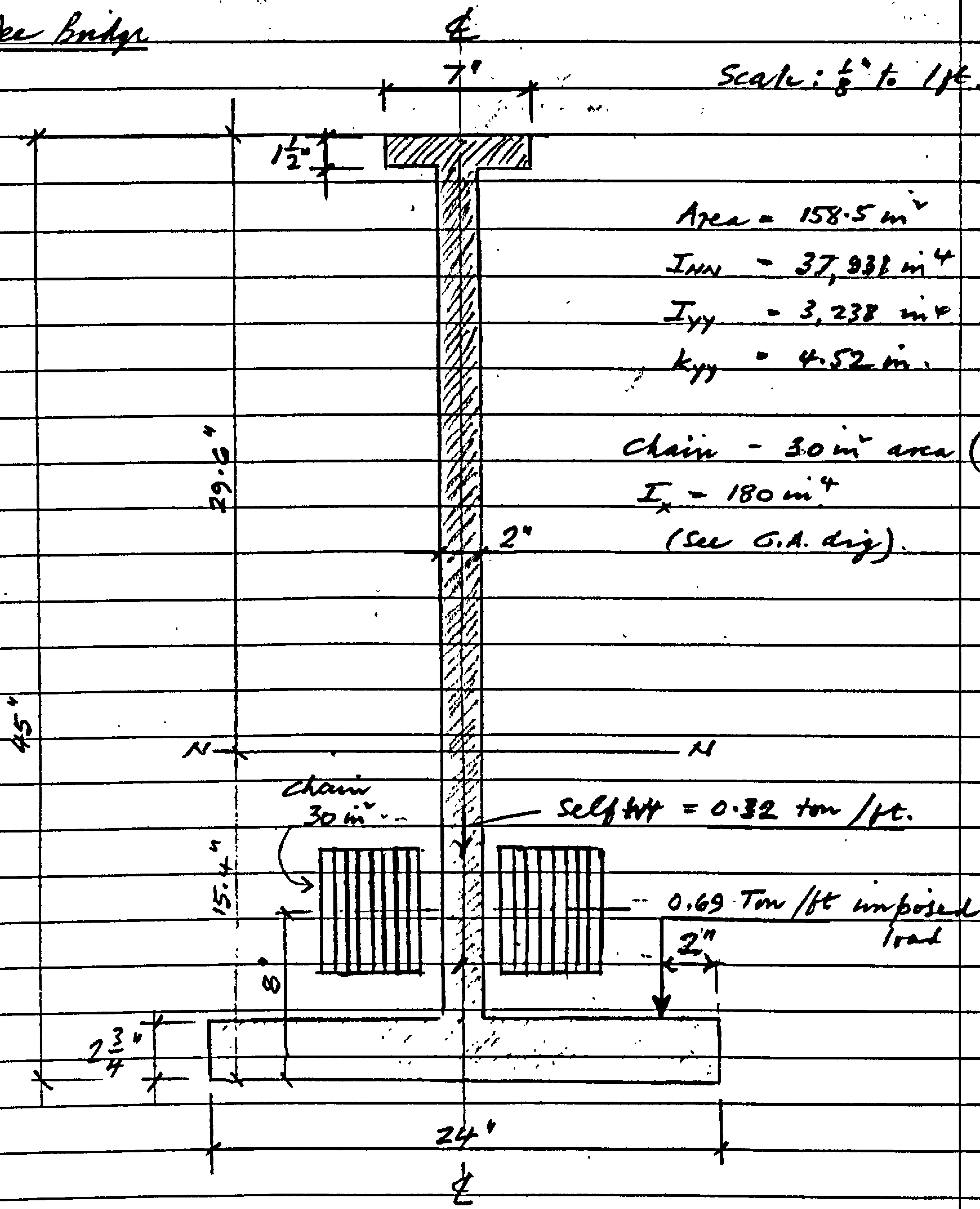
Dee Bridge

Scale: $\frac{1}{8}$" to 1ft.



Area = 158.5 in²

I_{NN} = 37,938 in⁴

I_{yy} = 3,238 in⁴

k_{yy} = 4.52 in.

Chain – 30 in² area ($8 \times \frac{5}{16}$" thk × 6" plates each side)

I_x = 180 in⁴

(See G.A. drg).

Span = 98 ft. w = 0.67 ton/ft. (eccentric)

Self wt = 0.30 ton/ft.

Note – k_{yy} is for whole section.

k_{yy} for top flg. alone will be less.

Calculations by	Concept checked by	Arithmetic checked by
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Chester Dee Bridge

Bending in bottom flange

Load is applied through timber cross-members @ 3 ft. centres (?).

Load per cross member: live load only = 0.69 T/ft.

∴ Load = 0.69 × 3.0 = 2.07 Tons.

Lever arm on flange to web (see sketch, p.8)

l_a = 7" − 1" to face of web = 6."

∴ BM = 2.07 × 0.5 = 1.03 Ton. ft.

Take width of flange action = 12 + 6 + 6

width = 24" at web face

∴ $Z = \frac{1}{6} \times 24 \times 2.75^2 = 30.25 \text{ in}^3$

∴ Bending stress = $\frac{M}{Z} = \frac{1.0 \times 12}{30.25} = 0.40 \text{ T/in}^2$

This is low: flange will not break.
but stress could be much higher if loco wheel load is taken as acting over a cross-member.

Also, eccentricity from girder ℄ is greater for the middle section where the chains are fixed.

Girder failed due to high compressive stress in top flange: l/k_{yy} : here k_{yy} = 4.52 in. l = 98 × 12 in.

∴ l/k_{yy} = 98 × 12 / 4.52 = 260. $\frac{D}{T} = \frac{45}{1.5} = 30$

Allowable stress (lateral bending) = 57 N/mm² or 3.69 T/in².

Actual (p.5) = 12.3 T/in². Highly overstressed.

Also, top flange was out of true, "wavy."

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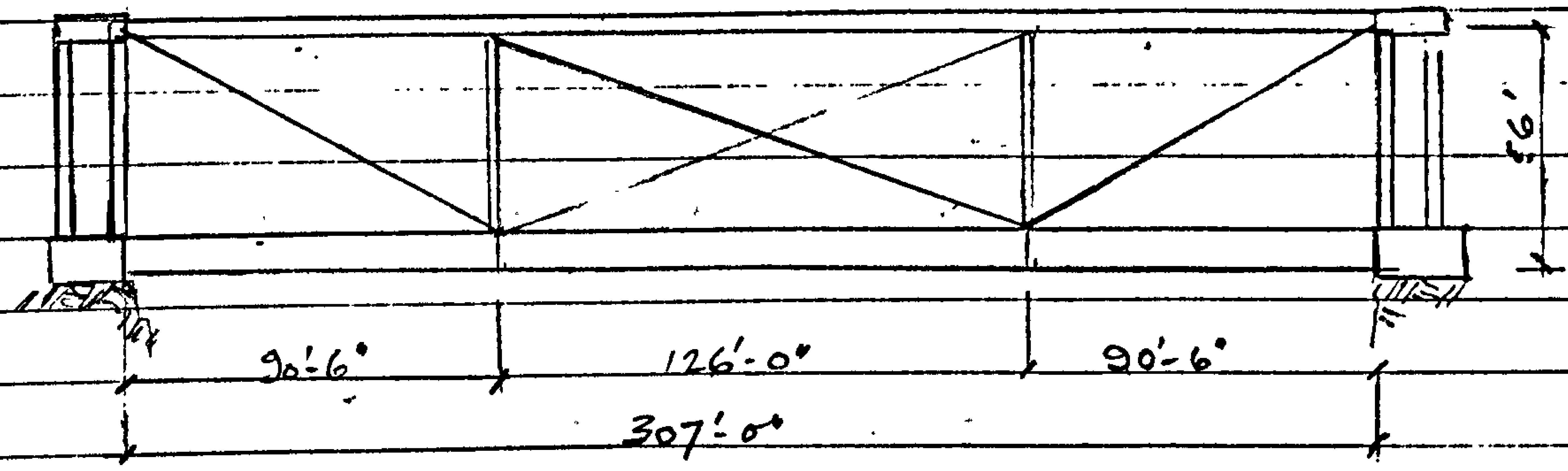
Chepstow Bridge - Main Span

Span 308 ft. Tube - 9'-0" dia. $\frac{5}{8}$" thick



Force in tube, end panel

Wt of wrought iron, allowing load shared with abutment in end spans = 427 tons.

Wt. of decking = 160 tons.

Live load, ($1\frac{1}{2}$ tons/ft) = 1.5 × 200 = 300 tons.

∴ Total D.L. + L.L = 887 tons.

Vertical reaction at bearing tube = 444 tons approx.

∴ Force in tube - Δ of forces = $\frac{444 \times 90.5}{43}$ = 934 tons

(Considering end panel)

Area of tube (c.s.a.) = 212 in². ($\frac{5}{8}$" thick)

∴ Stress in plating of tube = $\frac{F}{A} = \frac{934}{212}$ = 4.41 ton/in²

Force in chains, end panel

Length of chain = $\sqrt{90.5^2 + 43^2}$ = 100.2 ft

Area of chains (Binding, 1997) = 2 × 96 in² = 192 in²

∴ Force in chains = $\frac{444 \times 100.2}{43}$ = 1034 tons.

∴ Stress in chains = $\frac{F}{A} = \frac{1034}{192}$ = 5.39 ton/in²

Local Bending stress in tube

self wt of tube = 161 tons

Tube was continuous over the A-frame supports.

Assume they did not settle or move vertically:

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Sheet No.

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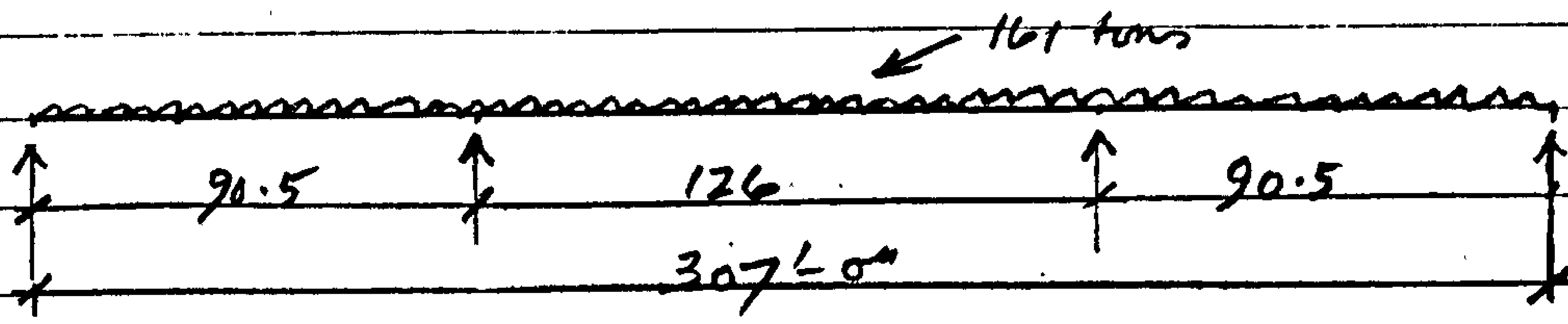
Chepstow Bridge Main Span – contd



Approx max moment over support = 0·100 WL

here W = 161/3 approx. = 54 tons

L = (mean) 102 ft.

∴ Max M = 0.10 × 102 × 54 = 540 tons. ft. (approx)

Z of 9'-0" dia, $\frac{5}{8}$" thick tube = $\frac{\pi}{4}\left(\frac{Re^4 - Ri^4}{Re}\right)$

here Re = 54.0 in. Ri = 53.375 in

∴ Z of tube = $0.785 \left(\frac{54.0^4 - 53.38^4}{54.0}\right)$

= 7108 × 0.785 = 5580 in^3

∴ Stress in tube = $\frac{M}{Z} = \frac{540 \times 12}{5580}$ = 1.15 ton in^2

Add this local stress (both tens. and comp.)
to overall comp. stress of 4.41 tons/in².
∴ Total stress at A-frame support:

f_c = 4.41 + 1.15 = 5.56 tons/in² (comp.)

acting on tube underside at A-frame.

Yielding of the A-frame under live load would reduce this.

Note: chains are not in a vertical plane.
Neglect small addition to stress resulting.

Calculations by	Concept checked by	Arithmetic checked by
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Sheet No. 1

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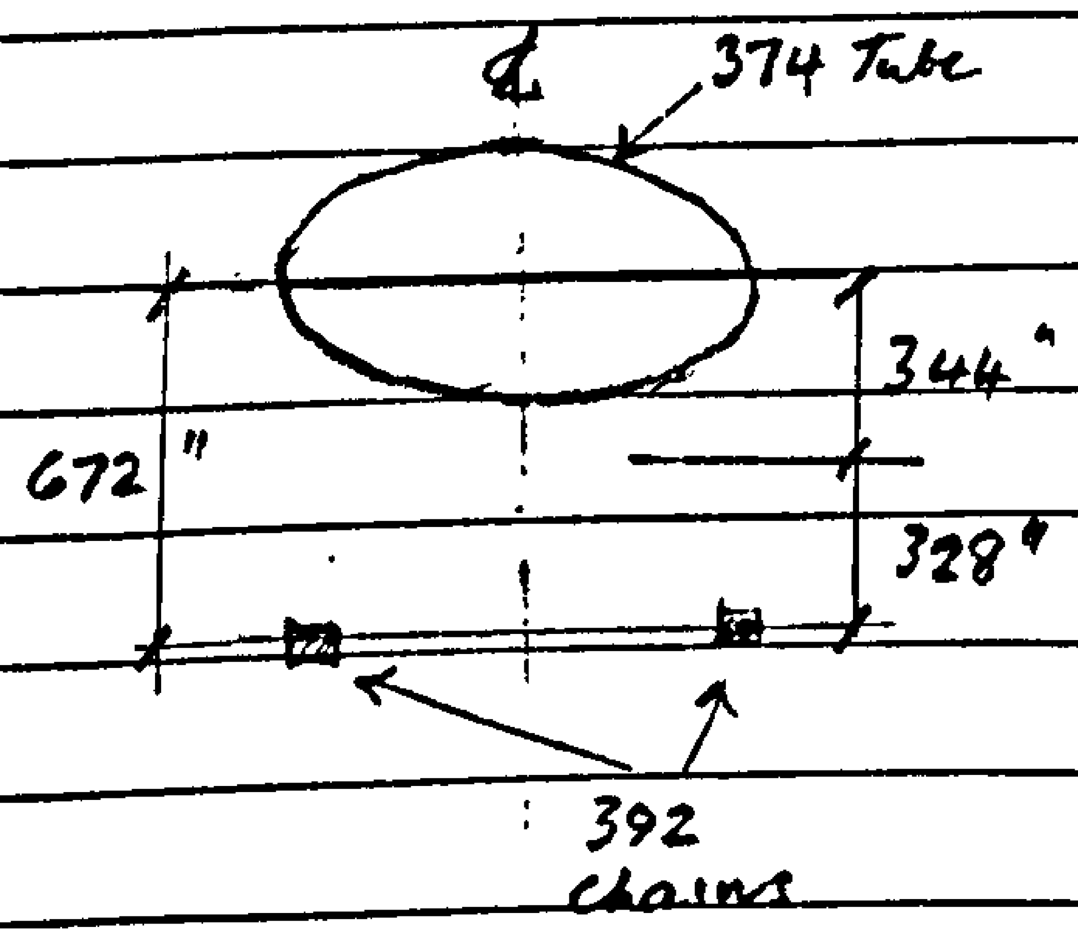
Saltash Bridge - Main Span

The span was 455 ft between bearings.
(For configuration see fig. 30 in text).
cross-section, elliptical 16'-9" x 12'-3"
along parabolic curve = 460 ft. in length
Internal longit. and annular stiffeners 12" x $\frac{1}{2}$"
Thickness varies - tube walls, $\frac{1}{2}$", $\frac{5}{8}$" and $\frac{3}{4}$" thick.
Take as $\frac{3}{4}$" & allow for stiffeners, laps, etc.

Approx weight of wrought-iron tube = 300 tons
" " of 2 tiers of chains, both sides = 310 tons

Forces, etc in Tube:
Area of annular ellipse $= \frac{\pi}{4}(bd - b_1 d_1)$
here b = 16'-9" (201 in) b_1 = 199.75 in
and d = 12'-3" (147 in) d_1 = 145.75 in
(assume t = 5/8" for stress calculation)
$\therefore bd = 29547 \text{ in}^2 \quad b_1 d_1 = 29114 \text{ in}^2 \quad \frac{\pi}{4} = 0.785$
$\therefore$ area $= 0.785 \times 433 = \underline{340 \text{ in}^2}$
If tube assumed $\frac{3}{4}$" thick, area = $\underline{408 \text{ in}^2}$
Take mean as area = $\underline{374 \text{ in}^2}$

Chains Two tiers, 7" deep × 14 no. × 1" thick = 196 in²
c.s.a = 2 × 196 in² = 392 in² (both sides)
Wt. approx = 2 × 0.33 × 465 = 307 tons. Say 310 tons



Position of N-N above c/l chains

$$N\text{-}N = \frac{374 \times 672}{374 + 328} = \frac{251328}{766}$$

$= \underline{328 \text{ in.}}$
$\underline{344 \text{ in below ℄ tube}}$

Calculate I_{xx} about neutral axis:

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Date 24.8.98	Date	Date

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Saltash Bridge - Main Span - contd

Calculate I_{NN} about neutral axis:

But, I_{NN} of tube alone about own axis:

$I_{NN} = \frac{\pi}{64}(d b^3 - d_1 b_1^3)$

here $d = 12'-3" = 147$ in and $b = 16'-9" = 201$ in.

assume thickness $= 5/8" = 0.625$ in

$\therefore d_1 = 145.75$ in and $b_1 = 199.75$ in.

$\therefore I_{NN} = 98 \times 10^4$ in^4 (tube alone)

Calculate I_{NN} of tube and chains as shown:

$I_{NN} = 374 \times 344^2 + 392 \times 328^2 + 98 \times 10^4$ in^4

$= 4425 \times 10^4 + 4217 \times 10^4 + 98 \times 10^4$

$= \underline{8740 \times 10^4 \text{ in}^4}$

$\therefore Z_u = I/y = 8740/344 \times 10^4 = 25.40 \times 10^4$ in^3

$Z_L = I/y = 8740/328 \times 10^4 = 26.65 \times 10^4$ in^3

Bending moment: dead load known to be 1060 tons

live load $= 1.50$ tons/ft over 460 ft $= 690$ tons

$\therefore$ Total load $= 1750$ tons.

$\therefore BM = \frac{WL}{8} = \frac{1750 \times 460}{8} = \underline{100,625 \text{ tons-ft}}$

$\therefore$ Stress in tube $= \frac{M}{Z_u} = \frac{100,625 \times 12}{25.40 \times 10^4} = \underline{4.75 \text{ T/in}^2}$

Stress in chains $= \frac{M}{Z_L} = \frac{100,625 \times 12}{26.65 \times 10^4} = \underline{4.53 \text{ T/in}^2}$

Suppose the BM to be resisted by a couple formed of the forces in tube and chain:

BM $= 100,625$ tons-ft. at centre of span

Lever arm of couple $= 56$ ft (see Fig 30).

$\therefore$ Force in tube and chains $= \frac{100,625}{57} = 1765$ tons.

$\therefore$ Direct stress in tube $= \frac{F}{A} = 1765/374 = 4.72$ T/in^2 (comp.)

" " in chains $= \frac{F}{A} = 1765/392 = 4.50$ T/in^2 (tens.)

Calculations by	Concept checked by	Arithmetic checked by
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Sheet No. 3.

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Saltash Bridge - Main Span - contd

These stresses in tube and chains, calculated by the two methods show a remarkable correlation. They also show the remarkable adequacy of the design - conservative, if anything, under the live load of 1.50 tons/ft.

Calculation for Test Load:

Pugsley and H. Shirley-Smith describe a test u.d.l. of 1190 tons before the truss was floated:

i.e. Total u.d.l = 1060 + 1190 = 2250 tons.

$$M = \frac{2250 \times 460}{8} = 129,375 \text{ tons-ft.}$$

$$\therefore \text{Stress in tube} = \frac{M}{Z_u} = \frac{129,375 \times 12}{25.40 \times 10^4} = \underline{6.11 \text{ Tons/in}^2} \text{ Comp}$$

$$\text{Stress in chains} = \frac{M}{Z_L} = \frac{129,375 \times 12}{26.65 \times 10^4} = \underline{5.83 \text{ tons/in}^2} \text{ Tens.}$$

(Smith wrote of stresses "about 10 tons/sq. in" !!)

Deflection: Take E for wrought iron = 11,000 T/in^2

$$\Delta = \frac{5WL^3}{384EI} = \frac{5 \times 2250 \times 460^3 \times 1728}{384 \times 11,000 \times 8740 \times 10^4} \text{ inches}$$

$$= \underline{5.13 \text{ in}}$$

(For E = 12,000 it would be 4.70 in)

Note that Smith quoted a deflection of 5 in, and the calculated figure above of 5.13 in. is in good agreement.

How flexible was the tube if required to span 460 ft. on its own? This has a bearing on the behaviour of the standards

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Saltash Bridge - Main Span - contd

Consider the tube spanning 460 ft on its own:

$I_{xx} = 98.0 \times 10^4 \text{ in}^4$ $\quad d = 12'3'' = 147 \text{ in}$

$y_u = y_L = 147/2 = 73.5 \text{ in}$.

$\therefore Z_u = Z_L = I/y = \frac{98.0 \times 10^4}{73.5} = \underline{13,300 \text{ in}^3}$

BM due to Self wt: (300 tons)

$M = \frac{WL}{8} = \frac{300 \times 460}{8} = 17,250 \text{ tons-ft}$.

$\therefore$ Bending stress $= \frac{M}{Z} = \frac{17,250 \times 12}{13,300} = \underline{15.56 \text{ T/in}^2}$

This is high. Tube could not span on its own.

Deflection

$\Delta = \frac{5 \times WL^3}{384 \times EI}$ $\quad$ take $E = 11,000 \text{ T/in}^2$ as before

$\therefore \Delta = \frac{5 \times 300 \times 460^3 \times 1728}{384 \times 11,000 \times 98 \times 10^4} = \underline{61 \text{ in}}$.

The tube would have sagged and buckled.
Its l/b ratio was $460/16.75 = 27.5$.

It was also flexible in such a way that when its centering was struck, the self weight would immediately load the standards, unless there was a hozl. end reaction to relieve the bending, provided by the chains.

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Name J.S. Shipway	Name JCI	Name [illegible]
Date [illegible]	Date	Date